# Mechanics (Flash Notes)

contain tough Formula's and derivations

$a_r = \ddot{r} - \omega^2 r$

$a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$

(and) $v = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

$\tan\theta = \dfrac{\sin\theta'}{\cos\theta' + \frac{m_1}{m_2}}$

$\sigma(\theta) = -\dfrac{b}{\sin\theta}\dfrac{db}{d\theta}$

$b = \dfrac{k}{2E_\alpha}\cot\theta/2$

$\left[\dfrac{dA}{dt}\right]_{Space} = \left[\dfrac{d'A}{dt} + \vec{\omega}\times\vec{A}\right]_{Body}$

$\vec{a} = \vec{a}' + 2(\vec{\omega}\times\vec{v}') + \vec{\omega}\times(\vec{\omega}\times\vec{R}) + \dfrac{d\vec{\omega}}{dt}\times\vec{R}$

$\dfrac{d^2u}{d\theta^2} + u = -\dfrac{m F(u)}{J^2u^2}$

$\frac{1}{2}mv^2 = \dfrac{J^2}{2m}\left[\left(\dfrac{du}{d\theta}\right)^2 + u^2\right]$

$l = J^2/km$ , $e = \sqrt{1 + \dfrac{2EJ^2}{mk^2}}$

planet - $a = -k/2E$

$I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 = N_1$

$\dfrac{I_1 - I_3}{I_1} = \dfrac{1}{300}$ ; $\omega_3 = 10^{-5}$ $\left(\frac{2\pi}{T}\right)$ ~ 24 hours

gyroscope: $\omega_p = \Omega = \dfrac{mgl}{I\omega}$

Poisson Ratio: $\sigma = \dfrac{\text{lateral strain}}{\text{longitudinal strain}}$

$Y = 3k(1 - 2\sigma)$

$Y = 2\eta(1+\sigma)$

($0 < \sigma < 1/2$ → lateral strain always less than 1/2 of longitudinal strain)

1 poise = 0.1 SI unit

1 stoke = $10^{-4}$ $m^2/s$ (SI unit)

1 dyne = $10^{-5}$ newton

1 erg = $10^{-7}$ Joule

(1 Gauss = $10^{-4}$ Tesla)

oersted ?

$V_{2/1} = \dfrac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}$

$\nu = \dfrac{c}{\lambda} = \dfrac{c}{(c-v)T}$ — $T = \alpha T_0$ — longer since moving

$\nu = \nu_0 \dfrac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}\cos\theta}$

(for sound: observer on top)

$\tan\theta = \dfrac{\sin\theta'\sqrt{1 - \frac{v^2}{c^2}}}{\cos\theta' + v/c}$

$$\begin{bmatrix} \alpha & 0 & 0 & i\alpha\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\alpha\beta & 0 & 0 & \alpha \end{bmatrix}$$

$\beta = v/c$

$x_\mu' = \alpha_{\mu\nu} x_\nu$

$\lambda = \Sigma \vec{p}\cdot\vec{r}$

$\bar{T}$ = virial of clausius $= -\frac{1}{2}\Sigma \vec{F}_i\cdot\vec{r}_i$ ⟨ virial Theorem ⟩

for $n = -2$ (inverse square force)

$2\bar{T} + \bar{V} = 0$

$\Delta x = c\Delta t = \dfrac{lv^2}{c^2}$ take ~~force~~ ×2 after rotation → $\dfrac{2lv^2}{c^2} = 2200\,A^\circ$

$l = 11m$, $v = 3\times10^4$ m/s

$F_x = F_x'$, $F_y = \dfrac{F_y'}{\gamma}$, $F_z = \dfrac{F_z'}{\gamma}$

$v$ is in dirn of $x$ thus ∥ ⊥

$F_\parallel = F_\parallel'$ & $F_\perp = \dfrac{F_\perp'}{\gamma}$

use $p' = \dfrac{dp'}{dt'}$ → Both INVERSE

## OPTICS

$v_g = v_p - \lambda\dfrac{dv_p}{d\lambda}$

$v_g = v_p\left[1 + \dfrac{\lambda}{n}\dfrac{dn}{d\lambda}\right]$

$\dfrac{b}{m} = 2c$ = damping constant

$\tau = \dfrac{c}{\omega_0}$ = damping ratio

$Q = 2\pi \times \dfrac{E}{\langle\text{energy loss per cycle}\rangle} = \dfrac{\omega_0}{2c}$

forced: $x = A\sin(pt - \theta)$

use $f_0\sin pt = f_0\sin(pt - \theta + \theta)$

$\tan\theta = \dfrac{2cp}{\omega_0^2 - p^2}$ $A = \dfrac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4c^2p^2}}$

Power $= \bar{F}\cdot\bar{v} = f_0\sin pt \times Ap\cos(pt - \theta)$

use $\sin\theta$ → △le from $\tan\theta$

Stress $= k\dfrac{\partial y}{\partial x}$ Bulk modulus (vol. strain $= \dfrac{\partial y}{\partial x}$)

longitudinal c.A. $= f_R - f_B = \omega f_Y$

$\omega = \dfrac{\mu_B - \mu_R}{\mu_Y - 1}$

$f = \dfrac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2}$

S.A. $= \dfrac{h^2}{f}\phi(k, \mu)$ ~ $\mu = 1.5$, $k = \dfrac{R_1}{R_2} = -\dfrac{1}{6}$ } Min S.A.

Min S.A – $\delta_1 = \delta_2$

$d = f_1 - f_2$.

$\begin{bmatrix} 1 & 0 \\ d/n & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix}$ $P = \dfrac{n_2 - n_1}{R}$

$d_1 = \dfrac{1 - \alpha}{\beta}$ , $d_2 = \dfrac{1 - \delta}{\beta}$

$\dfrac{E_r}{E_i} = \dfrac{n_1 - n_2}{n_1 + n_2}$

$\dfrac{E_t}{E_i} = \dfrac{2n_1}{n_1 + n_2}$

$2\mu\beta\theta = \lambda$

$(D_n)^2_{dark} = \dfrac{4n\lambda R}{\mu}$

$T = \dfrac{1}{1 + F\sin^2\delta/2}$

( $F = \dfrac{4R}{(1-R)^2}$ and

$\delta = \dfrac{2\pi}{\lambda}\Delta = \dfrac{2\pi}{\lambda}(2\mu t\cos\theta)$ )

$2x = \dfrac{\lambda^2}{2(\lambda_2 - \lambda_1)}$ ~ disappearance for discrete

$2x = \dfrac{\lambda^2}{\Delta\lambda}$ ~ disappearance for continuous

$l = \lambda Q$ purity

coherent length

$V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

visibility

$I_{max} \propto (\sqrt{I_1} + \sqrt{I_2})^2$

$I_{min} \propto (\sqrt{I_1} - \sqrt{I_2})^2$

$y_{P_0} = y_1/2$

$I_0 = \frac{y_1^2}{4}$

$y_{P_1} = \frac{3y_1}{2}$

$= \frac{9y_1^2}{4} = 9I_0$

$y_{P_2} = \frac{3}{2}y_1 - y_2$

$I_{P_2} = (\frac{3}{2}y_1 - y_2)^2$

$y_{P_3} = y_1 + y_3/2$

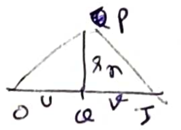
$y_{P_{-1}} = y_2/2$

$y_{P_{-2}} = y_3/2$

$x_n = k\sqrt{2n}$ where $k = \sqrt{\frac{b(a+b)\lambda}{a}}$

$OP' - OQ' = \frac{n\lambda}{2}$

$\sqrt{u^2+r_n^2}$ $\sqrt{v^2+r_n^2}$



$f = \frac{r_n^2}{n\lambda}$

circular fraunhofer. $J_1(z) = 0 \rightarrow z = 1.22\pi,\ 2.23\pi,\ 3.24\pi$

single slit : $I = I_0 \frac{\sin^2\alpha}{\alpha^2}$

double slit : $I = I_0 \frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$

diffraction grating : $I = I_0 \frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}$

$\Delta = (a - \beta\cos\phi)\sin\theta$

$R.P = \frac{d}{1.22\lambda}$

$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} - \frac{2E_xE_y\cos\phi}{ab} = \sin^2\phi$

$\theta = \phi/2 = \frac{\pi}{\lambda}(\mu_e - \mu_o)t$

$V \cdot N_0 < 2.404 \rightarrow$ single ~~slit~~ mode (step-index)

$N =$ max no. of modes $\frac{V^2 q}{2(2+q)}$ ( parabolic $q=2$, step $q=\infty$ )

$\tau = \frac{L}{v_g} \rightarrow \frac{dt}{d\lambda} \rightarrow dt = \left[-\frac{L}{c}\lambda\frac{d^2n}{d\lambda^2}\right]d\lambda$

$\frac{c}{n}\left[1+\frac{\lambda}{n}\frac{dn}{d\lambda}\right]$ $\quad \frac{dt}{L} = \left[-\frac{1}{c}\lambda_0\frac{d^2n}{d\lambda^2}\right]d\lambda$

$\frac{\delta t}{L} = \frac{n_1}{c}\left(\frac{n_1}{n_2}-1\right) \rightarrow \frac{n_1 L}{c}\Delta = \frac{L}{2n_1c}(NA)^2$

$\Delta = \frac{n_1-n_2}{n_1} \approx \frac{n_1-n_2}{n_2} \approx \frac{n_1^2-n_2^2}{2n_1^2}$

$r_{\text{laser}} = \frac{f\lambda}{a}$

## E&M

$E_\lambda = \frac{e_\lambda}{a_\lambda}$

incident = reflected + emitted

$dQ = (1-a_\lambda)dQ + e_\lambda d\lambda$

$a_\lambda dQ = e_\lambda d\lambda$

absorbed = emitted

$dQ = E_\lambda d\lambda$ (since $a_\lambda = 1$ for B.B)

$\int_0^\infty \frac{x^3}{e^x-1}dx = \frac{\pi^4}{15}$

$S = \frac{1}{4}cU$

$p = U/3$

$dQ = dU + PdV$

$dQ = 0$ & $U = uV$

$\rightarrow Vdu + udV + \frac{u}{3}dV$

$uV^{4/3} = k$

put $u = aT^4$

we get $VT^3 = \text{const.}$

wien distribution law : $\frac{f(\lambda T)}{\lambda^5}d\lambda = u(\lambda)d\lambda$

$\frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}}$

$\nabla f$ spherical : $\frac{\partial F}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial F}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial F}{\partial \phi}\hat{\phi}$

since $\nabla$ — cylindrical : $\frac{\partial F}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial F}{\partial \theta}\hat{\theta} + \frac{\partial F}{\partial z}\hat{z}$

$\nabla^2 V = 0$ (cylindrical) $\rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}$

$\Theta(\theta) = A\cos m\theta + B\sin m\theta \rightarrow \Theta(\theta) = \Theta(\theta + 2\pi)$ — m is integer

$R(r) = D r^m + E r^{-m}$

for $m=0 \rightarrow R = C\ln r + P$ & $\Theta = A$

$V(r,\theta) = a_0 + b\ln r + \sum_{k=1}^{\infty}\left[r^m(a_k\cos m\theta + b_k\sin m\theta) + r^{-m}(c_k\cos m\theta + d_k\sin m\theta)\right]$

Legendre D.E. $\rightarrow (1-x^2)\frac{d^2\Theta}{dx^2} - 2x\frac{d\Theta}{dx} + \lambda\Theta = 0$

(where $x = \cos\theta$)

soln $\rightarrow \lambda = n(n+1)$

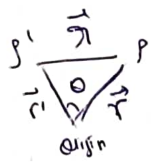
$\Theta_n(x) =$ legendre polynomial $= \frac{1}{2^n(n!)}\frac{\partial^n}{\partial x^n}(x^2-1)^n$

$V(r,\theta) = \sum_{n=0}^{\infty}\left(A_n r^n + \frac{B_n}{r^{n+1}}\right)\Theta_n(\cos\theta)$ $\quad \nabla^2 V = 0$ take $r$ & $\theta$ terms

conducting sphere in external field :

$V(r,\theta) = V_0 + \left(-E_0 r + \frac{E_0 R^3}{r^2}\right)\cos\theta$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right]$



$\frac{1}{\mathscr{r}} = \frac{1}{r}(1+\epsilon)^{-1/2}$, where $\epsilon = \left(\frac{r'}{r}\right)^2 - \frac{2r'}{r}\cos\theta$

$\boxed{\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^n\Theta_n(\cos\theta)}$ ★

$\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int_V \rho(r')r'^n\Theta_n(\cos\theta)\,d\tau'}$

( $\Sigma$ outside $\int$ inside since we get monopole, dipole, etc. terms)

$P_m = \alpha E_m$, $E_m = E_0 + P/3\varepsilon_0$, $P = nP_m$, $\vec{P} = (k-1)\varepsilon_0\vec{E_0}$

$\frac{k-1}{k+2} = \frac{n\alpha}{3\varepsilon_0}$

dielectric in uniform field. (3)

$$V_{in} = -\frac{3k_2 E_0}{k_1+2k_2} r\cos\theta$$

$$V_{out} = -E_0 r\cos\theta + \left(\frac{k_1-k_2}{k_1+2k_2}\right)\frac{E_0 a^3 \cos\theta}{r^2}$$

check this one.

$$b = \frac{R^2}{d}, \quad q' = -\left(\frac{R}{d}\right)q$$

$$d\phi_m = \frac{\mu_0 I}{4\pi} d\Omega = \frac{\mu_0 \vec{m}\cdot\vec{r}}{4\pi r^3}$$

for $E_{out}$ ~ use $p = n p_m = \alpha \epsilon_0 E_m = \left(\frac{k-1}{k+2}\right) 3\epsilon_0 \beta E_m = \left(\frac{k-1}{k+2}\right) 3\epsilon_0 F_0$ $= n \alpha E_m$

Now $E_{out} = F_0 + \frac{2kp\cos\theta}{r^3}\hat{r} + \frac{kp\sin\theta}{r^3}\hat{\theta}$

put $p = \frac{4}{3}\pi R^3 P$ and $P$

$$E_{out} = F_0 + 2\frac{(k-1)}{(k+2)}\frac{a^3}{r^3} F_0 \cos\theta\,\hat{r} + \left(\frac{k-1}{k+2}\right)\frac{a^3}{r^3} F_0 \sin\theta\,\hat{\theta}$$

For dielectric sphere ~ $V_{out}$ & $E_{out}$ like normal $\vec{p}$ ~ just put $p = \frac{4}{3}\pi R^3 P$

for $V_{in}$ ~ put $r = R$ in $V_{out}$

$$= \frac{P R\cos\theta}{3\epsilon_0} = \frac{Pz}{3\epsilon_0}$$ Just like how we did for magnetized sphere

$$E_{in} = -\nabla V_{in} = -P/3\epsilon_0$$

$$\phi_{in} = \frac{\mu M z}{3}$$

$$B_{in} = -\frac{\mu M}{3}\hat{z}$$

$$\vec{H}_{in} = \frac{B}{\mu} = -\vec{M}/3$$

Now $\vec{H}_{net} = H_0 - \vec{M}/3$ ( for $\vec{E}_m = \vec{F}_0 + P/3\epsilon_0$ )

$$\vec{M} = \frac{3(\mu_r - 1)}{(\mu_r+2)}\vec{H}_0 \quad \frac{k-1}{k+2} \Rightarrow k = \epsilon_r$$ for clausius mossotti also

$$U = \frac{1}{2}\vec{D}\cdot\vec{E} + \frac{1}{2}\vec{B}\cdot\vec{H}$$

$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(\mathbf{r})$$

$\tau = M \times B = \mu_0 M H \sin\theta$ $(B = \mu_0 H)$

$dW = -\tau d\theta = \mu_0 H dM$ (put $dM = -s\, m\sin\theta d\theta$) (counter-intuitive)

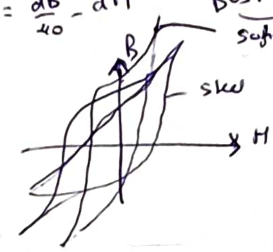
$$dW = \mu_0 \times H dM$$

Now $M = \frac{B}{\mu_0} - H \rightarrow dM = \frac{dB}{\mu_0} - dH$

$$dW = H dB$$

Steel - permanent magnet
Soft iron - electromagnet & transformer core

B vs H (B on y-axis)




$J_m = \nabla \times M$ ~ $m = IA$ ~ $M = \frac{m}{V} = \frac{I}{l}$ ~ $I = M \times l$ ~ $\Delta I = \Delta M_z \Delta z = \frac{\partial M_z}{\partial y}\Delta y \Delta z$ ~ $J_x = \frac{\Delta I}{\Delta y \Delta z} = \frac{\partial M_z}{\partial y}$

similarly $J_x = -\frac{\partial M_y}{\partial z}$

$J_p = \frac{\partial P}{\partial t}$

~~[illegible]~~:

$\nabla\cdot P = -\rho_p$ and $\nabla\cdot J + \frac{\partial \rho_p}{\partial t} = 0$

Thus $\nabla\cdot J = -\frac{\partial}{\partial t}\rho_p = \frac{\partial}{\partial t}(\nabla\cdot P) \rightarrow J_p = \frac{\partial P}{\partial t}$

Poynting: $\nabla\cdot(A\times B) = \bar{B}\cdot(\bar{\nabla}\times A) - A\cdot(\nabla\times B)$

$S = E\times H$

$$\phi = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2} = \frac{\mu_0}{4\pi}\left(\frac{4}{3}\pi R^3 M\right)\frac{\cos\theta}{r^2}$$

$$B_{out} = -\nabla\phi_{out} = \mu_0 H_{out}$$

Now $\phi_{in}$ ~ put $\mu_0 - \mu$, $r - R$

$\frac{\mu M z}{3} \rightarrow H_{in} = -M/3$

thus $H_{net} = H_0 - \frac{M}{3}$ ~ now $\chi = M/H_{net}$

$$A = \frac{\mu_0}{4\pi}\int \frac{I\,\vec{dl}}{r}$$

$$\left|\frac{J}{J_D}\right| = \frac{\sigma}{\omega\epsilon}$$

$\frac{\partial}{\partial t} = -j\omega$ ~~[illegible]~~

$\vec{\nabla} = j\vec{k}$

$-\bar{J}\cdot\bar{E} = \frac{\partial u}{\partial t} + \nabla\cdot S$

transferred into EM field; flowing out thru boundary surface.

from wave eq$^n$
$\nabla\times E = -\partial B/\partial t$
$\nabla\times B = \mu_0(J + \frac{\partial D}{\partial t})$

Now $\nabla\times(\nabla\times E)$

for wave eq$^n$ in non-homogeneous ($J = \sigma E \neq 0$) ($\sigma \neq 0$)

conducting : $\rho = 0, \sigma \neq 0$

$$\nabla^2\psi - \sigma\mu\frac{\partial\psi}{\partial t} - \epsilon\mu\frac{\partial^2\psi}{\partial t^2} = 0$$ ( $\vec{\nabla} = j\vec{k}$, $\frac{\partial}{\partial t} = -j\omega$ )

$$\rightarrow -k^2 + i\sigma\mu\omega + \mu\epsilon\omega^2 = 0$$

$k = \alpha + i\beta$ $$\rightarrow k^2 = (\alpha^2 - \beta^2) + 2i\alpha\beta = (\mu\epsilon\omega^2) + i(\sigma\mu\omega)$$

$$\beta = \sqrt{\mu\epsilon}\cdot\omega\left[\sqrt{1+(\sigma/\omega\epsilon)^2} - 1\right]^{1/2} \rightarrow [\text{Mug - can't derive}]$$
$2$

~ $\beta$ has units of $k$

$k = \frac{\omega}{v} = \sqrt{\mu\epsilon}\cdot\omega$

for good conductor ~ $\frac{\sigma}{\omega\epsilon} \gg 1$ ~ skin depth $= \delta = \frac{1}{\beta} = \sqrt{\frac{2}{\mu\sigma\omega}}$ NOTE!

same $\left|\frac{J}{J_D}\right|$

since $e^{-\bar{\beta}\cdot\bar{r}}$ is alternating factor

$Z = \sqrt{\frac{\mu}{\epsilon}} = \frac{Z_0}{\sqrt{K}} = \frac{Z_0}{n}$, where $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = (120\pi)\Omega$

$Z = \frac{|E|}{|H|} \rightarrow |H| = \frac{n|E|}{Z_0}$

comes from $B = \frac{k \times E}{\omega}$

for $t = \frac{2n_1}{n_1+n_2}$ & $r = \frac{n_1-n_2}{n_1+n_2}$

use $E_i - E_r = E_t$ and $\frac{n_1 E_i}{Z_0} - \frac{n_1 E_r}{Z_0} = \frac{n_2 E_t}{Z_0}$

$r_{\parallel} = \left|\frac{E_r}{E_i}\right|_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$

means $\vec{E}$ is $\parallel$ to plane of incidence

$r_{\perp} = \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$

(in QM: $\frac{k_1 - k_2}{k_1 + k_2}$, use $\psi_1 = \psi_2$ & $\frac{\partial \psi_1}{\partial t} = \frac{\partial \psi_2}{\partial t}$ in Ae..., etc )

$m\ddot{r} + b\dot{r} + kr = F = e\vec{E} = eE_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

put $D \rightarrow -j\omega$ (use $\psi = Ae^{i(\vec{k}\cdot\vec{r} - \omega t)}$)

$r = \frac{f_0 e^{-i\omega t}}{-\omega^2 - 2c\omega j + \omega_0^2}$

$= \frac{f_0 e^{-j\omega t}}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4c^2\omega^2}\, e^{-j\phi}}$ $\quad \phi = \tan^{-1}\frac{2c\omega}{\omega_0^2 - \omega^2}$

$P = \check{e}\check{r}$ $\quad$ $\omega$ is fixed as $\omega$ of $\vec{E}$

$\omega_0$ diff for diff $k$,

$P = \sum n_k p_k$

$P = \sum_k \frac{n_k e^2 \vec{E}}{m[(\omega_k^2 - \omega^2) - 2c\omega j]}$

$n^2 = 1 + \frac{\vec{P}}{\epsilon_0 E} = 1 + \frac{e^2}{\epsilon_0 m} \sum \frac{n_k}{(\omega_k^2 - \omega^2) - 2c\omega j}$

$n = 1 + \frac{e^2}{2m\epsilon_0} \sum \frac{n_k}{\omega_k^2 - \omega^2}$ ~ sellmier formula — put $\omega = \frac{2\pi c}{\lambda}$ to get cauchy formula

(when damping small: $c \rightarrow 0$) normal dispersion

for $\omega_k \approx \omega \rightarrow$ damping not neglible

(don't cancel out $\omega$ & $\omega_k$)

use ~~$n_2$~~ $n^* = n + ink$ $= 1 + \frac{e^2}{2\epsilon_0 m} \sum \frac{n_k}{(\omega_k^2 - \omega^2) - 2c\omega j} \times \frac{(\omega_k^2 - \omega^2) + 2c\omega j}{(\omega_k^2 - \omega^2) + 2c\omega j}$

refractive index (n), absorption coeffient (k)

~~now, rationalize~~.

~~and~~ equate real & imaginary.

now

to get $n = \ldots\ldots$

and $nk = \ldots\ldots$

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$B = \nabla \times A$, $\quad E = -\nabla\phi - \frac{\partial A}{\partial t}$

gauge transform $\rightarrow A' = A + \nabla\lambda$ $\longrightarrow$ $\lambda$ is Gauge fn

$\phi' = \phi - \partial\lambda/\partial t$

$\nabla \cdot A + \frac{1}{c^2}\frac{\partial\phi}{\partial t}$ — Lorentz gauge (note $\Box^2 \rightarrow \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$)

$A_\mu = (A_1, A_2, A_3, \frac{i\phi}{c})$ $\quad J_\mu = (J_1, J_2, J_3, ic\rho)$

$\Box^2 A_\mu = -\mu_0 J_\mu$ ~ covariant form of Maxwells equations

$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}$ $\quad \mu\nu$ right hand screw

EM field tensor

$$F_{\mu\nu} = \begin{bmatrix} 0 & B_z & -B_y & -\frac{iE_x}{c} \\ -B_z & 0 & B_x & -\frac{iE_y}{c} \\ B_y & -B_x & 0 & -\frac{iE_z}{c} \\ \frac{cE_x}{c} & \frac{iE_y}{c} & \frac{iE_z}{c} & 0 \end{bmatrix}$$

$z \; y \; x$

derive similar to $F' = \frac{dp'}{dt'}$ &

$E_{\parallel}' = E_{\parallel}$

$E_{\perp}' = \frac{E_{\perp}}{\sqrt{1 - v^2/c^2}}$ $\quad \left(E' = E_{\parallel} + \frac{E_{\perp}}{\sqrt{1 - \frac{v^2}{c^2}}}\right)$

To prove $I_c = I_D$

start $E = \sigma/\epsilon_0 = \frac{Q}{A\epsilon_0}$

$\frac{dQ}{dt} = I_c \sim = \epsilon_0 A \frac{dE}{dt}$ ↳ $I_D$

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## Thermo

$dQ = dU + PdV$

clausius inequality. $d\left(\frac{Q}{T}\right) \leq 0$

$\eta_{otto} = 1 - \frac{1}{(\rho_c)^{\gamma-1}}$

$\eta_{Diesel} : 1 - \frac{1}{\gamma} \frac{\left[\left(\frac{1}{\rho_e}\right)^{\gamma} - \left(\frac{1}{\rho_c}\right)^{\gamma}\right]}{\left(\frac{1}{\rho_e} - \frac{1}{\rho_c}\right)}$ $\rho_c > \rho_e$ this $\frac{1}{\rho_e} - \frac{1}{\rho_c} > 0$

use intermediate

$\frac{V_2}{T_2} = \frac{V_3}{T_3} = \alpha$ → $T_2 = \frac{V_1}{\alpha \rho_c}$ & $T_3 = \frac{V_1}{\alpha \rho_e}$

Joule-kelvin coefficient $\mu = \left(\frac{\partial T}{\partial P}\right)_H$ — enthalpy

$\mu = \frac{1}{C_P}\left[\frac{2a}{RT} - b\right]$, $T_i = \frac{2a}{Rb}$

$S = S(T, H)$ ~ adiabatic demagnetization

$\Delta T = \frac{CH^2}{2C_H T_{avg}}$

radn pressure ~~......~~ $p = I/c$ Intensity

at angle $\alpha$: $p = \frac{I}{c}\cos^2\alpha$

$\left(\alpha + \frac{3}{\beta^2}\right)\left(\beta - \frac{1}{3}\right) = \frac{8}{3}\gamma$

p from 100% reflection: $\frac{2I}{c}$

$P_{net} = P_{incident} + P_{emmitted} = \frac{2I}{c}$

$F = C - P + 2$

$P_{incident} = P_{emitted} = I/c$

$p = u/3$ energy density.

$n(E) = f(E)g(E)$ ~ $f_i = \frac{n_i}{g_i}$ opposites

also $\Omega = \left(\frac{g_i}{n_i}\right)$

also $f(E)$ for FD — $\frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$

thus $f_i = \frac{n_i}{g_i} = \left(e^{\alpha + \beta E_i} \pm \eta\right)^{-1}$

stirling approximation → $\ln(n!) = n\ln n - n$

MB: $d\ln\Omega = \sum \ln\left(\frac{g_i}{n_i}\right) dn_i$

FD: $d\ln\Omega = \sum \ln \frac{g_i - n_i}{n_i} dn_i$

BE: $d\ln\Omega = \sum \ln \frac{g_i + n_i}{n_i} dn_i$

also → to get $f(E)$ eqn. since $\alpha \sum dn_i = 0$, $\beta \sum E_i dn_i = 0$

use $\left[d\ln\Omega + [\alpha - \beta E_i] dn_i\right] = 0$

$d\ln\Omega = -(\alpha - \beta E_i)\, dn_i$

$Z = \sum g_i e^{-E_i/kT}$

$g(E)dE = \frac{2\pi V}{h^3}(2m)^{3/2} E^{1/2} dE$ (from $g(p)dp$)

for MB ~ $n(E)dE = \frac{2N}{\sqrt{\pi}(kT)^{3/2}} E^{1/2} e^{-E/kT} dE$

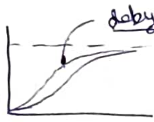
$(\Gamma(1/2) = \sqrt{\pi})$

$\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} = \sqrt{\frac{\pi}{\alpha}}\, e^{\beta^2/4\alpha}$

sommerfeld lemma $= \int_0^{\infty} f(x - x_0) x^n dx = \frac{x_0^{n+1}}{n+1}\left[1 + \frac{\pi^2}{6}\frac{n(n+1)}{x_0^2} + \ldots\right]$

$E_{f_0} = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$

$E(0) = \frac{3}{5} N E_{f_0}$



debye in middle (it is correct & Balanced) (not extreme)

Dulong Petit: $\langle E \rangle = kT$ → $C_V = 3R$

einstein: $\langle E \rangle = \frac{h\nu}{e^{h\nu/kT} - 1}$ → $C_V = 3R\left(\frac{\Theta_E}{T}\right)^2 \frac{e^{\Theta_E/T}}{(e^{\Theta_E/T} - 1)^2}$ don't miss this term on top!!

debye: $g(\nu)d\nu = 4\pi V\left(\frac{1}{V_l^3} + \frac{2}{V_t^3}\right)\nu^2 d\nu$ — analogous to $\frac{8\pi\nu^2 d\nu}{c^3}$

$\int_0^{\nu_m} g(\nu)d\nu = 3N$

$E = \int_0^{\infty} E(\nu)d\nu$

$E(\nu) = g(\nu)d\nu \times \langle E \rangle$

$= g(\nu)d\nu \times \frac{h\nu}{e^{h\nu/kT} - 1}$ — to give analogous to $\frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT} - 1}$

$= \frac{9R}{\Theta_D^3} \cdot T^4 \int_0^{\Theta_D/T} \frac{x^3 dx}{e^x - 1}$

$C_V = \left(\frac{\partial E}{\partial t}\right)_V = 9R\left[4\left(\frac{T}{\Theta_D}\right)^3 \int_0^{\Theta_D/T} \frac{x^3 dx}{e^x - 1} - \frac{(\Theta_D/T)}{e^{\Theta_D/T} - 1}\right]$

start with: $\Delta p = 2\lambda \frac{dp}{dz}$

end result: $\frac{d(\,)}{dt \cdot A} = 2\lambda \frac{d(\,)}{dz} \times \frac{1}{6} n\bar{v}$ ~ remember as:

in GBO: $\frac{n\bar{v}}{4}$ & $\pm\frac{2}{3}\lambda$

$(\,) \to P$ | $p = mv$

$(\,) = Q$ | $Q = mcT$

$(\,) = m$ | $m = \text{conc}/n$ (conc. $= mn$)

$\bar{v} = \sqrt{\frac{8kT}{\pi m}}$

$\lambda = \frac{1}{\sqrt{2}\, n\pi d^2}$ diameter

~~......~~: Force $= \frac{dp}{dt} = \eta A \frac{dv}{dx}$

$\frac{\partial Q}{\partial t} = kA \frac{\partial T}{\partial x}$

Fick's law: $J = -D\frac{\partial C}{\partial x}$ — conc. $= mn$ — analogous to density

Flux $= \frac{dm}{A\,dt}$

remove $mn$

$\eta = \frac{1}{3} mn\bar{v}\lambda$, $K = \frac{1}{3} mn\bar{v}\lambda C_V$, $D = \frac{1}{3}\bar{v}\lambda$

adder

$n(v)dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$

$f(v_x) = \frac{n(v_x)}{N} = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{mv_x^2}{2kT}} dv_x$

$n(q)dq = k e^{\frac{-aq^2}{kT}} dq$ ($q$ from $-\infty$ to $\infty$)

$\langle E_q \rangle = \frac{kT}{2}$

QM



Davisson - Germer:
$E = 54\,eV \longrightarrow \lambda = 1.66\,A^\circ$
$\phi = 50 \sim \theta = 65$
$d = 0.985\,A^\circ$

Fourier:
$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k)\, e^{ikx}\, dk$$ — normal $f(x)$ on LHS and $e^{ikx}$ on RHS

inverse fourier → $$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx}\, dx$$

infinite well → $\langle x \rangle = L/2$ $\quad \langle p_x \rangle = 0$

$$\langle x^2 \rangle = \frac{L^2}{3} - \frac{L^2}{2\pi^2 n^2} \qquad \langle p_x^2 \rangle = \frac{n^2\pi^2 \hbar^2}{L^2}$$

$$\Delta x = L\sqrt{\frac{1}{12} - \frac{1}{2\pi^2 n^2}}$$ ⇒ mug this, automatically one comes using $\langle x^2 \rangle$ and $\langle x \rangle = \frac{L}{2}$

SHO derivation → $y = \left(\frac{\sqrt{km}}{\hbar}\right)^{1/2} x = \left(\frac{m\omega}{\hbar}\right)^{1/2} x$

$\frac{d^2 v}{dy^2} - 2y\frac{dv}{dy} + 2nv = 0$ ⟶ Hermite's D.E.

$U_n(y) = (-1)^n e^{y^2} \frac{d^n}{dy^n}(e^{-y^2})$ ~ hermite's polynomials

Final $\psi_n(y) = C_n U_n(y)\, e^{-y^2/2}$ with $y = \left(\frac{m\omega}{\hbar}\right)^{1/2} x$

$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$

$\vec{J} = \frac{\hbar}{2mi}\left[\psi^* \nabla\psi - \psi \nabla \psi^*\right]$

$J = v|\psi|^2 = \rho v$

$R = \frac{|B|^2}{|A|^2} = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$

$T = \frac{k_2}{k_1}\left|\frac{C}{A}\right|^2 = \frac{k_2}{k_1} \cdot \frac{4k_1^2}{(k_1+k_2)^2} = \frac{4k_1 k_2}{(k_1+k_2)^2}$

$\sinh = \frac{e^{x} - e^{-x}}{2}$

rectangular Barrier ⎍ ~ $$T = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2 \sinh^2\left(\sqrt{\frac{2m(V_0-E)L^2}{\hbar^2}}\right)}$$

$\sinh^2(\gamma L)$, $\gamma = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

if $E > V_0$ ~ $V_0 - E \sim E - V_0$ and $\sinh \sim \sin$

for $\gamma L \gg 1$ ~ $T \approx e^{-2\gamma L}$ where $\gamma L = \sqrt{\frac{2m(V_0-E)}{\hbar}}\, L$

$\frac{\partial}{\partial t}\langle a \rangle = \frac{1}{i\hbar}\langle [A, H] \rangle + \left\langle \frac{\partial A}{\partial t} \right\rangle$

$[x, p_x] = i\hbar \qquad \frac{\partial}{\partial t}\langle x \rangle = \frac{\langle p_x \rangle}{m}, \quad \frac{\partial}{\partial t}\langle p_x \rangle = \left\langle -\frac{\partial V}{\partial x} \right\rangle$

$a_+ = \frac{1}{\sqrt{2m\hbar\omega}}(-ip_x + m\omega x), \quad a_- = \frac{1}{\sqrt{2m\hbar\omega}}(ip_x + m\omega x)$

$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$

$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$
$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$

$$\vec{L} = -i\hbar(\vec{r} \times \nabla) = -i\hbar \times \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ r & 0 & 0 \\ \frac{\partial}{\partial r} & \frac{1}{r}\frac{\partial}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} \end{vmatrix}$$

IMP

$L^2 = (-\hbar^2) \times [(\theta\ \&\ \phi$ component of $\nabla^2) \times r^2]$
$= -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$

$L_z = -i\hbar\, \partial/\partial\phi$

this is $\phi$

$L_\pm = \pm\hbar e^{\pm i\phi}\left[\frac{\partial}{\partial\theta} \pm i\cot\theta \frac{\partial}{\partial\phi}\right]$

↳ $L_x$ & $L_y$ will have $i\hbar$ on outside just like $L_z$ has $i\hbar$ outside

$\phi(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi}$

$$(1-x^2)\frac{d^2\Theta}{dx^2} - 2x\frac{d\Theta}{dx} + \left[\lambda - \frac{m^2}{(1-x^2)}\right]\Theta = 0$$

(associated Legendre D.E)

$\Theta_l(x) = \frac{1}{2^l (l!)}\frac{d^l}{dx^l}(x^2-1)^l$ (where $x = \cos\theta$)

$\Theta_l^m(x) \rightarrow (1-x^2)^{|m|/2}\frac{d^{|m|}}{dx^{|m|}}\Theta_l(x)$
(where $|m| \le l$)

$Y_l^m(\theta, \phi) = \Theta_l^m(\theta)\,\phi_m(\phi) \times \varepsilon$
$= |l, m_l\rangle$

$\varepsilon = (-1)^m$ for $m \ge 0$
$= 1$ for $m < 0$

$\int_0^{2\pi}\int_0^{\pi} |Y|^2 \sin\theta\, d\theta\, d\phi = 1$

For finite well → always use $-\frac{L}{2}$ to $+\frac{L}{2}$ ★

# AMP

$$H' = H_0 + H_{ls} + H_{relativistic}$$

$\propto (\vec{L}\cdot\vec{S})$

$$= -\frac{\hbar^4}{8m_0^3c^2}\nabla^4 \text{ — perturbation : } \langle n,l,m| \frac{-\hbar^4}{8m_0^3c^3}\nabla^4 |n,l,m\rangle$$

spin-orbit coupling — $E = -\vec{\mu}_s\cdot\vec{B}$ — $\vec{\mu}$ from $\vec{S}$, $\vec{B}$ from $\vec{L}$

$\frac{1}{2}\mu_B B'$ Thomson precession factor

$$\vec{B} = \frac{\mu_0}{4\pi}(ze)\frac{(-\vec{v})\times\vec{r}}{r^3}$$

$$= \cancel{\frac{\mu_0}{4\pi}\frac{ze}{mr^3}\vec{L}}$$

Now $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{ze}{r^3}\vec{r}$

Thus $\vec{B} = \frac{\mu_0}{4\pi}(ze)(-\vec{v})\times\frac{4\pi\varepsilon_0\vec{E}}{ze}$

$$= \frac{\vec{E}\times\vec{v}}{c^2}$$

$$E = -\vec{\mu}_S\cdot\vec{B}$$

$$= +\frac{e}{2m}g_s\vec{S}\cdot\left(\frac{\vec{E}\times\vec{v}}{c^2}\right)$$

$$= \frac{e}{2m}g_s\vec{S}\cdot\left(\frac{-\frac{\partial V}{\partial r}\hat{r}\times\vec{v}}{c^2}\right)$$

$$= \frac{-e}{2mc^2}g_s\,\vec{S}\cdot\left(\frac{\partial V}{\partial r}\frac{\vec{r}\times\vec{v}}{r}\right)$$

$$= \frac{-e}{2m^2c^2}\times 2\times\frac{1}{r}\frac{\partial V}{\partial r}\times(\vec{L}\cdot\vec{S}) = \alpha(\vec{L}\cdot\vec{S})$$

$$H_{ls} \simeq \frac{1}{2}\times\frac{-e}{m^2c^2}\langle n,l,m|\frac{1}{r}\frac{\partial V}{\partial r}(\vec{L}\cdot\vec{S})|n,l,m\rangle$$

( Thomson precession

putting $\langle\frac{1}{r^3}\rangle$ value :

and $\Delta E = E_{j=l+1/2} - E_{j=l-1/2}$

$$\Delta T = \frac{\Delta E}{hc} = \frac{5.84Z^4}{n^3 l(l+1)}\,\text{cm}^{-1}$$ — Dirac formula. (splitting due to spin-orbit coupling b/w $j=l+1/2$ & $j=l-1/2$)

combining spin-orbit coupling perturbation to relativistic perturbation

$$\Delta T = \frac{5.84Z^4}{n^3}\left(\frac{1}{j+1/2} - \frac{3}{4n}\right)\text{cm}^{-1}$$

Hartree for multi-$e^-$

$$H = \sum\frac{p_i^2}{2m} + \sum V_i(r) + LL + LS + SS$$

$V_i(r) = \frac{1}{4\pi\varepsilon_0}\frac{(ze)e}{r_i}$, $LL$: $\frac{e^2}{4\pi\varepsilon_0 r_{ij}}$

$$g_J = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}$$



$$\bar{\omega} = \bar{\omega}_v + (B'+B'')m + (B'-B'')m^2$$

P ~ $J''=1,2,3$, $m=-1,-2,-3$, thus $m=-J''$

R ~ $J''=0,1,2,3$, $m=1,2,3$, thus $m=J''+1$

ESR : $\nu_p = \frac{\mu_B B g_s}{h}$ — we use $E = -\vec{\mu}_s\cdot\vec{B}$ (spin)

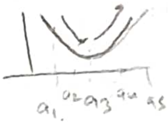
# NPP

Packing fraction — $\frac{M-A}{A}$ $\left(f = \frac{M}{A}-1\right)$

$$Q = \frac{1}{e}\int(3z^2-r^2)\rho\,d\tau$$

$$BE = -a_1A + a_2A^{2/3} + a_3\frac{Z(Z-1)}{A^{1/3}} + a_4\frac{(A-2Z)^2}{A} \mp a_5A^{-3/4}$$

$a_1 = 14$ MeV/$c^2$
$a_2 = 13.1$ MeV/$c^2$
$a_3 = 0.6$ MeV/$c^2$
$a_4 = 19.1$ MeV/$c^2$
$a_5 = 34$ MeV/$c^2$




predicted Q in shell model $= -\frac{3}{5}\left|\frac{2I-1}{2I+2}\right|R_0^2A^{2/3}$

Deuteron

$$V_0R^2 \geq \frac{\pi^2\hbar^2}{8m}$$ — $m$ = reduced mass $= \frac{m_p}{2}$

we know $r_d = 4.2$ Fermi, put $R = r_d$ for crude approximation

$$V_0 = \frac{\pi^2\hbar^2}{8mr_d^2} \approx 25\text{ MeV}$$ — put $m_p/2$ — for $\frac{3\pi}{5}$ — we set $9\times25 = 225$ MeV

Now $r_d$ = penetration depth $= \frac{1}{k_2} = \frac{\hbar}{\sqrt{2mB}}$

also $R^2 = \frac{\pi^2\hbar^2}{8mV_0}$ — $R = \frac{\pi\hbar}{\sqrt{8mV_0}}$

Thus $\frac{r_d}{R} = \frac{2}{\pi}\sqrt{\frac{V_0}{B}} \approx 2$ — $R \approx 2.1$ F

put to get $V_0 = 30$ MeV

$$\pi^0 \rightarrow \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$\eta^0 \rightarrow \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

$$\Delta\lambda = \frac{h}{m_ec}(1-\cos\theta)$$ — $\theta$ is scattering angle of photon since $\Delta\lambda$ is also of photon.

Mossbauer. ${}_{26}Fe^{57}$ — 14.1 keV | ${}_{77}Ir^{191}$ : 129 keV

$\phi$ is recoil — stationary initial
$\theta$ is scatter — moving initial & final with change in dirn

$i \to f + \beta + \bar{\nu}_e$ ~~[illegible]~~

$dw_{fi} = \frac{2\pi}{\hbar} |\psi(i|M|f)|^2 \rho$ → $\rho$ = density of state $= dN/dE$

transition probability. $M$ = Matrix Element

Kurie function $K = \left[\frac{N_p}{p^2 F}\right]$ transition probability ($N_p = dw_{fi}$)

momentum, Fermi fn

$\frac{dy}{dx} + Py = Q$ ~ $y e^{\int P dx} = \int Q e^{\int P dx} + C$

$Q = KE_0 - KE_{pot}$ ~ eliminate $E_Y$ & $\phi$ ~ ans in term of masses and $E_x, E_y, \theta$ ($KE$ of small)

$= E_y\left(1+\frac{m_y}{M_Y}\right) - E_x\left(1-\frac{m_x}{M_Y}\right) - \frac{2\sqrt{m_x m_y E_x E_y}}{M_Y}\cos\theta$

$E$ ~ kinetic energies

$(E_x)_{threshold} = \frac{1}{2} m_x v_x^2 = -Q\frac{(m_x+M_x)}{M_x}$

Geiger Nuttall; $\log\lambda = a\log R + b$

$\log\lambda = -a_1\frac{Z}{\sqrt{E}} + a_2$

also $\lambda = \frac{0.693}{t_{1/2}}$ → $\log t_{1/2} = c_1\frac{Z}{\sqrt{E}} + c_2$

$Q = [M(Z,A) - 2M(\frac{Z}{2},\frac{A}{2})]c^2$

take only $a_2, a_3$, rest cancel out — take $Z^2$ instead of $Z(Z-1)$

$Q = a_2 A^{2/3}[1 - 2^{1/3}] + a_3\frac{Z^2}{A^{1/3}}\left[1 - \frac{1}{2^{2/3}}\right]$

(pending easy ~ just get the correct values from PCT)

$k_\infty = \eta\,\epsilon\,p\,f$ ($k_\infty \geq 1$ for self-sustained)

$\eta$ → no. of fast neutron absorbed by fissile Nuclei

$\eta = \nu\frac{\sigma_f}{\sigma_a}$ no. of fast neutron produced per fission

$\epsilon$ = fast fission factor ($\epsilon > 1$)

$p$ — resonance escape probability

$f$ — ~~termal~~ thermal utilization Factor

SSPE

sum $= A\oplus B$, carry $= AB$ } H.A

sum $= (A\oplus B)\oplus C$, carry $= AB + C(A\oplus B)$ } F.A.

Half S. → Diff $= A\oplus B$, Borrow $= \bar{A}B$

Full S. → Diff $= (A\oplus B)\oplus C$, Borrow $= \bar{A}B + C\overline{(A\oplus B)}$ ~ $\overline{A\oplus B} = \bar{A}\oplus B$

$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$ critical magnetic field

$\psi_1 = \sqrt{n_1}\,e^{i\theta_1}$, $\nu = \frac{2eV_0}{h}$

Flux $= \phi = \bar{B}\cdot\bar{A} = s\left(\frac{h}{2e}\right)$, Fluxoid $= h/2e$

$H\psi = E\psi$ ~ $i\hbar\frac{\partial\psi_1}{\partial t} = -eV_0\psi_1 + \hbar T\psi_2$

Not $H\psi$! $i\hbar\frac{\partial\psi_2}{\partial t} = +eV_0\psi_2 + \hbar T\psi_1$

$\frac{dn_1}{dt} = -\frac{dn_2}{dt} = \frac{dn}{dt} = 2T\sqrt{n_1 n_2}\sin\delta$

$\frac{d\theta_1}{dt} = \frac{eV_0}{\hbar} - T\sqrt{\frac{n_2}{n_1}}\cos\delta$

$\frac{d\delta}{dt} = \frac{2eV_0}{\hbar}$

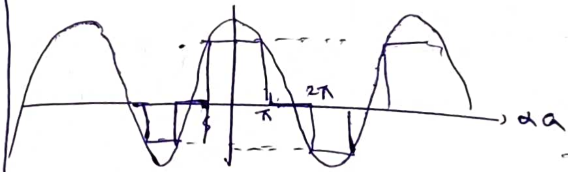
~~[illegible]~~ supercurrent → $J = \frac{dn}{dt} = J_0\sin\delta(t) = J_0\sin\left[\delta(0) - \frac{2eV_0}{\hbar}t\right]$

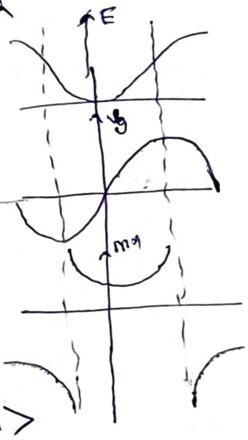
$\varepsilon = \frac{d\phi}{dt} = V_0$ ~ $\Delta\phi = V_0\Delta t$

Thus $J_1 = J_0\sin\left[\delta(0) + \frac{e\phi}{\hbar}\right]$ and $J_2 = J_0\sin\left[\delta(0) - \frac{e\phi}{\hbar}\right]$

Kronig-Penney Model ~ $P\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka$

$P = \frac{m}{\hbar^2}V_0 ab$, $\alpha = \frac{\sqrt{2mE}}{\hbar}$




$\alpha = \pm\frac{n\pi}{a}$ ~ forbidden

$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$, $v_g = \frac{1}{\hbar}\frac{dE}{dk}$

$d = \left(\frac{\vec{a}}{h}\right)\cdot\hat{n}$

$S_+, L_+, J_+|j,m\rangle = \sqrt{(j-m)(j+m+1)}\,\hbar\,|j,m+1\rangle$

$S_-, L_-, J_-|j,m\rangle = \sqrt{(j+m)(j-m+1)}\,\hbar\,|j,m-1\rangle$

$\sigma_x = \begin{bmatrix}0&1\\1&0\end{bmatrix}$, $\sigma_y = \begin{bmatrix}0&-i\\i&0\end{bmatrix}$, $\sigma_z = \begin{bmatrix}1&0\\0&-1\end{bmatrix}$

(they all have determinant $= -1$)

$a^* = \frac{2\pi(\bar{b}\times\bar{c})}{\bar{a}\cdot(\bar{b}\times\bar{c})}$

$\cos(90+\theta) = \sin\theta$ – give $2d\sin\theta = n\lambda$

$$2\vec{k}\cdot\vec{G} + G^2 = 0$$

$$\vec{a}^* = 2\pi \frac{\bar{b}\times\bar{c}}{\bar{a}\cdot\bar{b}\times\bar{c}}$$

$$I = J_s\left(e^{\frac{eV_D}{\eta kT}} - 1\right)$$

strength of magnetic shell

$$\phi = \frac{m}{A} = Mt$$

$$V = \frac{\mu_0}{4\pi}\times\phi\times\omega$$ – strength, – solid angle

---

$\bar{F} = -eE = \frac{dv_s}{dt}\times m$  ($F = m\times acc$)

$J_s = n_s e\vec{v}_s$

$\frac{dJ_s}{dt} = \frac{e^2 n_s \vec{E}}{m}$ – 1st London eqn

↳ take curl, & integrate

$\nabla\times J_s = -B\frac{n_s e^2}{m}$ – 2nd London eqn

now use $\nabla\times B = \mu_0 J_s$ ~ take curl.

$\nabla^2 B = \frac{\mu_0 n_s e^2}{m}B$ ~ $B(x) = B(0)\,e^{-x/\lambda_L}$

$\nabla^2 B = \frac{1}{\lambda_L^2}B$    penetration depth

---

$\sigma = e(n\mu_n + p\mu_p)$ ~ from $J = neV_d$ and $V_d = \mu E$

also $V_d = -\frac{eE\tau}{m}$ ~~Thus also~~

$$n_i = \frac{2(2\pi kT)^{3/2}}{h^3}(m_n^* m_p^*)^{3/4} e^{-\frac{E_g}{2kT}}$$

$$n = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} e^{-\frac{(E_c - E_F)}{kT}}$$

$$= N_c\, e^{-\frac{(E_c - E_F)}{kT}}$$

where $N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$

$p = N_v e^{-\frac{(E_F - E_v)}{kT}}$,  $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$

$n_i = N_c e^{-\frac{(E_c - E_i)}{kT}}$,  $p_i = N_v e^{-\frac{(E_i - E_v)}{kT}}$

$n = n_i e^{\left(\frac{E_F - E_i}{kT}\right)}$,  $p = n_i e^{\frac{(E_i - E_F)}{kT}}$

Diamagnetism – Langevin Theory : $\chi_{Dia} = \frac{M}{H} = -\frac{N\mu Z e^2}{6m}\langle r^2\rangle$  (no. density, $e^-$ per atom)

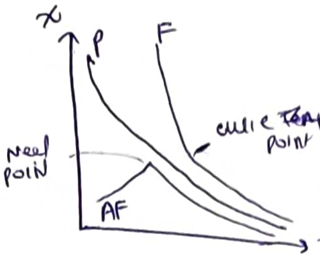
Paramagnetism – Langevin : $\chi_{para} = \frac{\mu_0 M}{B} = \frac{\mu_0 N \mu_B^2}{3kT} g^2 J(J+1) = \frac{C}{T}$  (no. density)

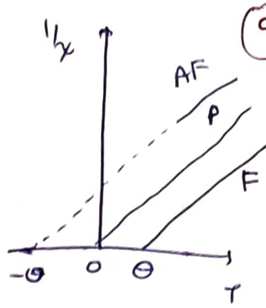
---

$\chi = \frac{C}{T}$ (paramagnetic)

Curie-Weiss law: $\chi = \frac{C}{T - T_c}$ (Ferro) $(T > T_c)$

$\chi = \frac{C}{T + \theta}$ (anti-ferro) $(T > \theta)$

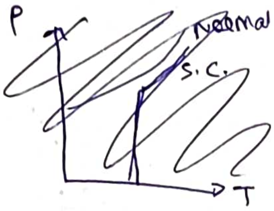


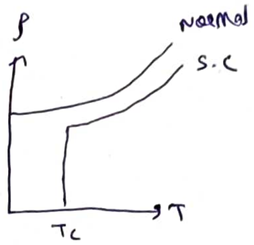




when $T > T_N$ & $T > T_c$ ~ paramagnetic behaviour.

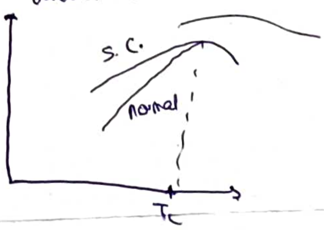
Superconductor : $T_c$ – transition ~~Semiconduct~~ temp – isotope effect. $T_c \propto \frac{1}{\sqrt{M}}$, also $\Theta_D \propto \frac{1}{\sqrt{M}}$.

Thus $\frac{T_c}{\Theta_D}$ = constant.

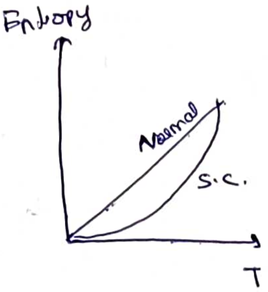



$\sigma_T$ (Thermal conductivity)

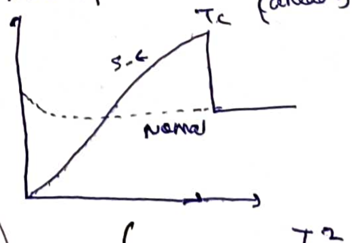



S.C has higher thermal conductivity, (& not just so electrical conductivity) ($\sigma_T$ opp to $\sigma$ dec with temp)




S.C state is more ordered than normal state. The thermally excited $e^-$ in normal state are ordered in S.C. state

Specific heat $C_T$   (ahead of $T_c$, SC becomes Normal)




Normal : $\frac{C}{T} = \frac{\gamma T}{T} = \gamma$ (const)

S.C : $C \propto e^{-\Delta/k_B T}$

Normal : $C = \gamma T$

S.C : $C = e^{-\frac{\Delta}{k_B T}}$

---

$J_n(\text{drift}) = en\mu_n E$, & $J_n(\text{diff.}) = eD_n\frac{dn}{dx}$

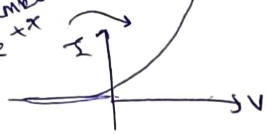
$I = J_s\left[e^{\left(\frac{eV}{\eta kT}\right)} - 1\right]$ ~ for Sn diode.

$V_B$ (Built in potential) $= \frac{kT}{e}\ln\left(\frac{N_d N_a}{n_i^2}\right)$

**Solar cell** → ~~[illegible]~~

(Aniket Final)

no -ve sign. remember as: $e^{+x}$




$$I = -I_{ph} + I_s\left(e^{\frac{qV_0}{\eta kT}} - 1\right]$$

never neglect "1".

short circuit: $I_{sc} = I_{ph}$

$I_s$ → always fixed (reverse sat. current)

$I_{sc} \propto$ intensity

(At $V_{oc}$ → $I = 0$)

Strength of mag. sheet $\phi = \frac{m}{A} = M \times t$

$V = \frac{\mu_0}{4\pi} \times \phi$ (same as $I$) $\times d\Omega$ (solid angle)

Intrinsic:

$n = N_c e^{-\frac{(E_c - E_F)}{kT}}$ $\quad N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$

$p = N_v e^{-\frac{(E_F - E_v)}{kT}}$ $\quad N_v = 2\left(\frac{2\pi m_p kT}{h^2}\right)^{3/2}$

Just like MB. $n \propto n_0 e^{-E/kT}$   E is +ve

$E_v < E_F < E_c$

Intrinsic: $n_i = (n \times p)^{1/2}$

for extrinsic:

replace $n \to N_D$ — $N_D$ donor is always $n$-type

$p \to N_A$

$j = \sigma E$ & $\frac{ne^2\tau}{m}$ derive from $J = nev_d$

$\mu = \frac{v_d}{E} = -\frac{e\tau}{m}$

$J_{drift} = (ne\mu_n + pe\mu_p)E$ (in all formula's $e$ → only magnitude! no sign)

small word ← Simple & small formula (easy)

(MINORITY)

$J_{diffusion}$ — $J_p = -eD_p \frac{dp}{dx}$   -ve sign for p (positive)

majority

$J_n = eD_n \frac{dn}{dx}$   diffusion constant

Thus $J_{electron} = ne\mu_n E + eD_n \frac{\partial n}{\partial x}$

$J_{hole} = pe\mu_p E - eD_p \frac{dp}{dx}$

Einstein reln: $\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{kT}{q} = V_T$

Diffusion length $L = \sqrt{D \cdot \tau}$

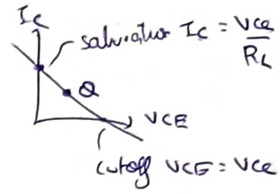
$D = \mu \times \frac{kT}{q}$

$\beta = \frac{I_C}{I_B}$, $\alpha = \frac{I_C}{I_E}$, $V_{BE} = 0$ (0.7V for silicon)

Cut-off: $I_C = 0$ → $V_{CC} = V_{CE}$

Saturation: $V_{CE} = 0$ → $I_C = \frac{V_{CC}}{R_L}$

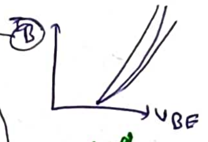
load line: $I_C$ vs $V_{CE}$



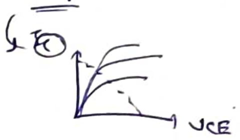

$V_{CC} = I_C R_L + V_{CE}$

2 'c's   2 'c's

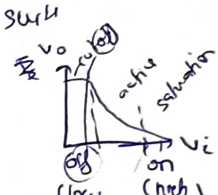
input characteristic




output:




switch



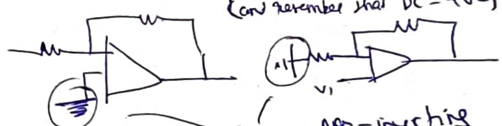

(low $V_i$) (high $V_i$)

op-amp → -ve feedback

amplifier works only on AC. Also remember AC = -ve

| | | | |
|---|---|---|---|
| Amplifier | -ve feedback | AC | |
| oscillator | +ve " | DC | no external input reqd ✓ |

doesn't DC → AC. Thus DC (can remember that DC = +ve)




inverting   only change!!   non-inverting

~~[illegible]~~ / differentiator — Both in inverting mode

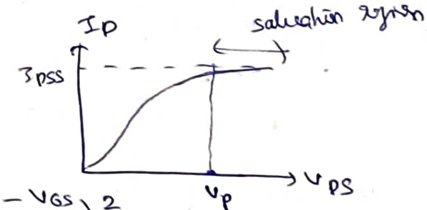
integrator


$V_0 = -\frac{1}{RC}\int V_i\,dt$

only chg

$V_0 = RC\frac{dV_i}{dt}$

JFET




$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$   pinch off voltage

$= I_{DSS}\left(1 - \frac{V_{GS}}{V_{GS(off)}}\right)^2$

$V_P = \frac{eN_D a^2}{2\varepsilon}$   'a' is height/width of channel

replace $N_D \to N_A$ for p-type channel

$\delta = \frac{1}{\beta} = \sqrt{\frac{2}{\mu\sigma\omega}}$

$g(E)dE = \frac{2\pi V}{h^3}(2m)^{3/2}E^{1/2}dE$

for MB: $n(E)dE = \frac{2N}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}E^{1/2}e^{-E/kT}dE$

$(\Gamma(1/2) = \sqrt{\pi})$

$f(v_x)_{MB} = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{1}{2}mv_x^2/kT} dv_x$

$\frac{n(v_x)}{N}$

**SHO:** $y = \left(\frac{m\omega}{\hbar}\right)^{1/2} x$

$\frac{d^2\psi}{dx^2} + \left(\frac{2E}{\hbar\omega} - y^2\right)\psi = 0$

$c_n e^{-y^2/2}\left[\frac{d^2u}{dy^2} - 2y\frac{du}{dy} + \left(\frac{2E}{\hbar\omega} - 1\right)u\right] = 0$

$\frac{a_{n+2}}{a_n} = \frac{(2n+1) - \frac{2E_n}{\hbar\omega}}{(n+2)(n+1)}$ , $\frac{d^2u}{dy^2} - 2y\frac{du}{dy} + 2nu = 0$

$u_n(y) = (-1)^n e^{y^2}\frac{d^n}{dy^n}(e^{-y^2})$

**Associated Legendre:** $x = \cos\theta$

$(1-x^2)\frac{d^2\Theta}{dx^2} - 2x\frac{d\Theta}{dx} + \left[\lambda - \frac{m^2}{1-x^2}\right]\Theta(x) = 0$

$\frac{a_{l+2}}{a_l} = \frac{l(l+1) - \lambda}{(l+1)(l+2)}$ , $\Theta_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l$ (where $x = \cos\theta$)

for $(1-x^2)\frac{d^2\Theta}{dx^2} - 2x\frac{d\Theta}{dx} + \lambda\Theta(x) = 0$

Now $\Theta_l^m(x) = (1-x^2)^{|m|/2}\frac{d^{|m|}}{dx^{|m|}}\Theta_l(x)$

$\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} = \sqrt{\frac{\pi}{\alpha}}e^{\beta^2/4\alpha}$

$\int_0^\infty e^{-x}x^{n-1}dx = \Gamma(n)$

$\int_0^\infty e^{-\alpha x^2}x^n dx = \frac{1}{2(\alpha)^{\frac{n+1}{2}}}\Gamma\left(\frac{n+1}{2}\right)$

(put $n=0$ to get $\frac{\sqrt{\pi}}{2}$, since $\frac{1}{2}$ of $-\infty$ to $\infty$) (verify)

**SHO** $\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-m\omega x^2}{2\hbar}}$

$\left[\left(\frac{\alpha}{\pi}\right)^{1/4}e^{-y^2/2}\right]$

$\psi_1 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\sqrt{\frac{2m\omega}{\hbar}}\,x\,e^{-\frac{m\omega x^2}{2\hbar}}$ $\left[\left(\frac{\alpha}{\pi}\right)^{1/4}\sqrt{2}y\,e^{-y^2/2}\right]$

$Y_{lm}$ $Y_0^0 = A = \sqrt{1/4\pi}$ $Y_1^1 = A\sin\theta e^{i\phi}$

$Y_1^0 = A\cos\theta$ $Y_1^{-1} = A\sin\theta e^{-i\phi}$ } Both have $A = \sqrt{3/8\pi}$

$R_{nl}$ $R_{10}(r) = \left(\frac{1}{a_0}\right)^{3/2} 2e^{-r/a_0} \rightarrow \psi_{100} = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$ ($P(r)dr = |R|^2 r^2 dr$)

$Y_0^0 = 1/\sqrt{4\pi}$

✓ $1s \sim R_{10}(r) \propto e^{-r/a_0}$ ✓ $2p \sim R_{21}(r) \propto re^{-r/2a_0}$ ✓ $3d \sim R_{32}(r) \propto r^2e^{-r/3a_0}$

for $Z$ — replace $r \rightarrow Zr$ — Thus Helium $2p \rightarrow \propto re^{-r/a_0}$

$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\psi_{z+} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\psi_{z-} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\psi_{x+} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\psi_{x-} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\psi_{y+} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$ $\psi_{y-} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

$(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) = (\vec{a}\cdot\vec{b})I + i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

$[\vec{a}\cdot\vec{\sigma}, \vec{b}\cdot\vec{\sigma}] = 2i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

$\sigma_x\sigma_y\sigma_z = iI = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

$(\hat{n}\cdot\vec{\sigma})^{2p} = I$

$(\hat{n}\cdot\sigma)^{2q+1} = \hat{n}\cdot\vec{\sigma}$

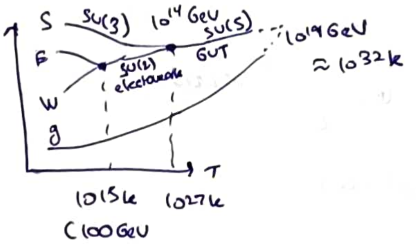
$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

$\cos x = 1 - x^2/2! + x^4/4! - \frac{x^6}{6!} + \cdots$

$\sin x = x - x^3/3! + x^5/5! + \cdots$

$e^{ia(\hat{n}\cdot\vec{\sigma})} = I\cos a + i(\hat{n}\cdot\vec{\sigma})\sin a$

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$




$10^{19}$ GeV $\approx 10^{32}$K

Transformer: $\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$

since $V_s = N_s\frac{d\phi}{dt}$ — divide Both.

$V_p = N_p\frac{d\phi}{dt}$

Euler angles: $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A = BCD$ (spin, Nutation, precession)

MM expt: $l = 11$m, $v = 3\times10^4$ m/s

$\Delta t = \frac{lv^2}{c^3}$ ; $\Delta x = c\Delta t = \frac{lv^2}{c^2}$

Now rotate 90°: $\frac{2lv^2}{c^2} = 2200$ A° (0.4 of $\lambda$ yellow fringe)

Blooming (anti-reflection glass)

$n_f = \sqrt{n_a n_g}$ → air / film / glass

Holography : object wave : $a(x,y)\cos[\phi(x,y)-\omega t]$

reference wave : $A\cos(kx-\omega t)$
reconstruction wave : $A\cos(kx-\omega t)$

recording : $O(x,y)$ + reference wave

$I = \langle y^2\rangle$

reconstruction : $I(x,y)\times R$

$V = \frac{2\pi}{\lambda} a\sin i_m = \frac{2\pi}{\lambda} a\sqrt{n_1^2-n_2^2}$

N = total no. of modes $\approx \frac{q}{2(2+q)}V^2$ — $\frac{V^2}{2}$ for large q (step-index)
$\frac{V^2}{4}$ for $q=2$ (parabolic)

p = radiation pressure : $U = \frac{I}{c}$ ~~xxx~~

($p = \frac{U}{3}$ for enclosure)

uncertainty : $\sigma_A^2 = \langle(\hat{A}-\langle A\rangle)\psi|(\hat{A}-\langle A\rangle)\psi\rangle = \langle f|f\rangle$

$f = (\hat{A}-\langle A\rangle)$

$\sigma_A^2\sigma_B^2 = \langle f|f\rangle\langle g|g\rangle \geq |\langle f|g\rangle|^2$

$|z|^2 \geq Re^2 + Im^2 \geq Im^2 \geq \left(\frac{1}{2i}(z-\bar{z})\right)^2$

$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}(\langle f|g\rangle - \langle g|f\rangle)\right)^2$

$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle\right)^2$ — eg: $[x,p_x] = i\hbar$

$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} a(p)\, e^{\frac{i}{\hbar}px}\, dp$ ~ $\psi(x)$ on LHS has +ve eikx on RHS

$a(p) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\psi(x)\, e^{-\frac{ipx}{\hbar}}\, dx$

Rydberg constant $R_M = \frac{R_\infty}{1+\frac{m}{M}}$ — m - mass of $e^-$, M - mass of Nucleus

$R_H = 109677\ cm^{-1}$ (not $R_\infty$!)

classical Raman — $\alpha_v = \alpha_{0v} + \alpha_{iv}\sin(2\pi\nu_{osc}t)$
— $\alpha_r = \alpha_{0r} + \alpha_{ir}\sin(2\pi(2\nu_{rot})t)$

$\vec{p} = \alpha\vec{E} = \alpha E_0\sin 2\pi\nu t = E_0\alpha_{0v}\sin 2\pi\nu t + \frac{1}{2}\alpha_{iv}E_0[\cos(2\pi(\nu-\nu_{osc})t) - \cos(2\pi(\nu+\nu_{osc})t)]$

$J_{max} = \sqrt{\frac{kT}{2B}} - \frac{1}{2}$ where $\frac{N_J}{N_0} = (2J+1)e^{-\frac{BJ(J+1)}{kT}}$

strong reaction (production of strange)

$\pi^+ + n^0 \to \Lambda^0 + k^+$
$\pi^- + p^+ \to \Lambda^0 + k^0$
$\pi^- + p^+ \to \Sigma^0 + k^0$
$\pi^- + p^+ \to \Sigma^- + k^+$

weak (decay of strange)

$\Lambda^0 \to p + \pi^-$
$\Lambda^0 \to n + \pi^0$
$\Sigma^+ \to p + \pi^0$
$\Sigma^+ \to n + \pi^+$
$\Sigma^- \to n + \pi^-$

(meson have spin 0, can fit in)

PSB - pg 41

LR DIP — Laue ($\theta$ diff, $\lambda$ same), rotating crystal ($\lambda$ same, $\theta$ diff), debye/powder ($\lambda$ same, $\theta$ diff, d diff)

GWS — Glashow Weinberg Salam

ASC ~ Anderson ($e^+$)
segre & chamberlain ($\bar{p}$)

$a_1 : 14.1$ $a_4 : 19.1$ (units of MeV/c²)
$a_2 : 13$ $a_5 : 34$
$a_3 : 0.6$
$n \sim 940$ MeV/c²
$p \sim 938$ MeV/c²

Lamb shift ant $n=2 \to 0.0353\ cm^{-1}$
$n=3 \to 0.0105\ cm^{-1}$

Davisson Germer : $E = 54eV \sim \lambda = 1.66\ A^\circ$
$\phi = 50^\circ \to \theta = 65^\circ$
$d = 0.915\ A^\circ$

stern Gerlach : use $v = \sqrt{\frac{4kT}{m}}$ (4 dof)
(thermal neutron ~~xxx~~)(thermal n means non-relativistic)
we set $g_s m_s = \pm 1$ → Thus $m_s = \pm 1/2$

Mossbauer : $Fe^{57} \to 14.4$ keV
$Ir^{191}_{77} \to 129$ keV

NMR : $g_p$ : ~~xxx~~ 5.56

13

$|S|_{scattering} = \frac{k'}{4\pi\epsilon_0} \frac{b^2\omega^4}{r^2}(1+\cos^2\theta)$ ; $\left(|S| \propto \frac{\omega^4}{r^2}(1+\cos^2\theta)\right)$

$\propto \frac{1}{\lambda^4}$ ~ since $\omega^4 \propto \frac{1}{\lambda^4}$ ; logical $S \propto \frac{1}{r^2}$ $\left(\frac{1}{4\pi r^2}\right)$

## Focault's Pendulum

$T = 2\pi\sqrt{\frac{l}{g'}}$ ~ $g' = g - \omega^2 R\cos^2\lambda$ *

$T_{precession} = \frac{2\pi}{\omega\sin\lambda}$

$B = \mu_0(H+M)$

Thus $H^{\perp}_{above} - H^{\perp}_{below} = -(M^{\perp}_{above} - M^{\perp}_{below})$

1) $B_{\perp}^{above} = B_{\perp}^{below}$ ~ since $\nabla\cdot B = 0$

2) $B''_{above} - B''_{below} = \mu_0 k$ ~ since $\oint B\cdot dl = \mu_0 I$

$H''_{above} - H''_{below} = \vec{k}_f \times \hat{n}$

surface current ; Normal to surface

$\vec{B}_{above} - \vec{B}_{below} = \mu_0(\vec{k}\times\hat{n})$

(Both 1 & 2 are coupled in this)

$S = Nk_B\ln Z + \frac{U}{T}$

$F = -Nk_BT\ln Z$

$U = Nk_BT^2\frac{\partial}{\partial T}(\ln Z)$

DAGAR FLASH on this pg

ideal gas : $Z = \frac{V}{h^3}(2\pi mk_BT)^{3/2}$

$\vec{J}_{bound} = \nabla\times\vec{M}$ $\quad (\rho_b = -\nabla\cdot\vec{P})$

$\vec{k}_{bound} = \vec{M}\times\hat{n}$ $\quad (\sigma_b = \vec{P}\cdot\hat{n})$

$\psi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(k)e^{ikx}dk = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}F(p)e^{\frac{ipx}{\hbar}}dp$

(replace $\rho$ by $\vec{J}$, $\sigma$ by $\vec{k}$ and $\cdot \to \times$ & all +ve signs)

volume current density ; surface current density

$p$ ~~&~~ $k = p/\hbar$

$[x^n, p_x] = ni\hbar x^{n-1}$

$[x, p_x^n] = ni\hbar p_x^{n-1}$

$H_n(y) = 1, y, (2y^2-1)$ (Hermite)

also $\nabla\times H = J$ (since $\nabla\times B = \mu_0 J$)

Radial Nodes = $n-l-1$

angular nodes = $l$

Total nodes = $n-1$

$Q = \frac{1}{e}\int(3\cos^2\theta - 1)r^2\rho(r)d\tau$

ostwald : $d_{hkl} = \frac{2\pi}{|G|}$

1 Becquerel = 1 disintegr/sec

1 Rutherford = $10^6$ " "

1 curie = $3.7\times10^{10}$ " "

$a_i\cdot b_j = 2\pi\delta_{ij}$

primitive vector of lattice ; primitive vector of reciprocal

21 cm line ~ $T_{1/2} = 10^7$ years

Mossbauer : $E_{recoil} = \frac{p^2}{2M} = \frac{E_\gamma^2}{2Mc^2}$ (since $p_\gamma = \frac{E_\gamma}{c}$)

$e^{i(\vec{G}\cdot\vec{T})} = 1$

reciprocal lattice vector ; normal lattice vector

## NMR proton

$\nu_p = \frac{\mu_N g B}{h} = g_p\frac{eB}{4\pi m_p}$

and $\Delta E = \frac{\hbar}{\Delta t}$

~~for resonance & Mossbauer absorption)~~

for $e^-$ orbital motion : (cyclotron)

$e v B = m\omega^2 R$ ) cancel ($v = \omega R$)

$\omega = 2\pi\nu = \frac{eB}{m_e}$

Thus $\nu_c = \frac{eB}{2\pi m_e}$ ~ take extra $\frac{1}{2}$

$\nu_c = \frac{eB}{4\pi m_e}$

Thus $g_p = \frac{\nu_p}{\nu_c}\frac{m_p}{m_e}$

HCP primitive vector:

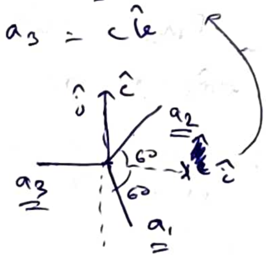
$a_1 = \frac{a}{2}(\hat{i} - \sqrt{3}\hat{j}) = a\cos60\hat{i} + a\sin60\hat{j}$

$a_2 = \frac{a}{2}(\hat{i} + \sqrt{3}\hat{j})$

$a_3 = c\hat{k}$

isotope effect : $T_c \propto \Theta_D \propto \frac{1}{\sqrt{M}}$

superconductor critical Temp



$W = \tau\theta$

$\phi = \frac{\tau L}{GJ}$ — Torque

$J = \frac{\pi R^4}{2}$

$G$ — modulus of rigidity (shear modulus)

shift $= L \times \phi$

$\eta$ - intrinsic impedance $= \dfrac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$

$\nabla^2 E + \nabla\left(\frac{1}{n^2}\nabla(n^2)\cdot \vec{E}\right) - \mu_0\epsilon_0 n^2 \frac{\partial^2 E}{\partial t^2} = 0$

$F_{\parallel}' = F_{\parallel}, \quad F_{\perp}' = \gamma F_{\perp}$ (use inverse)

$E_{\parallel}' = E_{\parallel}$

$B_{\parallel}' = B_{\parallel}$ — like $F = q(\vec{v}\times\vec{B})$

$E_{\perp}' = \gamma(E_{\perp} + \vec{v}\times\vec{B})$

$B_{\perp}' = \gamma\left(B_{\perp} - \frac{\vec{v}\times\vec{E}}{c^2}\right)$

put $v = v\hat{z}$ to get all 3 components

$\hat{n}\cdot\vec{S} = \frac{\hbar}{2}\begin{bmatrix}\cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta\end{bmatrix}$

This can be easily derived using $\hat{n}$ in $(r,\theta,\phi)$ & $\sigma_x, \sigma_y, \sigma_z$

Now use $(a,b)$ and $\lambda = \pm\hbar/2$ to get:

✓ $|\psi_{+1/2}\rangle = \cos\frac{\theta}{2}|\chi_{1/2}\rangle + e^{i\phi}\sin\frac{\theta}{2}|\chi_{-1/2}\rangle$

✓ $|\psi_{-1/2}\rangle = -\sin\frac{\theta}{2}|\chi_{1/2}\rangle + e^{i\phi}\cos\frac{\theta}{2}|\chi_{-1/2}\rangle$

★ first $\chi_{1/2}$, then $\chi_{-1/2}$ — even in $\langle S\rangle$ — 1st row $+\,+$ $\;+\,-$

- $e^{+i\phi}$ in both and in 2nd term both (unlike $\hat{n}\cdot\vec{S}$ where one is $e^{-i\phi}$ and diagonal opp.)
- $\begin{pmatrix}\cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}$ — similar to Euler angle ⟨but take $\frac{\theta}{2}$ !!!⟩

$\delta = \sigma_A - \sigma_B$ = chemical shift $= \frac{B_A - B_B}{B_0} = \frac{\Delta\nu}{\nu} = \frac{\nu_{sample} - \nu_{reference}}{\nu_{reference}} \times 10^6$

ESR — $E_{m_S m_I} = \mu_B B g_S m_S + a\, m_S m_I$

$\Delta m_S = \pm 1, \quad \Delta m_I = 0$

---

$\vec{\mu} = \mu_p\sigma_p + \mu_n\sigma_n + \frac{L}{2}$

$\sigma_p + \sigma_n = 2S = 2(I - L)$

$\sigma_p - \sigma_n = 0$

$\vec{\mu} = (\mu_p + \sigma_p)I - (\mu_n + \mu_p - \frac{1}{2})L$ — since $(\mu_n - \mu_p)$ term goes to zero since $\sigma_n - \sigma_p = 0$

Now $\langle I\rangle = 1$, $\langle L\rangle = \frac{L(L+1)}{4}$ — $\frac{L\cdot I}{I^2} I_z$

$I = 1^+$ for deuteron

$l = 0$ — $\langle L\rangle = 0$

$l = 2$ — $\langle L\rangle = 3/2$

$\mu_d = (\mu_n + \mu_p) - \frac{3}{2}P_D(\mu_n + \mu_p - \frac{1}{2})$

$\mu_{quark} = \frac{e_q \hbar}{2m_q}$ — $e_q = +\frac{2}{3}e$ or $-\frac{1}{3}e$

$\mu_{proton}$ (uud) $= \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$
$= 3x$

$\mu_{neutron}$ (udd) $= \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$
$= -2x$

$V(r) = V_1(r) + V_2(r)\,\bar{\sigma}_1\cdot\bar{\sigma}_2 + V_3(r)\hat{S}_{12}$

Nuclear potential — central force; spin dependent; Tensor force (non-central); Tensor force operator

triplet: $\sigma_1\cdot\sigma_2 = 1$

singlet: $\sigma_1\cdot\sigma_2 = -3$

in singlet state

$\hat{S}_{12} = \frac{3(\bar{\sigma}_1\cdot\bar{r})(\bar{\sigma}_2\cdot\bar{r})}{r^2} - (\sigma_1\cdot\sigma_2)$

$= -3 - (-3)$

$= 0$

Thus in singlet state: Nuclear force are central force

---

$E = (\Lambda + \frac{3}{2})\hbar\omega$

$\Lambda = 2n + l - 2$

$N_\Lambda = (\Lambda+1)(\Lambda+2)$

$N = \frac{1}{3}(\Lambda+1)(\Lambda+2)(\Lambda+3)$

$Q_{proton} = -\frac{3}{5}\left|\frac{2I-1}{2I+2}\right| R_0^2 A^{2/3}$

$Q_{single\ Nucleon} = \frac{Z}{A^2} Q_{proton}$

Q-value $\beta^+$

$[\underbrace{m(Z,A)}_{reactant} - (\underbrace{m(A,Z-1) + 2m_e}_{product})]$

$\langle V_{ls}\rangle = -\phi(r)\,\bar{l}\cdot\bar{s}$

$j = l + \frac{1}{2}$ : $E = -\frac{1}{2}l\langle\phi(r)\rangle$

$j = l - \frac{1}{2}$ : $E = \frac{l+1}{2}\langle\phi(r)\rangle$

$\Delta E = (l + \frac{1}{2})\langle\phi(r)\rangle$
$= 10(2l+1)A^{-2/3}$ MeV

Schmidt: $\mu_l = \sqrt{l(l+1)}\hbar$

$\mu = \frac{\mu_N}{\hbar}(\mu_l g_l + \mu_s g_s)$

$= \left(g_l\sqrt{l(l+1)}\cos(\hat{l}\cdot\hat{j}) + \ldots\right)\mu_N$

$= g_l\frac{(j(j+1) + l(l+1) - s(s+1))}{2\sqrt{j(j+1)}} + \ldots$

Now $j = I = l + \frac{1}{2}$ (stretch) & $j = I = l - \frac{1}{2}$ (jackknife)

write $l$ in terms of $I$ or $j$ ($I = j$) (Mug ★★) (cannot be derived)

from: $\mu\langle I\rangle$

$\mu = \left[(j - \frac{1}{2})g_l + \frac{1}{2}g_s\right]\mu_N$ (stretch)

$\mu = \left[\left(\frac{j}{j+1}\right)\left[(j + \frac{3}{2})g_l - \frac{g_s}{2}\right]\right]\mu_N$ (jackknife)

$I = 1/2$ for neutron/proton

small word: small equation thus (stretch is smaller than jackknife)

Now for $\hbar\omega = 41A^{-1/3}$ MeV ✓

$\frac{1}{2}m\omega^2 r^2 = \frac{1}{2}(\Lambda + \frac{3}{2})\hbar\omega$

get value of $r^2$

Now $\langle R^2\rangle = \frac{2}{A}\sum N_\Lambda\langle r^2\rangle \to \frac{2}{A}\frac{\hbar}{m\omega}\times\frac{1}{4}(\Lambda+2)^4$

use this $\omega$ to get $\hbar\omega = \frac{1}{A}\cdot\frac{\hbar^2}{m\langle R^2\rangle}\cdot\frac{1}{4}(\Lambda+2)^4$

Now write $(\Lambda+2)$ in terms of $A$ using $A = 2N = 2\times\frac{1}{3}(\Lambda+2)^3$

and put $\langle R^2\rangle = \frac{3}{5}R_0^2 A^{2/3}$ (don't miss 3/5)

Thus $\hbar\omega = 41A^{-1/3}$ $\boxed{\text{MeV}}$

✓ stretch (small word) — small eqn ✓

Jackknife (big word) — big eqn

Now $(j - \frac{1}{2})g_l + \frac{1}{2}g_s$

- $l$ first, then $s$ — just like ls sequence
- in next, invert $+\frac{1}{2}g_s \sim -\frac{1}{2}g_s$ & put $(j + 3/2)g_l$ ★
- don't forget $\mu_N$ outside.

Now $j - \frac{1}{2} + \frac{1}{2} = j$ //

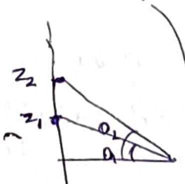
& $\frac{j}{j+1}(j + \frac{3}{2} - \frac{1}{2}) = \frac{j}{j+1}(j+1) = j$ //

$3N = 4\pi V\left(\frac{1}{v_l^3}+\frac{2}{v_t^3}\right) \times \frac{v_m^3}{3}$

1) put $v_{sound} = v_l = v_t$
2) $v_m = \frac{k\Theta_D}{h}$
3) and $\frac{N}{V} = n = \frac{\rho}{m}$ ($\rho = mn$)

Thus $\left(\frac{\Theta_{D_1}}{\Theta_{D_2}}\right) = \left(\frac{v_{s_1}}{v_{s_2}}\right)\left(\frac{\rho_1}{\rho_2}\cdot\frac{M_2}{M_1}\right)^{1/3}$

$A = \frac{\mu_0}{4\pi}\int \frac{I\,dl}{r} \rightarrow$ Finite wire $\Rightarrow \frac{\mu_0 I}{4\pi}\ln\left(\frac{z_2+\sqrt{r^2+z_2^2}}{z_1+\sqrt{r^2+z_1^2}}\right)$




$\rightarrow$ infinite wire $\Rightarrow A = -\frac{\mu_0 I}{2\pi}\ln r$

since $\nabla\times A = B = \frac{\mu_0 I}{2\pi r}$ (infinite wire $\vec{B}$)

$-\frac{\partial A_z}{\partial r}$

$n_{film} = \sqrt{n_a n_g}$ (Blooming ~ reducing reflectivity)

$P = u = \frac{I}{c}$ - radiation pressure ($P = \frac{u}{3}$ in cavity/enclosure)

Ruby : 6943 A°

He-Ne : 6328 A° - 20.66 eV & 18.70 eV

$(\bar{a}\cdot\bar{\sigma})(\bar{b}\cdot\bar{\sigma}) = (\bar{a}\cdot\bar{b})I + i(\bar{a}\times\bar{b})\cdot\vec{\sigma}$

$e^{ia(\hat{n}\cdot\vec{\sigma})} = I\cos a + i(\hat{n}\cdot\vec{\sigma})\sin a$

$\varepsilon_p = -N_p\frac{\partial\phi}{\partial t}$, $\varepsilon_s = -N_s\frac{\partial\phi}{\partial t}$ $\quad \frac{V_P}{V_S} = \frac{N_P}{N_S}$

$n = N_c e^{-\frac{(E_c - E_F)}{kT}}$

$N_d^+ = N_d e^{-\frac{(E_F - E_D)}{kT}}$

$E_c$
$E_F$ — $E_F$ b/w the levels always
$E_{donor}$
$E_V$

$V_B = \frac{kT}{e}\ln\frac{N_d N_a}{n_i^2}$

$\frac{D}{\mu} = \frac{kT}{e} = \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p}$ (Einstein reln)

$J_n = e n \mu_n \mathcal{E} + e D_n \frac{dn}{dx}$

$J_p = e p \mu_p \mathcal{E} - e D_p \frac{dp}{dx}$

$B_v = B_e - \alpha_e(v+\frac{1}{2}) + \ldots$

Dia : $\mu = I \times \pi\langle\rho^2\rangle = (IA)$

$\mu = I\times A$

e x rev per second

$\mu = e\times\frac{1}{2\pi}\frac{eB}{2m}\times\pi\langle\rho^2\rangle$

just add an extra $1/2$

(we also add extra $1/2$ in Thomas precession, in $\nu_c$ of cyclotron & here for rev-per second

same as $\nu_c$ $evB = m\omega^2 r$ then add extra $1/2$ to $\nu_c$.

$\mu = \frac{e^2 B}{4m}\langle\rho^2\rangle$

now $x^2+y^2 = \rho^2$
$x^2+y^2+z^2 = r^2$

Z is no. of $e^-$ per atom $\quad \mu = \frac{Ze^2B}{4m}\times\frac{2}{3}\langle r^2\rangle$

put $B = \mu_0 H$ & ~~[illegible]~~

$M = \mu\times N$ and $\chi = \frac{M}{H}$ to get:

$\chi_{dia} = -N\frac{\mu_0 Ze^2}{6m}\langle r^2\rangle$ — Z is atomic number. in Both, N is number density (number per unit volume)

$\boxed{\mu_0 N}$ in Both

$\rightarrow Ze^2\langle r^2\rangle$

$\rightarrow \mu_B^2 g^2 J(J+1) = \mu_B^2 P_{eff}^2$

one has $1/6m$ other $1/3kT$

$\chi_{para} = \frac{\mu_0 N P_{eff}^2 \mu_B^2}{3kT} = \frac{C}{T}$ where $P_{eff} = g\sqrt{J(J+1)}$

same C for $\chi_{Ferro} = \frac{C}{T-T_c}$ & $\chi_{AF} = \frac{C}{T+\Theta}$

Proof of para

$M = N\times\frac{\sum_{m_j=-J}^{+J}(\mu_B g m_j)e^{-\mu_B B g_j m_j/kT}}{\sum_{m_j=-J}^{+J}e^{-\mu_B B g_j m_j/kT}}$

now $e^{-x}\approx 1-x$, $\sum_{-J}^{+J} m_j = 0$, $\sum_0^n n_i^2 = \frac{n(n+1)(2n+1)}{6}$

we get $M = N\times\frac{g^2\mu_B^2 B}{3kT}J(J+1)$ — No need to show Math (we know the final result)

$\chi_{para} = \frac{\mu_0 M}{B} = \frac{\mu_0 N\mu_B^2}{3kT}g^2 J(J+1) = \frac{C}{T}$

Hund : $J = L-S$ (less than half filled)
$J = L+S$ (more than half filled)
$J = S$ ($L=0$) - half filled

Ferro : $B_E = \lambda M$ — exchange field; weiss field constant

---

$A = \frac{1}{2}(\bar{B}\times\bar{r})$ and $\phi = -\vec{E}\cdot\vec{r}$

$\nabla\times(\bar{a}\times\bar{b}) = \underbrace{\bar{a}(\nabla\cdot\bar{b}) - \bar{b}(\nabla\cdot\bar{a})}_{\text{normal}} + \underbrace{(\bar{b}\cdot\nabla)\bar{a} - (\bar{a}\cdot\nabla)\bar{b}}_{\text{last 2 opp. \& remember } \nabla \text{ in centre for all 4}}$

Non-Homogeneous pair of Maxwell eqn : $\sum_{\nu=1}^{4} \frac{\partial F_{\mu\nu}}{\partial x_\nu} = \mu_0 J_\mu$ — since RHS $\neq 0$; ② inhomogeneous: $\nabla\times B = \mu_0 J + \mu_0\epsilon_0 \frac{\partial E}{\partial t}$, $\nabla\cdot E = \rho/\epsilon_0$; just like $\Box^2 A_\mu = -\mu_0 J_\mu$

Homogeneous : $\frac{\partial F_{\lambda\mu}}{\partial x_\nu} + \frac{\partial F_{\mu\nu}}{\partial x_\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x_\mu} = 0$

RHS = 0

Clubbing ② Homogeneous: $\nabla\cdot B = 0$, $\nabla\times E = -\frac{\partial B}{\partial t}$

Lorentz Transform of $\vec{E}$ & $\vec{B}$ : use $B' = \nabla\times A'$ & $E' = -\nabla\phi' - \frac{\partial A'}{\partial t}$ — we had used $B = \nabla\times A$ & $E = -\nabla\phi - \partial A/\partial t$ to find $F_{\mu\nu}$ also

Now $B_x' = \frac{\partial A_3'}{\partial x_2'} - \frac{\partial A_2'}{\partial x_3'}$ and $E_x' = -\frac{\partial A_1'}{\partial t'} - \frac{\partial \phi'}{\partial x_1'}$

where $\frac{\partial}{\partial x_1'} = \alpha\frac{\partial}{\partial x_1} + \frac{\alpha v}{c^2}\frac{\partial}{\partial t}$ and $A_{\mu\nu}' = a_{\mu\nu}A_{\mu\nu}$

Thus $A_1' = \alpha A_1 + i\alpha\beta(A_4)$
$= \alpha A_1 + i\alpha\beta\left(\frac{i\phi}{c}\right)$

we can derive...

find ~ $B_x = \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}$ and $E_x = \frac{\partial A_1}{\partial t} - \frac{\partial \phi}{\partial x_1}$

Lorentz Force on charged particle.

$F_x' = F_x$, $F_y' = \gamma F_y$ also $F_x' = qE_x'$, $F_y' = qE_y'$ ...

Thus $F_x = F_x' = qE_x'$, $F_y = \sqrt{1-\frac{v^2}{c^2}}\,F_y' = qE_y'\sqrt{1-\frac{v^2}{c^2}}$ ...

also $E_x' = E_x$, $E_y' = \frac{E_y - vB_z}{\sqrt{1-v^2/c^2}}$, $E_z' = \frac{E_z + vB_y}{\sqrt{1-v^2/c^2}}$

Thus $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

take F on LHS. put F' on RHS, F' ~ E' on RHS, then E' ~ E & B on RHS

(i) $c^2B^2 - E^2$
(ii) $\bar{E}\cdot\bar{B}$
} Lorentz invariant ~ proof using E' & B' just like normal proofs

DIAS (2021) NPP Test 3 Q2(c)
Verification of General theory of relativity (Moessbauer effect)

$\gamma$-source height $d = 25m$, $m_1(\text{excited}) = \frac{E_1}{c^2}$, $m_0(\text{ground}) = \frac{E_0}{c^2}$ & P.E = mgd

Energy of $\gamma$-photon emitted = $E_1 + m_1gd - (E_0 + m_0gd)$ $= E_1\left(1+\frac{gd}{c^2}\right) - E_0\left(1+\frac{gd}{c^2}\right)$
($h\nu$)
$= \Delta E\left(1+\frac{gd}{c^2}\right)$

let $\Delta E = h\nu_0$. Thus $h\nu_{new} = h\nu_0\left(1+\frac{gd}{c^2}\right)$

Then shift = $\nu_{new} - \nu_0 = \frac{\nu_0 gd}{c^2}$ (shift in energy of photon)
(Red shift)

Now doppler shift Balance red shift.

Doppler shift : $\nu' = \nu_0\sqrt{\frac{1+v/c}{1-v/c}} \approx \nu_0\left(1+\frac{v}{c}\right)$ ~ shift $\nu' - \nu_0 = \frac{\nu_0 v}{c}$

Doppler shift = red shift → $\frac{\nu_0 gd}{c^2} = \frac{\nu_0 v}{c}$ — $v = \frac{gd}{c^2}$ — gravity = 9.8, d = 25m

Yukawa potential : $V = -Ce^{-r/R}/r$ 16

$T + 2T\cos\theta = mT$

$\cos\theta = \frac{m-1}{2}$ & $\theta = \frac{2\pi}{n}$

$|\cos\theta| \le 1$ ~ $m = 3, 2, 1, 0, -1$

L no 5 or 7 fold rotation

| structure | close packed plane | close packed direction |
|---|---|---|
| SC | — | <100> |
| BCC | {110} | <111> |
| FCC | {111} | <110> |

[ ] – line
< > – family of line
( ) – plane
{ } – family of plane
lit(edge) Bracket (linear)
curly

repeated   extended   reduced

(17)

$A \to B \to C$ : $\frac{dN_A}{dt} = \lambda_A N_A \to N_A = N_0 e^{-\lambda_A t}$

$\frac{dN_B}{dt} = \lambda_B N_B - \lambda_A N_A \to$ put $\lambda_A N_A = N_0 \lambda_A e^{-\lambda_A t}$

$\frac{dy}{dx} + Py = Q \to y e^{\int P dx} = \int Q e^{\int P dx} dx + C$

E is kE here
using $p = \sqrt{2mE}$
and $Q = kE_{prod} - kE_{re}$
$= E_y + E_Y - E_x$
since $E = kE$ here

kurie fn : $k = \left[\frac{N_p}{p^2 F}\right]$
(k)
momⁿ — Fermi fn — $N_p$ is transition probability

kurie plot : k vs E (energy)
kurie function

Q value (Q) : $E_y\left(1 + \frac{m_y}{M_Y}\right) - E_x\left(1 - \frac{m_x}{M_x}\right) - 2\frac{\sqrt{m_x m_y E_x E_y}}{M_Y}\cos\theta$

eliminate $\phi$ & $E_Y$

$x \to X$ (rest) $= \phi$

$(E_x)_{threshold} = \frac{1}{2}mv_x^2 = -Q\left(\frac{m_x + M_X}{M_X}\right)$ (use combined product)

For disintegration: daughter + $\alpha$-particle (rest) — $\alpha$ daughter

$Q_\alpha = k_\alpha\left(\frac{m_\alpha}{M_d} + 1\right)$ — daughter

$Q_\alpha = k_\alpha\left(\frac{A}{A-4}\right)$

$Q = kE_{pat} - 0 = kE_\alpha + kE_d$
(where $k_\alpha = \frac{1}{2}m_\alpha v_\alpha^2$)

**Geiger Nuttall law**

decay constant

$\log\lambda = a\log R + B$ (Range)

$\log\lambda = -a_1\log\frac{Z}{\sqrt{E}} + a_2$

$\log t_{1/2} = +a_3\log\frac{Z}{\sqrt{E}} + a_4$

~~For Zeeman~~:

NMR $= \nu_L = \frac{g\,\mu_N B g_J}{h}$ — $g = 5.56$ for proton & use $\mu_N$ ~ mass proton
$B = 0.1$ Tesla

$\frac{N_{-1/2}}{N_{+1/2}} = e^{-\Delta E/kT}$ — $m = +1/2$ more stable since $U = -\vec{M}\cdot\vec{B}$

Thus $\frac{N_{-1/2}}{N_{+1/2}} \approx 1 - \frac{\Delta E}{kT} \approx 1 - \frac{2\mu_p B}{kT} = 1 - \frac{h\nu_L}{kT}$ (since $h\nu_L = 2\mu_p B$)

$\mu_p = \mu_B g_J m_j$ ($m_j = \pm 1/2$)
$= \frac{1}{2}\mu_B g_J$

at $B = 0.1T$ and $T = 300K$ ~ factor diff in pm $\approx 10^{-7}$

$\nabla\cdot J_p + \frac{\partial}{\partial t}\rho_p = 0$ ; $\rho_p = -\nabla\cdot P$

Thus $J_p = \frac{\partial P}{\partial t}$ (polarization current)

$m = IA$ ~ $M = \frac{m}{V} \sim \frac{I}{l} \sim I = Ml$

$\delta I = \frac{\partial M_z}{\partial y}\delta y\,\delta z$ ~ $J_x = \frac{\delta I}{\delta y\,\delta z} = \frac{\partial M_z}{\partial y}$

$\vec{J} = \nabla\times\vec{M}$ — (magnetization currents)

$\vec{J}_{total} = \vec{J}_{true} + J_{polarization} + J_{magnetization}$
$= J_{true} + \frac{\partial\vec{P}}{\partial t} + \nabla\times\vec{M}$

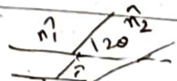
Clausius Mossotti : $E_m = E_0 + P/3\varepsilon_0$
magnetised sphere derivation gives $H = H_0 - \frac{M}{3}$

$f = \frac{hx}{a} + \frac{ky}{b} + \frac{lz}{c} = 1$

$\hat{n} = \frac{\vec{\nabla}f}{|\vec{\nabla}f|}$ — gradient is along Normal!

$d = \left(\frac{\vec{a}}{h}\cdot\hat{n}\right)$

Laue → $\Delta x = \hat{n}\cdot\vec{N}$



$\vec{N} = \hat{n}_1 - \hat{n}_2$ ~ $|N| = 2\sin\theta$

put $\frac{\vec{a}}{h}\cdot\hat{n} = d \to \frac{a}{h}\cos\alpha = d$

$\Delta\phi = \frac{2\pi}{\lambda}(\vec{a}\cdot\vec{N}) = \frac{2\pi}{\lambda}a|N|\cos\alpha = \frac{2\pi}{\lambda}a(2\sin\theta)\cos\alpha = 2n\pi h$

Thus $2d\sin\theta = n\lambda$

L — A(SC) – Anderson, Segre & Chamberlain
R
P/O — GSW – Glashow, Weinberg, Salam

$\sigma_{100} = \frac{1}{d_{100}}\hat{n}$ ; $a^* = 2\pi\left(\frac{b\times c}{a\cdot b\times c}\right)$

$(a^*\times b^*)\cdot c^* = \frac{[abc][abc]}{[abc]^3} = \frac{1}{[abc]} = \frac{1}{V}$

$e^{i\vec{G}\cdot\vec{T}} = 1$ — Translation, reciprocal vector

$G^2 + 2\vec{k}\cdot\vec{G} = 0$
vector form of Bragg law — wave vector — reciprocal vector

BCC : primitive : $a' = \frac{a}{2}(\hat{i} + \hat{j} - \hat{k})$

FCC : primitive : $a' = \frac{a}{2}(\hat{i} + \hat{j})$ → but reverse for FCC

$(-1,-1,-1)$
$(1,1,-1)$
$(-1,1,1)$
$(1,-1,1)$ ~ Bond angle $= \cos^{-1}(-1/3) = 109°28'$

Brillouin zones → use $2\vec{k}\cdot\vec{G} + G^2 = 0$ (vector form of Bragg law)

we set $a^* = b^* = \frac{2\pi}{a}$

1) and use $G = ha^* + kb^* + lc^*$ where h, k, l are integers and 2D

2) $a^* = 2\pi\frac{|b\times c|}{(a\cdot b\times c)}$ ... → $2\vec{k}\cdot\vec{G} + G^2 = 0$ → $\boxed{hk_x + kk_y = -\frac{\pi}{a}(h^2 + k^2)}$ — Just Mug this

3) $\vec{k} = k_x\hat{i} + k_y\hat{j}$ (don't take $k_z$ – only 2D)

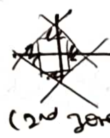
we get $hk_x + kk_y = -\left(\frac{\pi}{a}\right)(h^2 + k^2)$

+ 1st zone : $\left(\begin{matrix} h = \pm 1 \,\&\, k = 0 \\ k = \pm 1 \,\&\, h = 0\end{matrix}\right)$ → $k_x = \pm\pi/a$, → $k_y = \pm\pi/a$ (4 lines)

Brillouin zone is locus of h, k in $k_x$ vs $k_y$ space

(1st zone)

+ 2nd zone $(h = \pm 1 \,\&\, k = \pm 1)$ → $\pm k_x \pm k_y = \frac{2\pi}{a}$ (4 lines)



(2nd zone)

solid : $C \to 3R$, water : 1 cal gram⁻¹
gas $C_V = \frac{3}{2}R, \frac{5}{2}R$ ... etc

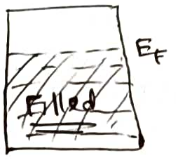
$C = C_{elec} + C_{lattice}$

Normal metal : $\gamma T$ — $\beta T^3$ for Both
Superconductor : $e^{-\Delta/k_BT}$

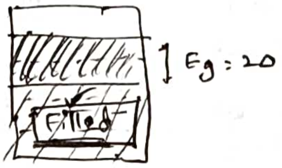
$E_g = 2\Delta$ ~ $\Delta$ ie energy gap — debye lattice specific heat

PTO →

$\Delta$ – energy gap – explained by BCS theory & cooper pair




normal metal




superconductor Band diagram


– similar to $H_c(T)$ graph *

energy gap at T = 0K ie $\Delta(0)$ *.

$T_c \propto \Theta_D \propto \frac{1}{\sqrt{M}}$ (isotope effect)

Inverted LS – High I (Nuclear) └ low energy

s p (d s) (f p) g d s h f p — only 2 overlap; No overlap for Next 6! *

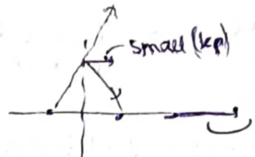
Hermitian operator – $\langle \psi_1 | A \psi_2 \rangle = \langle A\psi_1 | \psi_2 \rangle$

(and) $\langle \phi_1 | A | \phi_2 \rangle^* = \langle \phi_2 | A | \phi_1 \rangle$

for $-\frac{L}{2}$ to $+\frac{L}{2}$ – $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2n\pi x}{L}\right)$

– $\psi_n(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{(2n-1)\pi x}{L}\right)$

Dipole




FET – (V) – E-V, I

BJT – current (I)

FET – unipolar (only majority)
– voltage controlled device
– High input impedence
– High power gain
– thermal stability, less noisy, smaller, simple to fabricate

– superconductor: High $T_c$ (diff) semi-conductor
└ ceramic..
✓ └ (small) coherence length
✓ └ small Isotope Effect
└ liquid $N_2$

Same – Josephson Tunneling
└ energy Gap $\Delta$

MOSFET > JFET > BJT
highest input impedence, etc...

$E_r = \frac{2kp\cos\theta}{r^3}$ $\left(-\frac{\partial V}{\partial r}\right)$

$E_\theta = \frac{kp\sin\theta}{r^3}$ $\left(-\frac{1}{r}\frac{\partial V}{\partial \theta}\right)$

for $r = ke^{\alpha\theta}$ (spiral) – $F(u) = -\frac{J^2}{m}(1+\alpha^2)u^3 \propto \frac{1}{r^3}$

for $r = 2a\cos\theta$ – $F(u) \propto \frac{1}{r^5}$

total shift $\approx 2\Delta\theta = \frac{2v}{c}\sin\theta$ (Aberration)

$\lambda = \frac{2\pi\tau}{\sqrt{1-\tau^2}}$ – $\lambda = \ln e^{-cT} = cT = c \times \frac{2\pi}{\omega_d}$

now $\omega_d = \sqrt{\omega_0^2 - c^2}$

& $\tau = c/\omega_0$ – $c = \tau\omega_0$

Gabor – coaxial

Leith & upatnieks – LASER offset angle

$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x$ $\quad \int \frac{1}{1+x^2} = \tan^{-1}x$

$\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}x$ $\quad \int \frac{1}{x}dx = \log_e x$

$PV = RT(1-\frac{b}{V})^{-1} - \frac{a}{V}$ – $RT + \frac{RTb-a}{V} + \frac{RTb^2}{V^2} \ldots = \alpha + \frac{\beta}{V} + \frac{\gamma}{V^2} \ldots$

– $T_B = \frac{a}{Rb}$ (2nd virial coeff = 0) $\quad$ ($\alpha, \beta, \gamma$ are virial coefficients)

$\int a^x dx = \frac{a^x}{\log a}$ $\quad \int \tan x = \log|\sec x|$

$\int \sec x = \log|\sec + \tan|$

$\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$

$\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)$

$C_V = \frac{\partial E}{\partial T} = \frac{12}{5}\pi R^4\left(\frac{T}{\Theta_D}\right)^3$ – $C_V \propto \left(\frac{T}{\Theta_D}\right)^3$ at low temp (low temp)

$\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ $\quad \int \frac{dx}{\sqrt{x^2-a^2}} = \sin^{-1}\left(\frac{x}{a}\right)$

$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$ – $n = \frac{P}{kT}$ (since $PV = nRT$ (moles), thus $P = nkT$ (no. density))

$\int \frac{1}{\sqrt{a^2+x^2}} = \log|x + \sqrt{x^2+a^2}|$ $\quad \int \frac{1}{\sqrt{x^2-a^2}} = \log|x + \sqrt{x^2-a^2}|$

$\bar{T}$ ie virial of clausius (Ironic: Not $\bar{V}$!)

virial of clausius $\bar{T} = -\frac{1}{2}\sum \bar{F}\cdot\bar{r}$

for $F = kr^n$ – $\bar{T} = \frac{n+1}{2}\bar{V}$ (for $n = -2$ → $2\bar{T} + \bar{V} = 0$)

virial theorem: $2\bar{T} + \sum \bar{F}\cdot\bar{r} = 0$

$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x + \sqrt{x^2-a^2}|$

$\int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x + \sqrt{x^2+a^2}|$

$\int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

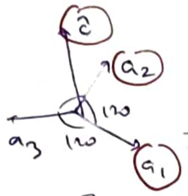
$\Delta E \approx \frac{\hbar}{2\Delta t}$ → $\Delta\nu = \frac{1}{4\pi\Delta t}$ (Natural width of spectral lines)

$\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$, $\tanh x = \frac{\sinh x}{\cosh x}$

From SPEM & Aniket sir, $S_{se} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{\omega^4}{8\pi c^3}|P_0|^2\sin^2\theta$ – now $\overline{\sin^2\theta} = \frac{1}{2}(1+\cos^2\theta)$ → $S \propto \omega^4 \propto \frac{1}{\lambda^4}$

$|P_0|^2 = \frac{(e^2E_0/m)^2}{(\omega_0^2-\omega^2)^2 + 4c^2\omega^2}$ $\left(\text{since } \bar{p} = er = \frac{\frac{e^2}{m}E_0 e^{i(\bar{k}\cdot\bar{r}-\omega t)}}{(\omega_0^2-\omega^2) - 2c\omega j}\right)$

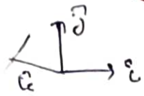
TEM → 2D, High Mag. (50 million x), All phases, Topography & internal structure. [ (TEM only 1 drawback → Tedious & complex preparation) ] (19)
SEM — 3D, low mag (2 million x), only solid, " & composition properties, low effect
BSE & 2nd ary




Planes (hkil)

make planes using $\underline{a_1}, \underline{a_2}$ and $\underline{c}$

(for $a_3$, just put $i = -(h+k)$)




vector is along normal to plane ★ — $a_1, a_2$ & c using normal to plane

$hkl \to hkil$

HCP – plane – easy – hkil where $i = -(h+k)$

$\cos\phi = \dfrac{H_1 \cdot H_2}{|H_1||H_2|}$ — always correct use 4 — $(xyz) \to (hkil)$ — then dot product $h_1h_2 + k_1k_2 + i_1i_2 + l_1l_2$

always use "4" for plane.

Primitive vector of HCP

✓ $a_1 = \frac{a}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$
✓ $a_2 = \frac{a}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$ } mug

$c = c\hat{k}$ ✓

$V = \bar{a}_1 \cdot (\bar{a}_2 \times \bar{c}) = \frac{\sqrt{3}}{2}a^2c$

Reciprocal lattice

Now $b_1 = 2\pi \dfrac{a_2 \times c}{a_1 \cdot (a_2 \times c)} = \dfrac{4\pi}{\sqrt{3}a}\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{\hat{j}}{2}\right)$ — don't plot, it won't be exactly correct.

$= \dfrac{4\pi}{\sqrt{3}a}\left(+\frac{\sqrt{3}}{2}\hat{i} + \frac{\hat{j}}{2}\right)$

$= \dfrac{2\pi}{c}(\hat{k})$

Just Mug

Thus reciprocal of HCP is HCP given by rotating direct lattice by $\pi/6$ (30°) (2009 Q2 5(c))

and scaling $a \to \dfrac{4\pi}{\sqrt{3}a}$ & $c \to \dfrac{2\pi}{c}$

even (hkil) had no balls!!

dir$^n$: $u'v'w' \to uvtw$
(complex)

$u = \frac{1}{3}(2u' - v')$ — similar to Quark M but instead of $4/3$, we have $2/3$
$v = \frac{1}{3}(2v' - u')$
$t = -(u+v)$ ✓
$w = w'$ ✓

since $i = -(h+k)$ &
$t = -(u+v)$ on LHS,
$t = -(u+v)$ on LHS like t
then u, v on LHS
and not u' or v' on LHS

HCP → $c = \sqrt{\frac{8}{3}}a$ — if radius = r, then $a = b = 2r$

$c = \sqrt{\frac{8}{3}}a = 2\sqrt{\frac{8}{3}}r$

A.P.F = 74%

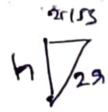
12 on edge – $12 \times 1/6 = 2$
3 in center – $3 \times 1 = 3$
2 on body face – $2 \times 1/2 = 1$
= 6

★ volume of atoms : $6 \times \frac{4}{3}\pi r^3$
★ volume of cell : Base × height $= 24\sqrt{2}r^3$

Thus APF = 74%


3 spheres in center of HCP ★

$h = \frac{2r\sqrt{3}}{2}$
centroid : 2:1
thus distance $= \frac{2}{3} \times \frac{2r\sqrt{3}}{2}$ — $\frac{2r}{\sqrt{3}}$




2r since touching!
$h = 2\sqrt{\frac{2}{3}}r$.

$P(|x| < a) = 0.84$

Thus $P(|x| > a) = 1 - \int_{-a}^{a}|\psi|^2$
$= 1 - 2\int_0^a |\psi|^2$

Now $y = \alpha x$ — $\left(\frac{m\omega}{\hbar}\right)^{1/2} x$

Now $\frac{1}{2}\hbar\omega = \frac{1}{2}m\omega^2 a^2$ — $a\alpha = 1$ ✓

ground state $E = \frac{1}{2}\hbar\omega$ ($E_0$) ($n=0$)

In $\psi$ — put $u = \alpha x$

$1 - \frac{2}{\sqrt{\pi}}\int_0^{\alpha x} e^{-u^2}\,du$

$= 1 - \frac{2}{\sqrt{\pi}}\int_0^1 1 - u^2 + \frac{u^4}{2!} - \frac{u^6}{3!} + \dots$

$= 1 - \frac{2}{\sqrt{\pi}}\left(1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \dots\right)$

$= 0.16$

H-atom

$-\frac{\hbar^2}{2m}\nabla^2\psi + \left(\frac{-e^2}{4\pi\varepsilon_0 r}\right)\psi = E\psi$

$\nabla^2 = -\frac{L^2}{\hbar^2 r^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)$

put $R(r) = u/r$ to get:

$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left(\frac{l(l+1)\hbar^2}{2mr^2} - \frac{e^2}{4\pi\varepsilon_0 r}\right)u = Eu$

★ $\rho = kr$ ($k = \sqrt{-2mE}/\hbar$)

$\frac{d^2u}{d\rho^2} = \left[1 - \frac{\gamma}{\rho} + \frac{l(l+1)}{\rho^2}\right]u$

$= \left[1 - \frac{\rho_0}{\rho} + \frac{l(l+1)}{\rho^2}\right]u$

take $u(\rho) = cv(\rho)e^{-\rho}\rho^{l+1}$

we get $\rho\frac{d^2v}{d\rho^2} + 2[\rho - (l+1)]\frac{\partial v}{\partial \rho} + [\rho_0 - 2(l+1)]v = 0$

↳ Laguerre D.E. ★

similar

Now $v(\rho) = \Sigma c_j \rho^j$

we get $\frac{c_{j+1}}{c_j} = \frac{\rho_0 - (2)(l+j+1)}{(j+1)[2(l+1)+j]}$

Thus $\rho_0 = 2(l+1+j) = 2(l+1+j)$
$= 2n$

we get $E_n = \frac{-13.6}{n^2}$ eV

Now $k \propto \sqrt{E} \propto \frac{1}{n}$

Thus let $k = \frac{1}{na_0}$ ★

$\rho = kr = r/na_0$

$R = \frac{u}{r} = \frac{u(\rho)}{r} = \frac{ce^{-\rho}\rho^{l+1}v_n^l(\rho)}{r}$

$= \frac{ce^{-r/na_0}}{r}\left(\frac{r}{na_0}\right)^{l+1}v_n^l\left(\frac{r}{na_0}\right)$

(2022 PYQ)

To solve: Radial D.E.

$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$ ($-\frac{e^2}{4\pi\varepsilon_0 r}$)

- put $\nabla^2$ in spherical and $\psi = R(r)Y(\theta,\phi)$
- multiply with $r^2$ and divide by RY
- ✓ split to r-terms and non-r-term
- we get:

$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2mr^2}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r}\right)$

$= -\frac{1}{Y}L^2 Y \times -\frac{1}{\hbar^2}$

$= l(l+1)$

★ now put: ! • $R = u(r)/r$ (last step)

240 V → RMS
$240\sqrt{2}$ V → peak

Sm = Siemens = (ohm)$^{-1}$

$\nabla \times (\bar a \times \bar b) = \bar a(\bar\nabla\cdot\bar b) - \bar b(\bar\nabla\cdot\bar a) + (b\cdot\nabla)a - (a\cdot\nabla)b$

Normal

$\nabla\cdot(\bar a \times \bar b) = \bar b\cdot(\bar\nabla \times \bar a) - \bar a\cdot(\bar\nabla\times\bar b)$

Now $L^2\psi = -\hbar^2(\bar r\times\bar\nabla)\cdot(\bar r\times\bar\nabla)\psi$

$-\hbar^2\bar r\cdot(\nabla\times(r\times\nabla\psi))$

$-\hbar^2\bar r\cdot(\nabla\times(r\times\nabla\psi))$

$= -\hbar^2\,\bar r\cdot[(\nabla\psi\cdot\nabla)r - (r\cdot\nabla)\nabla\psi - \nabla\psi(\nabla\cdot r) + r(\nabla\cdot\nabla\psi)]$

$= -\hbar^2\left[r\frac{\partial\psi}{\partial r} - r^2\frac{\partial^2\psi}{\partial r^2} - 3r\frac{\partial\psi}{\partial r} + r^2\nabla^2\psi\right]$

use $\nabla r = 1$, $\nabla\cdot r = 3$ (only 1 $\nabla^2$ term survive *)

$L^2 = -\hbar^2\left[r^2\nabla^2 - \frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\right]$ ✱

we know $n = N_c e^{-\frac{(E_c-E_F)}{kT}}$ — 1

for intrinsic, $E_F = E_i$

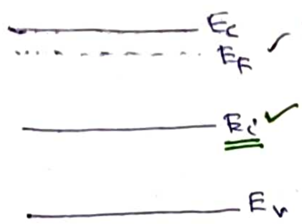
Thus $n_i = N_c e^{-\frac{(E_c-E_i)}{kT}}$ — 2

combining 1 & 2 ~ $\boxed{n = n_i e^{\frac{E_F-E_i}{kT}}}$ → For EXTRINSIC ($E_i \neq E_F$)

intrinsic $n_i$ level

$E_i$ is in center of $E_c$ & $E_v$

$E_F$ shifts when we dope.

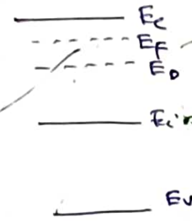



Thus $E_F > E_i$ & $n > n_i$ in N-type *

also
$p = n_i e^{\frac{(E_i-E_F)}{kT}}$ — for extrinsic
(since $\boxed{p\times n = n_i^2}$, we can easily get p ...)

for $E_F = \frac{E_d+E_c}{2}$ ~

$E_F$ b/w $E_D$ & $E_c$.




$E_F$ also in center of $E_c$ & $E_D$

$E_i$ in Absolute center

Now $N_d^+ = N_d e^{-\frac{(E_F-E_D)}{kT}}$

& $n = N_c e^{-\frac{(E_c-E_F)}{kT}}$

Thus $E_F = \frac{E_d+E_c}{2} + \frac{kT}{2}\ln\frac{N_d}{N_c}$

(equate n & $N_d^+$)

For QM → classical

$e^{\alpha-\beta E_i} \longrightarrow e^{-\alpha} \gg\gg 1 \rightarrow \boxed{\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2} \gg 1}$ (High Temp)

do $n(E) = \int f(E)g(E)\,dE$ where $f(E) = \frac{1}{e^{E/kT-\alpha}\pm 1}$

$\approx e^{\alpha - E/kT}$

$N = \int_0^\infty \frac{2\pi V}{h^3}(2m)^{3/2}E^{1/2}e^{\alpha-E/kT}\,dE$

Thus $e^\alpha = \frac{N}{V}\left(\frac{2\pi mkT}{h^2}\right)^{-3/2}$ (similar term as $n_i$ derivation in semi-conductor *)

Laguerre polynomials ~ $V_n^l(r)$ in our $R(r)$

(refer prev pg 19 bottom right)

always $e^{-x}$ on RHS of $\frac{d}{dx}$

$L_q(x) \equiv e^{x}\left(\frac{d}{dx}\right)^q e^{-x}x^q$ ~ Basically all cancels out...

$L_q^p(x) = (-1)^p\left(\frac{d}{dx}\right)^p L_{p+q}(x)$

Both 'p' added, already $\left(\frac{d}{dx}\right)^q$ used so 'p' must be next.

$\psi = R_n^l(r)\,Y_l^m(\theta,\phi)$

$R_n^l(r) = \frac{C e^{-r/na_0}}{r}\left(\frac{r}{na_0}\right)^{l+1} \times V_n^l(r/na_0)$

laguerre polynomial

U