HCS - 2021 ★

1(a) use $n_x^2 + n_y^2 + n_z^2 = n^2 = \left(\frac{2L}{\lambda}\right)^2$ ~ $\underset{(P.O.S)}{dN} = \frac{1}{8} \times 4\pi n^2 dn$ ⟶ $\frac{8\pi}{\lambda^4} d\lambda = \frac{8\pi \nu^2 d\nu}{c^3} \times V$

$\left(\frac{2L_x}{\lambda}\right)^2$ $\left(\frac{2L_y}{\lambda}\right)^2$ $\left(\frac{2L_z}{\lambda}\right)^2$

use $n^2 = \left(\frac{2L}{\lambda}\right)^2$

1(e) simple ~ diatomic - classical equipartition, 3T+2R ~ $C_v = \frac{5}{2}R$ (monoatomic only 3T - Thus $\frac{3}{2}R$) (No rotation)

Q2) Damn Tough → Chegg soln can see - V.V. Tough. using homework help

Q3) easy → use $V_{A/B} = \frac{V_A - V_B}{1 - \frac{V_A V_B}{c^2}}$ ~ L_B in B frame is L_0

find L_A in B frame using $V_{A/B}$ ~ $L_A = L_0\sqrt{1 - \frac{V_{A/B}^2}{c^2}}$

Q4(a) Easy ⟶ covariance of Maxwell's eqn : $\Box^2 A_\mu = -\mu_0 J_\mu$

Q4(b) ~ ? contravariant EM Tensor?

Q5(a) Easy

Q5(b) ~ angular harmonic express ?

Q6 ~ easy - use $F = \frac{\Delta p}{\Delta t}$ and $p = h/\lambda$ and $\Delta p = 2p$ ⇄ (reflection

~ absorbed photon contribute to temp rise

$\ln(1+x) \approx x$ as $x << 1$

Q7 ~ (a) $\Delta S = C_p \ln\frac{T_f}{T_i} - C_p\left(\frac{T_f - T_i}{T_f}\right)$

(b) Now as $T_f \to T + \Delta T$ ~ $\Delta S = C_p \ln\left(1 + \frac{dT}{T}\right) - C_p\left(\frac{dT}{T}\right) = C_p \ln\frac{dT}{T} - C_p \ln\frac{dT}{T} = 0$

$(\Delta T \to 0)$

Thus $\Delta S = 0$ for each of the tiny quasi-static cycle

(c) since $\Delta S \geq 0$ ~ equality for reversible

Q8 ~ Power = 0.5D = $\frac{1}{f}$. ~ Now $\frac{1}{f} = \left(\frac{\mu_{lens}}{\mu_{surr}} - 1\right)\left(\frac{1}{R} - \left(-\frac{1}{R}\right)\right)$ ~ if $\mu_{glass} = 1.5$ ~ $\frac{1}{f} = \frac{1}{R}$

$\frac{1}{f} = \frac{1}{R}$ and $\mu_{air} = 1$

Now $D_{n\,dark}^2 = 4n\lambda R/\mu_{air}$

for 10th dark ring → put n = 10 (n=0 dark point not a ring!)

(b) for water ($\mu = 1.33$) ~ calculate again reln of f & R

find R then ~~use~~ $D_n^2 = 4n\lambda R/\mu_{water}$ — $\mu = 1.33$

Q9(a) ~ Molecular orbital Theory ?

Q9(b) periodic: $\underline{\psi(x+2\pi R)=\psi(x)}$ (use ?) ~ let $\psi=e^{ikx}$ ~ $e^{ik(x+2\pi R)}=e^{ikx}$ ~ $e^{i2\pi Rk}=1=e^{i(2n\pi)}$

Thus $k=n/R$

Now $p\psi=-i\hbar\frac{\partial}{\partial x}e^{ikx}=\hbar k=\hbar n/R$ (momⁿ eigenvalue) - $p=n\hbar/R$ ~ $L=p\times R=n\hbar$ (as desired by Bohr)

Basically we had $x-x+2\pi R$ ~ ~~...~~

Thus use $x+2\pi R$ and periodicity to get $k=n/R$, $p=\hbar k=\frac{n\hbar}{R}$

Then $L=pR=n\hbar$

Q9 (c) & (d) ~ Easy.

if $\psi(x+2\pi R)=e^{i\phi}\psi(x)$ ~ $e^{ik(x+2\pi R)}=e^{i\phi}e^{ikx}$ ~ $e^{i2\pi Rk}=e^{i\phi}=e^{i\phi}\times 1=e^{i\phi}\times e^{i2n\pi}$

Thus $2\pi Rk=2n\pi+\phi$ ~ $k=\frac{n}{R}+\phi$

Q9(e) ~ solⁿ in saaathak.com ~ can see it ?

Q10) ~ (a), (b) simple

(c) ? ~ $\frac{\hbar k}{m}(|A|^2-|B|^2)=\frac{\hbar q}{m}(|C|^2-|D|^2)$ ~ even for simple

$I=R+T$

$k_1(|A|^2-|B|^2)=|C|^2k_2$

$k_1|A|^2=k_1|B|^2+k_2|C|^2$

(c) Doubt ?

and create unitary?

easy proof is

$2\pi r=n\lambda$ (standing wave)

and $\lambda=h/mv$

to get: $mvr=n\hbar$

Q11) $\psi(r)=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$ (1s) ~ since $Y_0^0=\frac{1}{\sqrt{4\pi}}=\frac{1}{2\sqrt{\pi}}$

$R_{1,0}$ $2\left(\frac{1}{a_0}\right)^{3/2}e^{-r/a_0}$

(a) $r_{mp}=a_0$ (using $\frac{d}{dr}|R|^2r^2=0$)

(b) $\langle r\rangle=\frac{3}{2}a_0$ (✗ Bohr theory) ⇒ since Bohr gives $r=a_0$.

(c) $\langle\frac{1}{r}\rangle=\frac{1}{a_0}$ (✓ Bohr theory)

Q12) Doubt ?

Q13) verified answer ✓ ~ (a) use: $\chi=\chi_{dia}+\chi_{para}=\chi_{dia}+\frac{C}{T}$ (negative & constant) ~ from data, we can find slope to get C (χ vs $\frac{1}{T}$)

can see a paramagnetic contamination

(b) $C=\frac{\mu_0 n p^2\mu_B^2}{3k_B}$ ~ for Mn^{2+} - $3d^5$ ~ $S=5/2$, $L=0$ ~ since M_l, $-2,-1,0,1,2$ ~ sum $=0$

Thus $J=5/2$ ~ Landé's $g\Rightarrow 2$

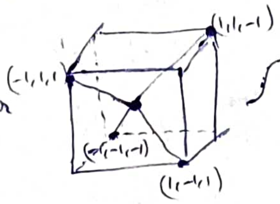
for half filled: $L=0$

Thus $J=S$

Now using data ~ slope of χ vs $1/T$ gives C ⇒ Thus $n=2.1\times10^{24}\,m^{-3}$ (concentration of Mn^{2+} ions)

($C=1.9\times10^{-4}K$)

(c) To find χ_{dia} ~ intercept of plot of χ vs $\frac{1}{T}$ gives χ_{dia} ~ $\chi_{dia}=-9.53\times10^{-6}$



3 '-' sign, '1' & 2 '1'

$(-1,-1,-1),(1,1,-1),(1,-1,1),(-1,1,1)$

Now $\cos\theta=\frac{\vec a\cdot\vec b}{|a||b|}$

$=-1/3$

$\theta=\cos^{-1}(-1/3)=109°28'$

Q14) easy

Q15) - check ✗ (solⁿ not uploaded) - V. V. Difficult!

Q16) Weigner Seitz cell ?

(d) To show diamond bond angle $=109°28'$ ($\cos^{-1}(-1/3)$)

But Note!!

in primitive cell: FCC: $\frac{i+j}{2},\frac{j+k}{2},\frac{k+i}{2}$

& BCC. $\frac{i+j-k}{2},\frac{j+k-i}{2},\frac{k+i-j}{2}$

here we have done opp. & used $(1,1,-1),(-1,1,1)\ldots$ etc ✓

for FCC lattice ✓

diamond cubic is FCC in ZnS-style (alternate Tetra voids)

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DIAS (2021) Test 1 - QM

Q1(a) Easy

Q1(b) $\langle V\rangle = \frac{1}{2}(n+\frac{1}{2})\hbar\omega$

Q1(c) $\psi(x,0) = Ax(a-x)$

Normalise $\int_0^a |\psi|^2 \rightarrow A = \sqrt{\frac{30}{a^5}}$

Now $\psi(x,0) = \sum c_n \phi_n$ ~ ϕ_n is orthogonal Basis

for infinite well $\phi_n = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a}$

Now $c_n = \langle \psi | \phi_n\rangle = \int \psi^* \phi_n(x)\, dx$

$= \sqrt{\frac{30}{a^5}}\left(\sqrt{\frac{2}{a}}\right)\int_0^a \sin\frac{n\pi x}{a} \cdot x \cdot (a-x)\, dx$

solving by integration by parts

$c_n = \frac{4\sqrt{15}}{(n\pi)^3}\left[\cos(0) - \cos(n\pi)\right]$ ~ $c_n = \begin{cases} 0 \text{ if } n \text{ is even} \\ \frac{8\sqrt{15}}{(n\pi)^3} \text{ if } n \text{ is odd} \end{cases}$

Now $\psi(x,t) = \sum c_n \phi(x,t)$ ~ $\phi(x,t) = \phi(x,0) \times e^{-\frac{iE_n t}{\hbar}}$

Thus $\psi(x,t) = \sum_{\substack{n=2,4,\ldots \\ (\text{even},\ldots)}}^{\infty} \frac{8\sqrt{15}}{(n\pi)^3}\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) e^{-\frac{iE_n t}{\hbar}}$.

Q2(b) $L^2 = -\hbar^2 [\ldots r^2 \partial^2)$ don't miss "$-\hbar^2$"

Q4(a) Lattice spacing $d \approx 0.3$ nm

for e^- ~ QM : $\lambda \sim d$

Thus $\lambda = \frac{h}{p} = 0.3$ nm ~ we get p

Now $E = p^2/2m \approx kT$ ~ we can get Temp at which e^- show QM

Q4(c) use $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$

Now $k' = 4k$ ~ $\omega' = 2\omega$

Now Ground state probability $= |\langle \psi_0 | \psi_0'\rangle|^2$

Both shud be Normalized.

Q4(d) for e^- in H atom ~ degree of degeneracy ~ $\sum 2(2l+1) = 2n^2$

since l from 0 to $n-1$

and m_l has $-l$ to $+l$

and $g_{spin} = 2$.

Q5(b) $P(r) = |R|^2 r^2\, dr$

for 2p ~ $A r\, e^{-r/2a_0}$

$|R|^2 r^2$ ~ $A^2 r^2 e^{-r/a_0} \times r^2 = A^2 r^4 e^{-r/a_0}$

$\frac{dP(r)}{dr} = 0 \rightarrow r = 4a_0$ ~ logical since $r = 0.529\frac{n^2}{Z}$ ~ $[r \propto n^2]$

to get $P(r)|_{r=r_0}$ ~ first Normalise $P(r)$!! ~ $\int |R|^2 r^2 dr = 1$ ~ get A. Then put value of r_0 to get max probability.

Q6(a) Always for finite well - we use from $-\frac{L}{2}$ to $+\frac{L}{2}$!! ~ use B.C. at only one of the places

$x = \frac{k_1 L}{2}$

$y = \frac{k_2 L}{2}$ ~ $r^2 = x^2 + y^2 = \frac{L^2}{4}(k_1^2 + k_2^2) \rightarrow \frac{L^2}{4} \times \frac{2mV_0}{\hbar^2} = \frac{mV_0 L^2}{2\hbar^2}$

~ now cos ~ $x\tan x = y$  (always atleast 1 Bound state ~ This is ground (COSINE) : 

sin ~ $x\tan(\frac{\pi}{2}+x) = y$ ~ 

DIAS (2021) Test 2 - AMP

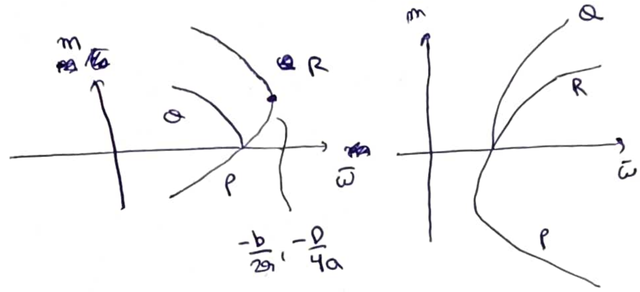
Q1(b) Number of Modes of Vibration — linear : $3N-5$ → for HCl : $N=2$ linear, Thus Modes $=1$ — symmetric stretch ($2886\ cm^{-1}$)
— Non-linear : $3N-6$

Q1(d) In $I = \mu r^2$ $\mu = \frac{m_1 m_2}{m_1+m_2}$ amu — 1.67×10^{-27} kg

Q1(e) Bohr model is derived from 2 equations — $mvr = nh/2\pi$ and $\frac{mv^2}{r} = \frac{kq^2}{r^2}$

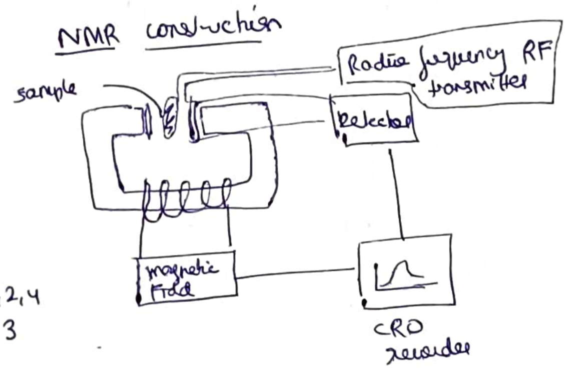
Q2(a) Significance of Q-Branch in V-R spectra:

- Finding band origin is important to find vibrational constant but is difficult to find.
- Head of Q-band coincides with Band-origin & is used to find vibrational constant
- Band origin ~ $m=0$ ~ gap b/w $v'=0$ and $v''=0$



Q2(b) for proton — always use μ_N ~ Not μ_B
~ $g_p = 5.56$

NMR construction



Q3(a) p^3 ~ easy from RJK (Pg 151/152)

p^2/d^2 ~ use Breit's scheme (pg 150 RJK)

for p^2 — $S=1$ & $L=1$
$S=0$ & $L=0,2$

d^2 — $S=0$ and $L=0,2,4$
$S=1$ and $L=1,3$

Basically odd-odd & even-even

for p^3 → refer pg 151 RJK (EASY) ★★

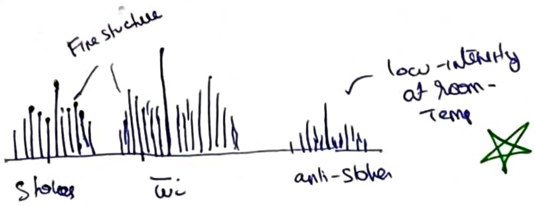
Q3(c) 3 rot. lines - 84, 101, 118 ~ now assume $D \ll B$ ~ Thus $2B = 17$ (spacing b/w lines)
$\underbrace{84, 101}_{17}, \underbrace{\ }_{17}118$
Now $84 = 2B(J+1) = 17(J+1)$ — Thus 84 is $J: 4\to5$
101 is $J: 5\to6$
118 is $J: 6\to7$

Q4(d) for D, $^2D_2$ → spin-orbit coupling ~ Thus $E = -\mu_s \cdot B$ — μ_s ~ spin, B ~ L
$= \mu_B B\, g_s m_s$ ✓
not $m_J g_J$!

also use put 1/2 for Thomson precession
Thus $E = \pm\frac{1}{2}\mu_B B$
Thus $\frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = \mu_B B$ ~ $B = 37$ Tesla

Q5(b) pure rotational spectra only in gaseous state (not solid/liquid) because in condensed states (solid/liquid) ~ rotational energy levels are not quantized. Another reason is that in solid & liquid phase ~ The intermolecular interaction hinder rotation.
— In gaseous state; atoms have low force of attraction b/w them and as a result have no restriction in movement and can rotate easily

Pg 46/54 diagram ~



Q6(c) easy
just use $B_1 g_1 = B_2 g_2$

Q6(d) Tough — can see soln of DIAS (can be skipped ✓) — have used formula of Biot-Savart for spin-orbit coupling — $B = \frac{\mu_0 I}{2r}$ ~ put $I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{e}{2\pi}\times 2\pi f$ ~ put $f = \frac{c}{\lambda}$ ~ given
we get B ~ now solve normally

DIAS 2021 Test 3 – NPP

Q1(e) B.E. of mirror Nuclei are same → Thus charge independence of Nuclear Force $F_{nn} \approx F_{np} \approx F_{pp}$

Radius using mirror Nuclei ~ $E_b - E_b' = a_3 \frac{Z(Z-1)}{A^{1/3}} - a_3 \frac{(Z')(Z'-1)}{A^{1/3}}$ (remaining cancel since A is same)

Thus we can find a_3

Now $E_c = \frac{e^2}{4\pi\varepsilon_0 R} \times \frac{Z(Z-1)}{2} = \frac{a_3 Z(Z-1)}{A^{1/3}}$ ~ Thus $a_3 = \frac{e^2}{8\pi\varepsilon_0 R_0}$ ~ $a_3 \propto \frac{1}{R_0}$

Q2(a) : $M(Z,A) = \alpha A + \beta Z + \gamma Z^2 \pm \delta$

$= m_p Z + (A-Z)m_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z(Z-1)}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} \pm \delta$

club Z & Z^2 terms in β & γ respectively

(remaining goes to α) ~ α won't have any Z terms

Now $Z_0 = -\beta/2\gamma$ easy.

put values $a_1 = 14.1$ MeV
$a_2 = 13$ MeV
$a_3 = 0.6$
$a_4 = 19$
$a_5 = 34$

Q2(e) Refer soln by Akash Dagar ~ Easy derivation for verification of general theory of relativity using Moessbauer Effect.

Q3(b) To measure helicity ~ explain the expt. (with magnetic field) and explain schematic diagram of experimental set-up

Q4(a) In my RG-xerox A4 append for GUT

Q4(c)(ii) My silly mistake : → $\Delta S \neq 0$ ~ but still not allowed → Forbidden – because prodn of strange particle can only proceed by strong !!!

Q5(c) use Non-relativistic simple steps → $Q = \Delta KE$, $mx = (m_x + M_x)V_x$, $E = \frac{1}{2}M v_x^2$ → $E_x = -Q\frac{(m+M)}{M}$

Q5(e) we use α-particle since $\lambda_\alpha = \frac{h}{\sqrt{2mE}} \approx$ comparable to size of Nucleus

Q6(b) Fermi theory of β-decay → same as Pauli, etc.. etc... (Nicely written by Aniket Hinde)

Q6(c) $f = \frac{M}{A} - 1$ ~ usually $M < A$ (where $A = N + Z$) ~ when M not given → find using $M(Z,A) = \underbrace{m_p}_{938\, MeV/c^2} Z + \underbrace{m_n}_{940\, MeV/c^2}(A-Z) - a_1 A + a_2 A^{2/3} \ldots$ / .... put suggested values of $a_1, a_2 \ldots a_5$ MeV/c^2

Q5(b) Always find $E_{recoil} = \frac{p^2}{2M} = \frac{E_\gamma^2}{2Mc^2}$ (since $P_M = P_\gamma = \frac{E_\gamma}{c}$)

and we get $E_{recoil} >> \Delta E \left(\sim \frac{\hbar}{\Delta t}\right)$

(Not $E_\gamma > \Delta E$)

DIAS 2021 - Test 4 SSPE

Q1(a) use $\frac{hx}{a} + \frac{ky}{b} + \frac{lz}{c} = 1$

Q1(c) Note $L_{series} = L_1 + L_2$ and $C_{series} = \frac{C_1 C_2}{C_1 + C_2}$ (capacitor) $C = \frac{\epsilon_0 A}{d}$ and $L = \mu_0 n^2 A l$

frequency of Hartley $\nu = \frac{1}{2\pi\sqrt{LC}}$ where $L = L_1 + L_2 + 2M$

Q2(c) V. V. easy ~ just like in P&B

take $g(E) = \frac{(4)\pi V}{h^3}(2m)^{3/2}(E-E_c)^{1/2}\, dE$ and $f(E) = \frac{1}{e^{\frac{E-E_F}{kT}}+1} \approx \frac{1}{e^{\frac{E-E_F}{kT}}}$

4 since e^- is F.D.

we had split $E-E_F$ and taken $E_F - E_c$ out to have $E-E_c$ in

Now $n = \int_{E_c}^{\infty} f(E) g(E) = \frac{4\pi V}{h^3}(2m)^{3/2} e^{\frac{E_F-E_c}{kT}} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-\frac{(E-E_c)}{kT}}\, dE$

put $\frac{E-E_c}{kT} = x$ ~ $dE = (kT)dx$ ..

we get $\frac{n}{V} = 2\left(\frac{2\pi mkT}{h^2}\right)^{3/2} e^{-\frac{(E_c-E_F)}{kT}}$

Q3(b) even though question just asked for truth table, if is a 20 mark Q and thus also draw logic diagram -▷- ▷-, etc...

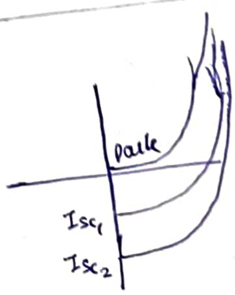
always draw logic diagram...

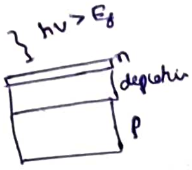
For c^n ~ Aniket also wrote 1-line application ~ "logic gates are used to design memory chips and in computers"

Q4(c) Refer soln in my DIAS(S) pg 160 ~ for POS → use $X + YZ = (X+Y)(X+Z)$

also. $(A+B) = (A+B+C.\bar{C})$
$= (A+B+C)(A+B+\bar{C})$

Q4(a)





If circuit is shorted $I_{ph} = -I_{sc} = C_1 I$ (constant; intensity)

Thus if Intensity is doubled:
$I_{sc_2} = 2 \times I_{sc_1} = 300\,mA$

$I_{ph} = I_{sc}$
I at V=0

Now $I = -I_{ph} + I_s\left[e^{qV_0/\eta kT} - 1\right]$

I_s → reverse saturation current.

$I_{ph} = eAG_0[L]$, G_0 → generation rate of e^--hole pair

A → area exposed
L → active region length

Now at $V = V_{oc}$ ~ $I = 0$ — I_{sc} & I_{ph} are same

Thus $V_{oc} = \frac{\eta kT}{q}\ln\left[\frac{I_{sc}}{I_s} + 1\right]$

we know value of V_{oc}, I_{sc}, etc

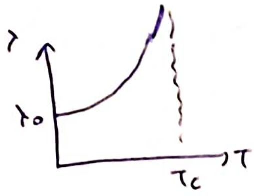
we get $I_s = 0.17\,\mu A$ (Fixed and constant value)

Now at double intensity: $I_{sc_2} = 2 \times I_{sc_1}$

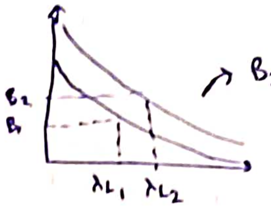
$V_{oc_2} = \frac{\eta kT}{q}\ln\left[\frac{I_{sc}}{I_s} + 1\right] = 0.556\,V$

Q5(b) same as PSB numerical, 1st find all values for electron → then put $F_h = -F_e$, $v_h = -v_e$, $P_h = -P_e$, $m_h = -m_e$

Q5(d) variation of penetration depth with Temperature ~ $\lambda = \lambda_0 \left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-1/2}$ (λ_0 is penetration depth at T = 0 kelvin)

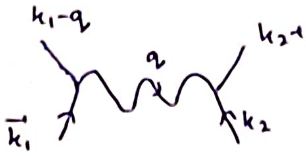


$B(x) = B_0 e^{-x/\lambda_L}$



$\nearrow B_2 > B_1 \Rightarrow \lambda_{L_2} > \lambda_{L_1}$

Q5(e) IMP diagram ~



(electron - phonon interaction) → BCS theory

Q6(a) significance of Debye Temp (Θ_D) ~ It is temperature of crystal's highest mode of vibration - $\frac{h\nu_F}{k}$

↳ correlates elastic properties with thermodynamic properties such as thermal expansion, thermal conductivity, specific heat, Latent Heat

above $T > \Theta_D$, all phonon modes have been activated.

$C_v \propto \left(\frac{T}{\Theta_D}\right)^3$ (for low T)

$C_v = 3R$ (for high Temp)

Isotope effect $\Theta_D \propto \frac{1}{\sqrt{M}}$ and $T_c \propto \frac{1}{\sqrt{M}}$ (critical temp for superconductivity)

$\frac{\Theta_D}{T_c}$ = constant

[above debye temp ~ all phonon modes have been activated and C_v → dulong petit (3R)]

DIAS (2021) Test 5 Mechanics

Q1(b) ~ silly – can skip

Q1(c) → Don't assume combined rest mass = $2m_0$ ✗

assume combined rest mass = M_0 & velocity = V

Total Energy conservation (Total E always conserved) = $m_0c^2 + \frac{m_0c^2}{\sqrt{1-v^2/c^2}} = M c^2$ ← moving mass $M = \frac{M_0}{\sqrt{1-V^2/c^2}}$

similarly do momn conservation

← then we can get both V and M_0

Q1(e) $\vec{P_\mu} = (\vec{p}, \frac{iE}{c})$ ~ $P_\mu^2 = p^2 - \frac{E^2}{c^2} = -m_0^2c^2$

Q3(a) take $u^n = \sec n\theta \times \frac{1}{a^n}$

$$nu^{n-1}\frac{du}{d\theta} = \frac{1}{a^n}\sec n\theta \tan n\theta . n$$

$$nu^{n-1}\frac{du}{d\theta} = u^n \tan n\theta . n$$

$$\frac{du}{d\theta} = u \tan n\theta$$

← for $r^n = a^n \cos n\theta$ ↳ $F \propto \frac{1}{r^{2n+3}}$

$$\text{Now } \frac{d^2u}{d\theta^2} = nu\sec^2 n\theta + \tan n\theta . \frac{du}{d\theta}$$

$$= nu\sec^2 n\theta + u\tan^2 n\theta$$

$$\frac{d^2u}{d\theta^2} = (nu+u)\sec^2 n\theta - \underline{u}$$

← this cancels ★ with $\frac{d^2u}{d\theta^2} + \underline{u}$ ...

↳ write $\sec^2 n\theta = u^{2n}a^{2n}$

Now Easy ★

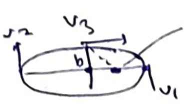
$$F(u) = -\frac{J^2}{m}\frac{(n+1)}{r^{2n+3}}a^{2n-3}$$

(if $r = 2a\cos\theta$ → $n=1$ ~ $F \propto 1/r^5$ ~ circular motion)

Q5(d) My steps are correct (v. Easy only ★) (wrong answer given in soln) ~ I even verified from doubtnut & toppr.com ~ $\underline{57}$ in Numerator

Q6(c) Easy = work = $\Delta KE = KE_f$ (since $KE_i = 0$)

Q6(b) Easy



J is always conserved from Jin

$J = mv_3 b$ (b is ⊥r distance in cross product)

$J = mv_1 a(1-e)$

$J = mv_2(a)(1+e)$

and use $e^2 = 1 - b^2/a^2$

for clean soln refer pg 54/54 of DIAS soln PDF 2021

DIAS (2021) - Test 6 optics

1(a) and 1(c) → Irrelevant

2(b) Fring of equal width same as fringe of equal ~~inclination~~ thickness such as wedges
other defn : YDSE, etc...

3(a) Molecular origin of optical activity → interaction of EM radiation of polarized light with electric fields produced by e^- in chiral molecule

Shown by chiral molecules (non-symmetric mirror images)
optically active

Dipole due to differential displacement interacts with EM field ~~~ (temporary / oscillating) (my made up answer)

5(c) - $n_1 = 1.48$
$n_2 = 1.46$ ~ $\sin i_m = \sqrt{n_1^2 - n_2^2}$
$= 0.24$

Q6(b) & (c) → Easy

Q6(a) ~ use λ_{av} (decay rate) $= \frac{1}{t_{av}}$ ~ where $\frac{dN}{dt} = \lambda N$

~~where~~ we have $\frac{dN_2}{dt} = A_{21} N_2$ ~ thus $\boxed{A_{21} = \frac{1}{t}}$ ★

Now easy. use $\frac{A_{21}}{B_{21}} = 8\pi h\nu^3/c^3$ to find B.

DIAS (2021) - Test 7 E&M

~~Q1(a)~~ see soln ~~...~~ once
Q1(b)

Q1(c) use $n(\nu)d\nu = f(\nu)g(\nu)$ — photons & EM have 2 modes ★

$$= \frac{1}{e^{h\nu/kT}-1} \times \frac{8\pi\nu^2 d\nu}{c^3}$$

Q1 (a) → (e) : Just see soln once easy

Q2 (a) → Tough Numerical ★ ~ write $E = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ — $k\cos\theta x + k\sin\theta y \ldots$ also $B = \frac{\vec{k}\times\vec{E}}{\omega}$

Q2 (c) → see the limitation of Clausius Mossotti →
1) only valid for linear isotropic solids
2) we assumed Polarization is proportional to $\vec{E}$ and $\parallel$el
3) Not applicable to strong soln & solids
(only applicable to gases and neutral liquids)
4) inapplicable to substances with significant permanent dipole. (coz we assumed $p_m = \alpha E_m$)

2(b) easy — avg power → $\langle P\rangle = \frac{V_0^2}{2R}$ (since $X_L = X_C$ cancel out)

$Q = \frac{\omega_0 L}{R}$

Bandwidth $\Delta\omega = \omega_2 - \omega_1 = R/L$

3(b): poynting vector $\bar{S} = \bar{E}\times\bar{H}$ → represents energy per unit time per unit area transported by EM field

Q3 (c) ~ Amazing soln pg ~~...~~ 27 & 28
(he did textbook way ~ take $c \neq 0$ but dont put $\omega_0 = \omega_k$
multiply $\frac{a-jb}{a-jb}$ ~ and get separate n_+ & n_- in $n = n_+ + jn_-$

Q4 (a)

$$F_{\mu\nu} : \begin{bmatrix} 0 & B_z & -B_y & -\frac{iE_x}{c} \\ & & B_x & \\ & & & \end{bmatrix}$$

~ it is anti-symmetric

$F_{\mu\nu}^2$ ~ sum of all terms $= 2(B^2 - E^2/c^2)$

Q4(b) & 4(c) → we only need to get answer
(choose 4/10 tough & New vs 0-5 for VAM)
~ Aniket Hinde wrote 1.5/4 Qs for 20 marks.

Q5(a) use $\left|\frac{J}{J_D}\right| = \left|\frac{\sigma E}{\varepsilon j\omega E}\right| = \left(\frac{\sigma}{\varepsilon\omega}\right)$

Q5(d) use $\vec{D} = \frac{Q_{free}}{4\pi r^2}\hat{r}$

and $\bar{D}_{in} - \bar{D}_{out} = \sigma_{free}$

Q5(e) use $S = \frac{1}{2}c\varepsilon_0 E_0^2$ — proof: $S = \frac{E\times B}{\mu_0}$ — put $B = E\sqrt{\mu_0\varepsilon_0}$

Thus $S = E^2\sqrt{\frac{\varepsilon_0}{\mu_0}} = c\varepsilon_0 E^2$

$\langle S\rangle = c\varepsilon_0\langle E^2\rangle = c\varepsilon_0\frac{E_0^2}{2}$

or

use $S = c\times u$

$= c\times\frac{1}{2}\varepsilon_0 E^2$

$\langle S\rangle = \frac{1}{2}c\varepsilon_0\langle E^2\rangle = \frac{1}{2}c\,\varepsilon_0\frac{E_0^2}{2}$

Now multiply by 2 since $U_E = U_B$ — thus $\langle S\rangle = \frac{1}{2}c\varepsilon_0 E_0^2$

to derive $u_E = \frac{1}{2}\varepsilon E^2$ → use $u = \frac{U}{V} = \frac{\frac{1}{2}CV^2}{A\times d}$ (volume) and put $C = \frac{\varepsilon_0 A}{d}$ → $u = \frac{U}{V} = \frac{1}{2}\varepsilon_0\frac{V^2}{d^2} = \frac{1}{2}\varepsilon_0 E^2$ ✓ $(E = V/d)$

to derive $u_B = B^2/2\mu$ → we know $L_{solenoid} = \mu_0 n^2 l.A.$ ✓ $(\phi = LI)$

Both follow same steps.

Thus $u_B = \frac{U}{V} = \frac{\frac{1}{2}LI^2}{A.l} = \frac{\frac{1}{2}\mu_0 n^2 l.A\,I^2}{A.l} = \frac{1}{2}\mu_0 n^2 I^2$ ☆ put $B = \mu_0 nI$

Thus $u_B = \frac{B^2}{2\mu_0}$

$u = \frac{U}{V}$ ~ $\frac{1/2\,CV^2\,(or)}{1/2\,LI^2}$ ($A\times d\,(l)$) → Then put C (or) L value → Then $E = V/d$ (or) $B = \mu_0 ni$

Q6(c) — Refer Akash Dayal 2021 T7 Soln last page (31/31) ☆

$B_{due\ to\ Ring} = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$ (put $z=0$ → to get $B = \frac{\mu_0 I}{2a}$ — verified ✓)

$B_{ring} = \frac{\mu_0 I}{2a}$

$B_{infinite\ wire} = \frac{\mu_0 I}{2\pi r}$ — infinite wire — can also get using Ampere law: $\oint B.dl = \mu_0 I$

same in ring & infinite wire

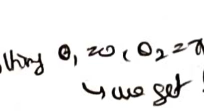
actually $B = \frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$

$= \frac{\mu_0 I}{2\pi r}$ (for infinite)

$dB = \frac{\mu_0 IR^2}{2(R^2+z^2)^{3/2}}\times n\,dx$ — $n = \frac{N}{l}$

$B = \frac{\mu_0 nIR^2}{2}\int\frac{dx}{(R^2+x^2)^{3/2}}$

put $\boxed{x = R\cot\theta}$ ☆ — from Diagram θ, x, R relation ☆



$\tan\theta = \frac{R}{x}$

$B = \frac{\mu_0 nI}{2}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$

$= \frac{\mu_0 nI}{2}(\cos\theta_2 - \cos\theta_1)$ — putting $\theta_1 = 0, \theta_2 = \pi$ → we get $B = \mu_0 ni$ (Thus verified.

now for (i) — $\theta_1 = \tan^{-1}(R/L_2)$

and $\theta_2 = \pi - \theta_1$

for (ii) — $\theta_1 = \pi/2$

$\theta_2 = \tan^{-1}(R/L)$

Aakash Dayal 2021 T7 soln pg 30 & 31

DIAS (2021) Test 8 - Thermo

$e^{\frac{-\alpha}{kT}} >> 1 \rightarrow$ for $e^{\frac{E-\alpha}{kT}} \leftarrow e^{\frac{E-\alpha}{kT}} \pm 1$ (Quantum to classical) ||

Now $N = \int_0^\infty n(E)dE = \int_0^\infty f(E)g(E) = \int_0^\infty e^{-\frac{E-\alpha}{kT}} \times \frac{2\pi V}{h^3}(2m)^{3/2} E^{1/2} dE$

we get $e^{-\alpha} = \frac{V}{N}\left(\frac{2m\pi kT}{h^2}\right)^{3/2} >> 1$

put $\frac{E}{kT} = u$

and $\Gamma(3/2) = \frac{1}{2}\sqrt{\pi}$

Q1 (a) ~ Beautifully solved by Akash Dagar

⌐ see my DIAS(2022) soln

(b) ~ use 1 poise = 0.1 SI unit (Dynamic viscosity) (Don't forget unit poise ~ 0 (1 SI unit)

refer 2022 DIAS Test 8 soln//

1 stoke (cm²/s) = 10^{-4} m²/s (kinematic viscosity = $\frac{\text{Dynamic}}{\rho}$)

(c) use $\frac{\partial C_p}{\partial p} = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_p$

(d) Take T as initial T (Ti) = $-38.87°C$ in the formula

and use $v = \frac{1}{\rho}$ (specific volume = $\frac{1}{\text{density}}$)

(e) To get $PV^\gamma = \text{const}$ ~ we use $dS = C_p \ln V + C_v \ln P + k$

and we put $dS = 0$

here also take S in terms of T & V coz answer in terms of T and V

$TdS = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_v dV$ ~> $S = S(V,T)$ and $\frac{\partial S}{\partial V} = \frac{\partial P}{\partial T}$

Now $\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v-b}$ for van der waals

since final request in terms of T & V

Thus $TdS = C_v dT + \frac{TRdV}{v-b} = 0$

Thus $T(v-b)^{R/C_v} = \text{constant}$

Q2(a) use $dP = \rho g dh = \frac{PM}{RT} g dh$ (since $\rho = \frac{PM}{RT}$)) here we are not given isothermal!! Thus T not constant.

and $P^{1-\gamma}T^\gamma = k \rightarrow (1-\gamma)\frac{dP}{P} + \gamma\frac{dT}{T} = 0$ —①

put dP in ① to get answer (both P & T will get cut)

(since we need T vs h for adiabatic lapse rate.

Q 2(c) for $g(p)dp = \frac{4\pi p^2 dp V}{h^3} \rightarrow \frac{2\pi V}{h^3}(2m)^{3/2}E^{1/2}dE$ ~ for MB – $n(E)dE = \frac{2N}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}E^{1/2}e^{-E/kT}dE$

Note we only have $\frac{4\pi V}{h^3}(2m)^{3/2}$... for photon & EM wave, not here!!

don't put extra 2 in gases, MB, etc....

Q3(b) Mug C_v for debye & Einstein

$$(C_v)_{\text{Einstein}} = 3R\left(\theta_E/T\right)^2 \frac{e^{\theta_E/T}}{(e^{\theta_E/T}-1)^2}$$

$\boxed{\begin{array}{l} 1 \text{ calorie} = 4.186 \text{ Joule} \\ R = 1.98 \text{ cal/k/mol} \end{array}}$ ~ remember that $R = 8.314\ J k^{-1} mol^{-1}$ and $R = 1.98\ cal\ k^{-1}\ mole$

Thus $1\ cal = 4.186$ Joule

< also $1 eV = 1.9 \times 10^{-19} C \times J/C = 1.6 \times 10^{-19} J$ >

(1 HP Horsepower = 746 watt)

(1 atm = 760 mm of Hg)

1.013×10^5 Pa

5(a) $(C_v)_{water} = \underline{1\ cal\ gram^{-1}}\ k^{-1}$ or 1 kilo calorie/kg

5(b) To prove $\frac{E_S}{E_T} = \frac{-V\left(\frac{\partial P}{\partial V}\right)_S}{-V\left(\frac{\partial P}{\partial V}\right)_T}$ ~ professional & easy method as in GBG

LHS = $\frac{C_P}{C_V}$

we start with $\frac{C_P}{C_V} = \frac{\left(\frac{\partial S}{\partial T}\right)_P}{\left(\frac{\partial S}{\partial T}\right)_V}$ ✱

$\frac{\partial x}{\partial y} = \left(\frac{\partial x}{\partial z}\right)_u \cdot \left(\frac{\partial z}{\partial y}\right)_u$

start with $\frac{C_P}{C_V} = \frac{\left(\frac{\partial Q}{\partial T}\right)_P}{\left(\frac{\partial Q}{\partial T}\right)_V} = \frac{\left(\frac{\partial S}{\partial T}\right)_P}{\left(\frac{\partial S}{\partial T}\right)_V} = \frac{\left(\frac{\partial V}{\partial T}\right)_P\left(\frac{\partial S}{\partial V}\right)_P}{\left(\frac{\partial S}{\partial P}\right)_V\left(\frac{\partial P}{\partial T}\right)_V} = \frac{\left(\frac{\partial V}{\partial T}\right)_P\left(\frac{\partial T}{\partial P}\right)_V}{\left(\frac{\partial S}{\partial P}\right)_V\left(\frac{\partial V}{\partial S}\right)_P}$ (using $\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\partial y/\partial x\right)_z}$)

same

Now $\left(\frac{\partial y}{\partial x}\right)_z\left(\frac{\partial x}{\partial z}\right)_y\left(\frac{\partial z}{\partial y}\right)_x = -1$

diff

$= \frac{-\left(\frac{\partial P}{\partial V}\right)_S}{-\left(\frac{\partial P}{\partial V}\right)_T}$ = Hence proved

we split with V and not P since P is const.

similarly in denominator V is constant.

5(e) - gram-atom $\equiv$ mol < Both mean the same thing >

Q6(a) $\frac{dL}{dT} - \frac{L}{T} = C_2 - C_1$ ~ note in $\frac{L}{T}$ ~ put T in kelvin!

but for $\frac{dL}{dT}$ ~ since $T = 273 + t$

Thus $dT = dt$ (linear term)

and thus simple differentiation with t in °C

Q6(b) $T_{triple\ point} = 273.15\ k$

$P_{triple\ point} = 4.58$ mm of Hg

Q6(c) we can put 'T' in formula as Tinitial

1 Joule = 10^7 erg (easy derivation)

1 Newton = 10^5 Dyne

1 Tesla = 10^4 oersted/Gauss

use $F = i l B$

$kg\ m/s^2 = A \times m \times B$

Thus $B = \frac{kg}{Ampere} \times \frac{1}{s^2}$

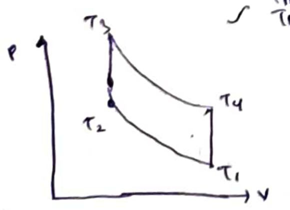
Now MKSA → cgs → Biot for current

1 kg = 10^3 gram

1 A = 0.1 Biot (opp. to expected if ↓ rather than ↑)

Thus $B = 10^{+4}$ Gauss

Q2(b) (of 2022) DIAS



~ $T_{max} \to T_3$, $T_{min} \to T_1$ } since adiabatic is steeper than isothermal

$\eta_{carnot} = 1 - \frac{T_1}{T_3}$

Now $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$\hookrightarrow T_1 = \frac{T_2}{\rho_c^{\gamma-1}}$

$\eta_{carnot} = 1 - \left(\frac{T_2}{T_3}\right)\frac{1}{\rho_c^{\gamma-1}} > 1 - \frac{1}{\rho_c^{\gamma-1}} > \eta_{otto}$ (since $\frac{T_2}{T_3} < 1$)

DIAS (2022) Test 1 - QM

Q2(b) $Y_1^0(\theta,\phi) = A\cos\theta$ ∽ $\psi = AY_1^0 + BY_1^1$ — $L_z\psi = \hbar BY_1^1$

$Y_1^{\pm1}(\theta,\phi) = B\sin\theta e^{\pm i\phi}$ $L^2 = 2\hbar^2\psi$ not eigen fn

Thus it is eigenfn since both had same l (diff m_L values)

Q2(c) For 1D Box ∼ $\langle p\rangle = 0$!! we never write $\langle p\rangle = \frac{n\pi}{L}\hbar$ X

we can integrate using $\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$ and $p_x = -i\hbar\frac{\partial}{\partial x}$ to prove.

For $\psi(x) = \frac{1}{\sqrt{5}}\phi_1 - \frac{2}{\sqrt{5}}\phi_2$ ∼ $\langle p\rangle = 0$ ∽ if we integrate we will get zero!

Akash Dagar - $\langle p_x\rangle$ is always zero if ψ is Real function ??

Q4(b) in light - we use $\vec{E}$ and $\vec{H}$ } to get the coefficients

in QM - use 2 B.C - $\psi_1 = \psi_2$, $\frac{\partial\psi_1}{\partial x} = \frac{\partial\psi_2}{\partial x}$

here take Flux of particles $\vec{J}$ ∽ we $Ae^{ikx} + Be^{-ikx}$ and Ce^{ikx}

$R = \left(\frac{k_1-k_2}{k_1+k_2}\right)^2$, $T = \frac{4k_1k_2}{(k_1+k_2)^2}$ ∽ $p_1 = \hbar k_1 = mv_1$, $p_2 = \hbar k_2 = mv_2$ ∽ put values of k ∽ $R = 1/9$ and $T = 8/9$

incoming current = 1 mA

Thus $I_{Reflected} = 1/9$ mA

$I_{transmitted} = 8/9$ mA

Q4(c)(iii) ∽ since 1D - well !! ∼ Thus use 1D. D.O.S (Not 3D value)! ($1D \propto E^{-1/2}$)

take 1/2 (just like 1/4 & 1/8) and later multiply $g_i \times 2$.

Q5(b) Vajpayee sir gave same Numerical answer as Akash Dagar in 2022 Akash checked Test1 Q5(b) ∽ $\Delta x\Delta p_x \geq \hbar/2$, $\Delta x \sim r$, $\Delta p_x \sim p\sin\theta \sim p\tan\theta \sim \frac{rp}{d}$ ∫ Thus $r \approx \sqrt{\frac{\hbar d}{2p}}$

Q5(a) For Finite well ∼ always from $-\frac{L}{2}$ to $+\frac{L}{2}$ ∼ $\frac{k_1L}{2} \sim x$, $\frac{k_2L}{2} \sim y$ ∽ $x^2+y^2 = \frac{L^2}{4}\frac{2mV_0}{\hbar^2}$ (R^2) ∽ cos : $x\tan x = y$, sin : $x\tan(\frac{\pi}{2}+x) = y$

$R^2 = x^2+y^2$ → Thus find R^2 using $x^2+y^2 = R^2$

Q5(c) Skip

Q6(a) Avg energy of e^- ∼ they are Fermions.

take $T = 0K$; $n = \int 2f(E)g(E)$ ∼ $\langle E\rangle = \frac{3}{5}NE_{F0}$

Q6(d) : $(n_x^2+n_y^2+n_z^2)\frac{h^2}{8mL^2} = 66E_0$ ∼ (1,1,8) ∼ 3, (1,4,7) ∼ 6, (4,5,5) ∼ 3 ∽ Thus 12 states ✓ ∽ Total number of e^-: $24e^-$ if we take $g_i = 2$

DIAS (2022) Test 2 – AMP

Q1(c) $H_\beta \to 4 \to 2 \to n=4$ ~ s, p, d, f levels

Q1(e) $N_J = N_0 (\underline{2J+1}) e^{-BJ(J+1)}$ → easy derivation by differentiating: $J_{max} = \sqrt{\frac{KT}{2B}} - \frac{1}{2}$

pg 11/30 ~ selection rule for zeeman: $\Delta m_j = 0, \pm 1$ & $\underline{m_j = 0 \not\leftrightarrow m_j = 0}$ ~~for~~ $\underline{\text{if } \Delta J = 0}$

P8 11 ~ applications of zeeman → Laser cooling. eg : zeeman slower
→ plotting magnetograms : to find magnetic field distribution of sun

Q3(a) N^+ ~ $1s^2 2s^2 \boxed{2p^2}$ → $S = 0$ – $L = 0, 1, 2$
~ $S = 1$, – $L = 1$

Q3(b) $\Delta T = \frac{5.84 Z^4}{n^3 (l(l+1))} \underline{cm^{-1}}$ → spin-orbit interaction ⟨No relativistic included⟩ → also $\Delta E = hc\Delta T$
→ $\underline{\text{linear reln}}$ for $cm^{-1} \to$ Joule

DIAS (2022) Test 3 - NPP

~~Q1(a) ΔE = ...~~ — always use μ_N for Nucleus! $\mu_N = \frac{\mu_B}{1836}$

Q1(a) $\Delta E = \underline{\mu_N} B g_i \underline{m_i}$ — $\Delta m_i = \pm 1$ (given)

Q1(c) Most stable Isobar: $Z_0 = -B/2\gamma$; my values of a_1, a_2, a_3, a_4, a_5

Pg 23 ~ GUT ~ 10^{15} GeV ~ <u>String theory</u> further adds ~~that~~ <u>Gravitation</u> for "Theory of everything" at <u>10^{19} GeV</u>

Q5(b) Electroweak dominates at high energies of 10^2 GeV

Pg 33 - Examples of strange prod^n & strange decay.

prod^n of strange

$\pi^+ + n^0 \rightarrow \Lambda^0 + K^+$

$\pi^- + p^+ \rightarrow \Lambda^0 + K^0$

$\pi^- + p^+ \rightarrow \Sigma^0 + K^0$

$\pi^- + p^+ \rightarrow \Sigma^- + K^+$

Strangeness = 0 ★ Strangeness = 0 ★

decay of strange

$\Lambda^0 \rightarrow p + \pi^-$

$\Lambda^0 \rightarrow n + \pi^0$

$\Sigma^+ \rightarrow p + \pi^0$

$\Sigma^+ \rightarrow n + \pi^+$

$\Sigma^- \rightarrow n + \pi^-$

★ Strangeness $\neq 0$

$\boxed{\text{one side has} \sim n/p \,\&\, \pi \text{ only}}$ ★ — meson

$\boxed{\text{other has} \sim \Lambda, \Sigma, K}$ ★ — meson

(all are particles ~ no anti-particles here)

(<u>K</u> & <u>π</u> are <u>mesons</u> ~ Baryon No. = 0)

decay: 1 strange on LHS (<u>not K meson !!</u>) — since we have 1 Baryon p/n on RHS

Prod^n: 2 strange ; 1 will be K ★ always

Q) → $\hbar\omega = 41 A^{-1/3}$ <u>MeV</u> → it is in my PYQ soln

Q) → $\mu = (\mu_n + \mu_p)\vec{I} - (\mu_n + \mu_p - 1/2)\vec{L}$

DIAS Test 4 - SSPE (2022)

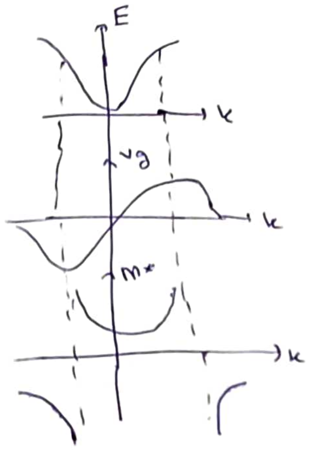
Q1(c) $n_i = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4} e^{-\frac{E_g}{2kT}}$

$\sigma = e n_i (\mu_n + \mu_p)$

Q1(d) ~ $2eV_0 = h\nu$, 2e cos cooper pair ~ $\nu = \frac{2eV_0}{h}$

Q1(e) put $x = \frac{h\nu}{kT}$ ~ Then differentiate to get $C_v = \partial E/\partial T$ ~ Then put limit of $T \to 0$

Q2(a)



Q2(c) use $2T\cos\theta + T = mT$

$\cos\theta = \frac{m-1}{2}$ and $\theta = \frac{2\pi}{n}$

Q3(b) superconductor vs conductor

↳ 1st ~ draw graphs of S, C_v, σ, etc.... for superconductor vs conductor

↳ 2nd ~ superconductor / perfect Diamagnetic, obey Meissner effect, cooper pair

Draw the P&B figure of perfect conductor vs superconductor with Flux & Magnetic Field.

Q3(c) - KCl ~ FCC structure (just like NaCl ~~[illegible]~~)

thus n=4 per cell.

$d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$ — 'a' is the lattice parameter of unit cell.

Q6(a) Easy op-amp

Q5(c) Easy reln : $\alpha = \beta/1+\beta$

Q5(b) Wien oscillator - skip

(~~[illegible]~~ $f = \frac{1}{2\pi\sqrt{R_1C_1R_2C_2}}$ ~ $= \frac{1}{2\pi RC}$ (If $R_1 = R_2$ and $C_1 = C_2$)

frequency of wien Bridge oscillator

Note $RC \equiv \sqrt{LC}$ (in dimensions)

$R = \frac{1}{\omega C} = \omega L$

~ $\omega = \frac{1}{RC} = \frac{1}{\sqrt{LC}}$

Q5(a) RC-phase shift oscillator ~ skip

Q4(a) $X + YZ = (X+Y)(X+Z) = XX + XY + XZ + YZ$, $X(1+Y) + YZ$ ~ $X(1+Z) + XY + YZ$

Now $A + \bar{A}B = (A+\bar{A})(A+B) = A+B$ (use above relation)

Q4(d) ~ $\alpha = \frac{I_C}{I_E}$, $\beta = \frac{I_C}{I_B}$ and $I_C = \alpha I_E + I_{CO}$ ★

value of α & I_B given & I_{CO} given → use α to get β ~ then get $I_C = \beta I_B$ ~ now use $I_C = \alpha I_E + I_{CO}$

$I_E = \frac{I_C - I_{CO}}{\alpha}$ ~ to get value of I_E

Q6(b) P&B Pg 143 Ex 4.3 and also a PYQ!

$3N = \int_0^{\nu_m} g(\nu)d\nu = \int_0^{\nu_m} \frac{4}{3}\pi\nu^2 d\nu \left[\frac{1}{v_l^3} + \frac{2}{v_t^3}\right]V$

Thus $3N = 4\pi\left[\frac{1}{v_l^3} + \frac{2}{v_t^3}\right] \times \frac{\nu_m^3}{3} \times V$

let $v_{sound} = v_l = v_t$

Thus $3N = \frac{4\pi \nu_m^3}{v_s^3} V$ (also $\nu_m = \frac{k\Theta_D}{h}$)

and $\frac{N}{V} = n = \left(\frac{\rho}{M} \times N_a\right)$

$\nu_m^3 = \left(\frac{k\Theta_D}{h}\right)^3 = \left(\frac{3\rho N_a}{M}\right) v_s^3 \times \frac{1}{4\pi}$

$\frac{(\Theta_D)_{gold}}{(\Theta_D)_{Cu}} = \frac{(v_{sound})_{gold}}{(v_{sound})_{Cu}} \left(\frac{\rho_{gold}}{\rho_{Cu}} \cdot \frac{M_{Cu}}{M_{gold}}\right)^{1/3}$

DIAS (2022) Test 5 - Mechanics

Q1(d)  assume $h_1 = h_2$

Lift $= \Delta P \times A = \frac{1}{2}\rho(v_1^2 - v_2^2) \times$ Area

This is ρ of air (Bernoulli's is applied on the Fluid → air here!)

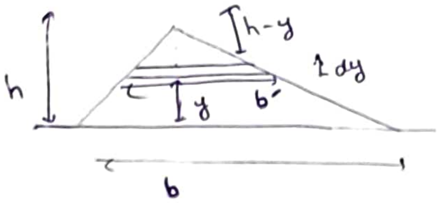
$\rho_{air} = 1.2 kg/m^3$

Q2(c) ~~V. Tough~~ → but see Akash Pagar soln → ★ → use $m\frac{d^2x}{dt^2} = mv\frac{dv}{dx}$ ★ for easy $a = v\frac{dv}{dx}$ ★

Q4(c) Tough → you need to use Euler Angles ★

On a side Note:

MoI for △le



Now $dm = \frac{M}{\frac{1}{2}bh} \times b'dy$ ★

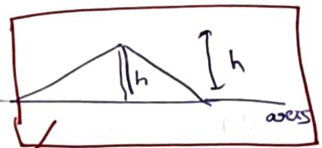
also $\frac{b'}{b} = \frac{h-y}{y}$ (similar triangles)

$I = \int \left(\frac{2M}{bh} b' dy\right) \times y^2 \to (dm \times y^2)$

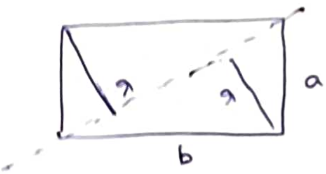
$I = \int \frac{2M}{bh} b' y^2 dy$

$= \frac{2M}{bh} \int_0^h b\left(\frac{h-y}{y}\right) y^2 dy$

$= \frac{Mh^2}{6}$ ★★



Thus for ▭le



$I = 2 \times$ (MI of △le)

$= 2 \times \left(\left(\frac{M}{2}\right) x^2 / 6\right)$

$= \frac{1}{6} M \times \left(\frac{ab}{\sqrt{a^2+b^2}}\right)^2$

$= \frac{1}{6} M \frac{a^2 b^2}{(a^2+b^2)}$

Q5(b) - Take $F = -\frac{\partial U}{\partial r} = -2kmr$

Now $F_c = \frac{mv^2}{r} = 2kmr$ → now easy

DIAS (2022) Test 6 – optics

Q1(b) ~ In Matrix – for Translation Matrix ~ all +ve ~ if we get v is –ve, then it means v formed behind the lens

for P or f ~ put proper sign eg: $\begin{bmatrix} 1 & -1/f \\ 0 & 1 \end{bmatrix}$ for convex – f is +ve, concave – f is –ve

Now don't see soln of DIAS or Akash – I've done correctly

Just do last operation first to get $\begin{bmatrix} -2.2 & 1/100 \\ 2.2v+32 & v/100+0.6 \end{bmatrix}$

Now $2.2v + 32 = 0$ ~ $v = -14.5\,cm$ ~ Magnification is $0.6 - 0.145 \approx 0.45$

Q1(c) use condition of appearance (distinct) + disappearance (indistinct) for discrete : $\frac{2x}{\lambda_1} - \frac{2x}{\lambda_2} = 1/2$

Q3(a) Molecular origin of optical Activity ~ chirality (non-super-imposable mirror images)

↳ non-symmetry of molecules causes net charge in oscillation of electric fields

< Fresnel Theory for optical activity > ~ split cos & sin into LCP & RCP

Q6 (c) $\Delta t_{total} = \sqrt{\Delta\tau^2_{pulse} + \Delta\tau^2_{material}}$

Max Bit Rate $= \frac{0.7}{\Delta\tau_{total}}$

DIAS (2022) Test 7 – E&M

Q1(d) To show dipole moment is independent of ~~the~~ origin if charge is free

$\vec{p} = \int \vec{r}\,\rho(\vec{r})\,d\tau$

if new origin at $\vec{r}_0$
Now let $r' = r_0 + \vec{r}'$

Thus $\vec{p}' = \int r'\,\rho(r)\,d\tau$

$= \int (r - r_0)\,\rho\,d\tau$

$= \int r\,\rho\,d\tau - r_0 \int \rho\,d\tau$ — r_0 comes out as it is fixed

$\vec{p}' = \vec{p} - r_0\,Q_{total}$

$V_{dip} = \frac{k\,p\cos\theta}{r^2} = \frac{k}{r^2}\int r'\cos\theta\,\rho(r)\,d\tau$

Thus $\vec{p} = \int r\,\rho(r)\,d\tau$

($\cos\theta$ is not carried in)

Q1(e) $n(\nu)d\nu = f(\nu)\,g(\nu)$ (for photon)
(photon)

$= \frac{1}{e^{h\nu/kT}-1} \times \frac{8\pi\nu^2 d\nu}{c^3}$

Now $N = \int_0^\infty n(\nu)d\nu$ ~ put $\frac{h\nu}{kT} = x$

$N = \frac{8\pi}{c^3}\left(\frac{kT}{h}\right)^3 \int_0^\infty \frac{x^2 dx}{e^x - 1}$

$= 8\pi/c^3 \left(\frac{kT}{h}\right)^3 \times [2.404]$

$N \propto T^3$

Q3(b) To get Wien's law ~ use λ-scale and club $\frac{hc}{\lambda kT}$ ~ $\frac{xe^x}{e^x-1} = 5$ ~ $x = 4.95$

and remove the common terms on both sides

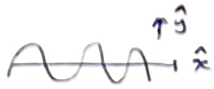
if we use ν-scale eqn

$\frac{xe^x}{e^x-1} = 3$ ~ $x = 2.82$
with $x = h\nu/kT$ (different parameterization → diff. results)

Q4(a) Use $\nabla^2 V = 0$ (cylindrical coordinate) ~ $V = A\ln r + B$ ~ Now $E = -\nabla V$, easy

(V_1 & V_2 give value of A and B)

Q6(c) $A\sin(kx - \omega t)\hat{y}$ ~ correct since



~ wave along x dirn $(x - vt)$
$A\sin(kx - \omega t)$ and displacement in $\hat{y}$.

Q6(b) – Easy: but very long ~ Just use a small Hint → $\int e^x (f(x) + f'(x)) = e^x f(x) + C$

when we differentiate $e^x f(x)$ we get $e^x f'(x) + e^x f(x)$
$= e^x (f(x) + f'(x))$

Q5(a) ! Given $\sigma = k\cos\theta$ (Easy) ✱

$V_{in} = \left(A_0' + \frac{B_0'}{r}\right) + \left(A_1' r + \frac{B_1'}{r^2}\right)\cos\theta$ — Final: $A_1' r\cos\theta$ ✱

$V_{out} = \left(A_0 + \frac{B_0}{r}\right) + \left(A_1 r + \frac{B_1}{r^2}\right)\cos\theta$ — Final: $B_1/r^2 \cos\theta$ ✱ (Mug this, NO&O simple!)

~ ignore higher terms since we only have uptil $\cos\theta$

if $\sigma = \sin^2\frac{\theta}{2}$ we would split as $\sin^2\frac{\theta}{2} = 1 - \cos\theta$ since now in terms of Laplace.

Now ~~Approx~~ take $A_0 = A_0' = 0$ (by our choice of reference frame)

$B_1' = B_0' = 0$ since V_{in} must be finite at $r \to 0$

$A_1 = 0$ since V_{out} must be finite at $r \to \infty$

we $P_n(\cos\theta)$ → Basis (so match) and use finiteness at $r \to 0$ and $r \to \infty$

Basically just take $\cos\theta$ term only since $\sigma = k\cos\theta$

Now only 2 B.C on
$V_{in} = A_1' r\cos\theta$
$V_{out} = \frac{B_1}{r^2}\cos\theta$

1) $V_{out} = V_{in}|_{r=R}$

2) $\frac{\partial V_{in}}{\partial r} - \frac{\partial V_{out}}{\partial r}\Big|_{r=R} = \frac{\sigma}{\varepsilon_0}$

Basically: $(\vec{E}_{out} - \vec{E}_{in}) = \sigma(\ldots$

Thus $V_{in} = A_1' r\cos\theta$
$V_{out} = \frac{B_0}{r} + \frac{B_1}{r^2}\cos\theta$

Now doing $V_{in} = V_{out}$ (at $r = R$)
$B_0 = 0$ by comparing coefficients

Now $\frac{\partial V_{in}}{\partial r}\Big|_{r=R} - \left(\frac{\partial V_{out}}{\partial r}\Big|_{r=R}\right) = \frac{\sigma}{\varepsilon_0} = \frac{k\cos\theta}{\varepsilon_0}$

$A_1'\cos\theta + \frac{2B_1\cos\theta}{R^3} = \frac{k\cos\theta}{\varepsilon_0}$

and $V_{in} = V_{out}|_{r=R}$ → $A_1' R = \frac{B_1}{R^2}$ ~ solving we get A_1' and B_1

soln

$$V(r,\theta) = \begin{cases} k\cos\theta\, r/3\varepsilon_0 & \sim r \le R \\ k\cos\theta\, R^3/3\varepsilon_0 r^2 & \sim r \ge R \end{cases}$$

DIAS (2022) Test 8 - Thermo

Q1(d) = as $P\uparrow$ ~ melting point $\uparrow$ just like Boiling ~ since more heat reqd to overcome pressure.

use $\frac{dP}{dT} = \frac{L}{T(v_2-v_1)}$ ~ put $v = \frac{1}{\rho}$

and put $T \to T_{\text{initial}}$

(all calculations in kelvin only)

(1 cal = 4.186 Joule

since $R \approx 1.98$ cal $mol^{-1}k^{-1}$)

$\frac{dP}{dT}$ → +ve $\frac{\Delta P}{\Delta T}$ ~ ΔP ~~[illegible]~~ +ve mean ΔT also +ve

Q1(e)(i) assume $g_i = 1$, $Z = \sum g_i e^{-E_i/kT}$

Q1(a)

$f = \frac{1}{e^{E/kT - \alpha} \pm 1}$ ~ for QM → classical: $e^{-\alpha} \gg 1$

α doesn't have kT in denominator $\alpha - \beta E_i$ ~ $\beta = 1/kT$!

we later chose $\alpha = \frac{E_F}{kT}$... etc.

or μ/kT, etc ~ but if we alter $\alpha - \beta E_i \to \alpha - \frac{E_i}{kT}$

approximate

Now $n(E)dE = f(E)g(E) = e^{\alpha - E/kT} \times \frac{2\pi V}{h^3}(2m)^{3/2}E^{1/2}dE$ (for Maxwell Boltzmann)

take MB case

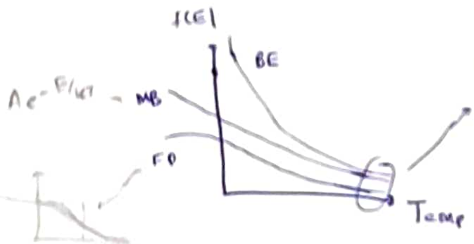
eg: MB: $e^{\alpha}e^{-E_i/kT} = Ae^{-E/kT}$

$N = \int_0^\infty \frac{2\pi V}{h^3}(2m)^{3/2}E^{1/2} e^{\alpha - E/kT} dE$ → is given $e^{\alpha} = \frac{N}{V}\left(\frac{2m\pi kT}{h^2}\right)^{-3/2}$

Note: this term also came in semi-conductor derivation: there also we had used $N = \int f(E)g(E)dE$.

Now $e^{-\alpha} \gg 1$ → $\frac{V}{N}\left(\frac{2m\pi kT}{h^2}\right)^{3/2} \gg 1$

for convergence of QM → classical.



Q2(a) ~ refer 2(a) in DIAS 2021 Soln (Mine)

Q4(b) ~ $k = \frac{1}{\text{Bulk Modulus}} = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ ~

Now $dW = PdV$

put $dV = -kVdP$

Thus $\int dW =$ ~~[illegible]~~

$= -kV\int PdP$

$kV\frac{P^2}{2}$ $V = m/\rho$

Note: we just took V as a constant & took it out of the bracket. This is NOT correct but just assume and do it. There is no other correct way. NO overthinking

we are allowed to make assumptions and approximations like this. Just like we take T_i in ΔT instead of doing full integration, etc...

Q5(b) ~ refer to DIAS 2021 Soln for $\frac{E_S}{E_T} = \frac{C_P}{C_V}$

Q6(a) - $(C_v)_{water}$ = 1 cal gram^{-1} k^{-1} (where 1 cal = 4.186 J since $R = 1.98$ cal $k^{-1}mol^{-1}$)

also 1eV = 1e × 1 J/C = 1.6×10^{-19} C × J/C = 1.6×10^{-19} J

Q6(b) refer my DIAS 2021 Soln 6(a)

Q2(b) - I've written in Bottom of DIAS 2021 soln and also in DIAS(3) Notes.

DIAS (2021) – Test 9 (Test 10 ~~corrections~~/additions were made in DIAS Notes itself)

Q1(b) α-particle : $Z=2$, $A=4$
AU (gold : $Z=79$, $A=179$

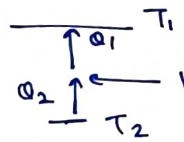
Now $b = \frac{k}{2E'} \cot\frac{\phi}{2}$ → $\phi' = \phi$ and $E' = E - E_{CM}$

$= \frac{m_2}{m_1+m_2} E_\alpha$

$= \cancel{\frac{179}{179+4} E_\alpha}$ $\frac{197}{197+4} E_\alpha$

Q5(a) coefficent of performance Fridge → $\beta = \frac{Q_2}{W}$ (η was $= \frac{W}{Q_1}$)

for carnot Fridge : $\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$



Fridge takes Q_2 from low T_2, adds W, and releases Q_1 at high T_1.

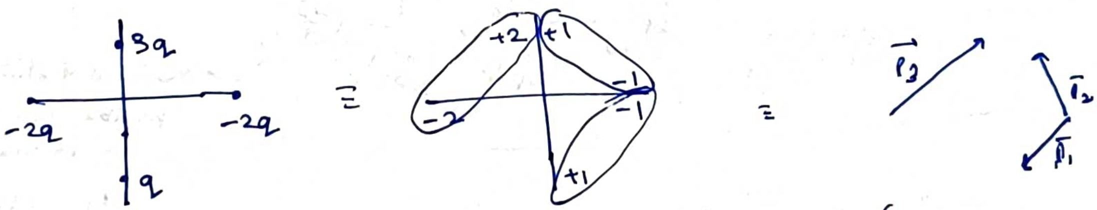
Q6(a) It matches with SPEM proof.

$P = np_m = n\alpha E_m \approx n\alpha E_0$

put $\frac{n\alpha}{3\varepsilon_0} = \frac{k-1}{k+2}$ – and $p = \frac{4}{3}\pi a^3 P$ – Then $V = \frac{kp\cos\theta}{r^2}$ – our answer matches

$V(r,\theta) = -\left[1 - \left(\frac{k-1}{k+2}\frac{a^3}{r^3}\right)\right] E_0 r\cos\theta$

Q6(b)



($\vec{p}$ from − → +)

$\vec{p} = \vec{a}q$ ★

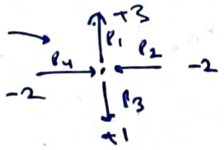
doing $\vec{p}_1 + \vec{p}_2 + \vec{p}_3$ → $\underline{2aq\,\hat{z}}$ (Now easy ...)

(x & y component cancel)

$\uparrow p = 2aq\,\hat{z}$

(or) add 4 charge in center → $-3+2+2-1 = 0$ (it cancels out)

so we get 4 dipoles →



– we add all $\vec{p}$ → $2aq\,\hat{z}$

Q7 (c) $\frac{dE}{dt} = \text{Power} = \sigma A T^4$ $\quad A = \cancel{\pi R_E^2}\ \underline{4\pi R_E^2}$ ★

NOTE – in GBG 11.7 – even for $P = \frac{I}{c}$ – Force $= P \times A$

– $\underline{A = 4\pi R_E^2}$ & not πR^2 (app. to expected) (verified in GBG & in net & Nucleade also)

Q8 (b) pressure coeff. of expansion : $\frac{\Delta P}{P} = \alpha \Delta T$ (Basically we have, P, V, T, S, since isochoric & isentropic mentioned, Thus we have P & T left)

~ $\alpha = \frac{1}{P}\frac{\partial P}{\partial T}$

To show: $\dfrac{\left(\frac{\partial P}{\partial T}\right)_s}{\left(\frac{\partial P}{\partial T}\right)_v} = \dfrac{\gamma}{\gamma-1}$

we work Backwards:

(soln from Dayar)

~ $\dfrac{\gamma}{\gamma-1} = \dfrac{C_p}{C_p - C_v}$, Now $C_p = T\left(\frac{\partial S}{\partial T}\right)_p$ & $C_v = T\left(\frac{\partial S}{\partial T}\right)_v$

Now to get : $C_p - C_v$:

$$dQ = dU + PdV$$

$$dQ = C_v dT + \left(\frac{\partial U}{\partial V}\right)_T dV + PdV$$

put $\partial U = \partial Q - P\partial V$

$$dQ = C_v dT + T\left(\frac{\partial S}{\partial V}\right)_T dV - \cancel{PdV} + \cancel{PdV}$$

$$C_p = \left(\frac{dQ}{dT}\right)_p = C_v + T\left(\frac{\partial S}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p$$

$$C_p = C_v + T\left(\frac{\partial P}{\partial T}\right)_v\left(\frac{\partial V}{\partial T}\right)_p$$

Thus $C_p - C_v = T\left(\frac{\partial P}{\partial T}\right)_v\left(\frac{\partial V}{\partial T}\right)_p$

Other way to prove is:

$S = S(V,T)$: $dS = \left(\frac{\partial S}{\partial V}\right)_T dV + \left(\frac{\partial S}{\partial T}\right)_v dT$

$$T\left(\frac{\partial S}{\partial T}\right)_p = \cancel{\partial S}\, T\left(\frac{\partial S}{\partial T}\right)_v + T\left(\frac{\partial S}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p$$

$$C_p = C_v + T\left(\frac{\partial P}{\partial T}\right)_v\left(\frac{\partial V}{\partial T}\right)_p$$

Now $\dfrac{\gamma}{\gamma-1} = \dfrac{C_p}{C_p - C_v} = \dfrac{T\left(\frac{\partial S}{\partial T}\right)_p}{T\left(\frac{\partial P}{\partial T}\right)_v\left(\frac{\partial V}{\partial T}\right)_p} = \dfrac{1}{\left(\frac{\partial P}{\partial T}\right)_v} \times \dfrac{\left(\frac{\partial S}{\partial T}\right)_p}{\left(\frac{\partial V}{\partial T}\right)_p} = \dfrac{1}{\left(\frac{\partial P}{\partial T}\right)_v} \times \left(\frac{\partial S}{\partial V}\right)_p$ ~ maxwell of this is not possible, but we can do maxwell of $\left(\frac{\partial V}{\partial S}\right)_p$ (Thus take inverse)

$$\frac{1}{\left(\frac{\partial P}{\partial T}\right)_v\left(\frac{\partial V}{\partial S}\right)_p} = \frac{1}{\left(\frac{\partial P}{\partial T}\right)_v\left(\frac{\partial T}{\partial P}\right)_s} = \frac{(\partial P/\partial T)_s}{(\partial P/\partial T)_v} \quad //$$

Q4 (b) I have done correctly : $\lim_{\beta \to m\pi} \dfrac{\sin^2 N\beta}{\sin^2 \beta} = N^2$

and $I_{secondary} = \dfrac{I_{primary}}{1 + (N^2-1)\sin^2\beta_0}$ ~ using $\tan N\beta_0 = N\tan\beta_0$

for secondary : put $\sin^2\beta_0 \simeq 1$

$$I_{secondary} = \frac{I_{primary}}{N^2}$$

DIAS (~~2022~~ 2022) - Test ~~10~~ 9 ~~(Test 10 soln, I have written in DIAS Notes)~~

Q 1(a) Max energy for satellite ~ $E=0$ ~ Thus parabola is satellite ✓
(since Bound is $E \leq 0$) (Hyperbola means escape)

Q 1(b)
$2\mu\beta\theta = (2n-1)\lambda/2$ ~ for coinciding ~ at same β $\frac{2n-1}{2n+1} = \frac{\lambda_2}{\lambda_1}$ ~ we get $n = 932$ and thus β can be found.
(minima)

Q 2(a) Tough → see soln ! (DIAS soln)

Note: θ must be in Radians!
(degree $\times \frac{\pi}{180}$)

$T = \frac{Mg}{1+R^2/k^2}$ and $v^2 = \frac{2gh}{1+k^2/R^2}$ ~ $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$ → $v^2 = \frac{2gh}{1+k^2/R^2}$
put $\omega = \frac{v}{R}$ and $I = mk^2$

Q 3(c) circular Pinhole
($J_1(z)$ - minima ~ $1.22\pi, 2.23\pi, 3.24\pi$
($1.43\pi, 2.46\pi$ ~ maxima of single slit diffraction)

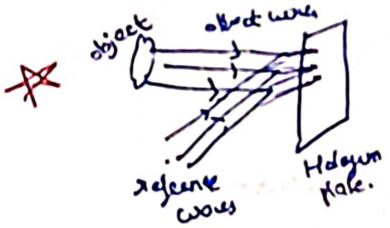
in $2d\sin\theta = n\lambda$
$R = f \times \theta$

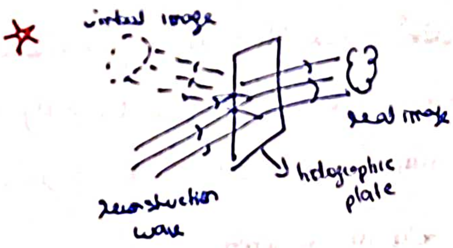
* $\sin^{-1}$ gives θ in degree. convert to radian for $r = f\theta$
rad = degree $\times \frac{\pi}{180}$

[Always, Temp → kelvin ✓
angle → Radian]

Q 4(c) Hologram recording

Hologram viewing/ reconstruction





Q 5(a) $\nabla \cdot D = \rho_{free}$ ~ Now divergence in cylindrical coordinates ~ $\nabla \cdot A = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial \phi}(A_\phi) + \frac{\partial A_z}{\partial z}$
& $\oint D \cdot ds = Q_{free}$ ~ in question we have $\hat{z}$ only ~ thus $\frac{\partial A_z}{\partial z} = \rho_{free}$

use $\int_0^{2\pi}\int_0^{R} r^2\cos^2\phi \; r\,dr\,d\theta$

Q 5(b) for $\rho = \frac{\rho_0 x}{a}$ and $E = 0$ at $x = 0$ and $V = 0$ at $x = a$. Find E & V
↳ $\nabla^2 V = -\rho/\epsilon_0$ → use this * (put $\rho = \frac{\rho_0 x}{a}$) (use Laplace for 1-D : x-dirn)

* Q 6 (a) Find $\bar{E}$ & $\bar{V}$ for $\rho = \begin{cases} \frac{\rho_0 r}{R}, & 0 \leq r \leq R \\ 0, & r > R \end{cases}$ → Here don't use Laplacian, it'll be too complex ! (use Gauss law)

(for $\rho = \rho(x)$ - laplace : $\partial^2 V / \partial x^2$ ✓
for $\rho = \rho(r)$ - cylindrical → avoid laplace : do Gauss law ✓ *

PTO

first for $r>R$ ~ $\rho=0$ ~ $E_{out} = \frac{\rho_0 R^3}{4\varepsilon_0 r^2}\hat{r}$ for $r>R$ ~ Now $V = -\int E.d\vec{l}$

$= \frac{\rho_0 R^3}{4\varepsilon_0 r}$ ~ assume $V=0$ as $r \to \infty$

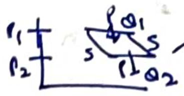
Now for $r<R$ ~ $E_{in} = \frac{\rho_0 r^2}{4R\varepsilon_0}\hat{r}$ for $r \le R$ ~ Now $V_{in} - V_{out} = V_{in}(r) - V_{in}(R) = -\int_r^R \vec{E}.d\vec{l}$

(see Dagar soln or DIAS official soln same for all steps)

we can find $V_{in}(r)$ since $V(R)$ is known from $r>R$ expression of $V(r)|_{r=R}$.

Q6(b) easy : use $\phi = LI$ and $\phi = MI$

Q6(c) Energy in inductor Rate $= \frac{dE}{dt} = LI\frac{dI}{dt}$

Q8(a) for  SPSP process ~ same process as SVSV except instead of using $TV^{\gamma-1}=k$, use $P^{1-\gamma}T^{\gamma}=k$

subtract two for T_3-T_2 & T_4-T_1 like in SVSV process

$\eta = 1 - \frac{C_v(T_3-T_2)}{C_p(T_4-T_1)} = 1 - \left(\frac{P_1}{P_2}\right)^{\gamma-1/\gamma}$

in SVSV we had $1 - \frac{1}{\rho_c^{\gamma-1}}$

$\rho_c = V_1/V_2$

Q5(c) $\vec{H} = -0.1\cos(\omega t - z)\hat{i} + 0.5\sin(\omega t - z)\hat{j}$ and $\vec{k} = 1\hat{k}$ (since $\omega t - kz$ is $\omega t - z$, $k=1$)

Now use $\vec{B} = \frac{\vec{k}\times\vec{E}}{\omega}$ ~~[illegible]~~

$\vec{k}\times\vec{E} = \begin{vmatrix} i & j & k \\ 0 & 0 & k \\ E_x & E_y & E_z \end{vmatrix}$ ~ $\hat{i}(-E_y) + \hat{j}(E_x) = \omega\mu_0 H = \mu_0\omega\left[-0.1\cos(\omega t - z)\hat{i} + 0.5\sin(\omega t - z)\hat{j}\right]$

we can compare $\hat{i}$ & $\hat{j}$ to get E_x & E_y

$E_y = 0.1\mu_0\omega\cos(\omega t - z)$ & $E_x = \mu_0\omega(0.5)\sin(\omega t - z)$

Note : $y = a\sin(kx - \omega t)\hat{j}$ ~ x is dirn of wave, y & $\hat{j}$ → displacement

Q7(c) $|S|_{scatt} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{\omega_0^4}{8\pi c^3}(P_0\sin\theta)^2$ ~ experiment for Rayleigh : 

(same as sunset phenomena)

(class X NCERT)

Q4(b) Answered in DIAS

DIAS (2022) - Test 10

Q1(c) $S_x|\tfrac{1}{2}\rangle = \frac{\hbar}{2}\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix}0\\1\end{bmatrix} = \frac{\hbar}{2}|-\tfrac{1}{2}\rangle$ where $|\tfrac{1}{2}\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ and $|-\tfrac{1}{2}\rangle = \begin{bmatrix}0\\1\end{bmatrix}$ (eigenvectors of S_z)

$S_x|-\tfrac{1}{2}\rangle = \frac{\hbar}{2}\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix}1\\0\end{bmatrix} = \frac{\hbar}{2}|+\tfrac{1}{2}\rangle$

similarly: $S_y|\tfrac{1}{2}\rangle = \frac{i\hbar}{2}|-\tfrac{1}{2}\rangle$, $S_y|-\tfrac{1}{2}\rangle = -\frac{i\hbar}{2}|\tfrac{1}{2}\rangle$, $S_z|\tfrac{1}{2}\rangle = \frac{\hbar}{2}|\tfrac{1}{2}\rangle$, $S_z|-\tfrac{1}{2}\rangle = \frac{\hbar}{2}|-\tfrac{1}{2}\rangle$

Q1(d) Banwell pg 168: $\varepsilon_v = (v+\tfrac{1}{2})\omega_e - (v+\tfrac{1}{2})^2\omega_e x_e$ — using both terms

Now ΔE between levels $= \varepsilon_{v+1} - \varepsilon_v = \bar{\omega}_e[1 - 2x_e(v+1)]\ \text{cm}^{-1}$

at $v_{max} \rightsquigarrow \bar{\omega}_e(1 - 2x_e(v_{max}+1)) = 0 \longrightarrow v_{max} = \frac{1}{2x_e} - 1$; for HCl: $x_e = 0.0174$
(since $\Delta E \to 0$) Thus $v_{max} = 27$

(since spacing b/w levels is decreasing)

Q1(e) Banwell pg 117:

Q2(b) use $L_xL_y = \frac{(L_+ + L_-)}{2}\frac{(L_+ - L_-)}{2i} = \frac{1}{4i}[L_+^2 - L_-^2 + L_-L_+ - L_+L_-]$ — free operators: (don't cancel) $L^+L^- - L_-L_+ \neq 0$ ★ IMP

Now $\langle l,m|L_+^2|l,m\rangle = 0$
$\langle l,m|L_-^2|l,m\rangle = 0$; and use $L_+|l,m\rangle = \sqrt{(l-m)(l+m+1)}$
$L_-|l,m\rangle = \sqrt{(l+m)(l-m+1)}$
(b/w l & m)

derivation
$L_+ = L_x + iL_y$
$|c_1|^2 = \langle\psi|L_-L_+|\psi\rangle$
$= \langle\psi|L^2 - L_z^2 + i[L_y, L_x]\rangle$
★ IMP ★ $L_yL_x - L_xL_y \neq 0$ ($= -i\hbar L_z$)
$|c_1|^2 = l(l+1) - m_l^2 + m_l$

Q3(a) $L_x = yp_z - zp_y$

$[p^2, L_x] = [p_x^2 + p_y^2 + p_z^2, L_x]$

$= \cancel{[p_x^2, yp_z - zp_y]}^{\,0} + [p_y^2, yp_z - zp_y] + [p_z^2, yp_z - zp_y]$

(operators of diff. variables)

~ now just keep splitting using product rule of operators

$[p_y^2, yp_z - zp_y] = [p_y^2, yp_z] - \cancel{[p_y^2, zp_y]}^{\,0} = y\cancel{[p_y^2, p_z]}^{\,0} + [p_y^2, y]p_z$
$= -2i\hbar\, p_yp_z$

similarly $[p_z^2, yp_z - zp_y] = 2i\hbar\, p_zp_y$.

(a) Thus $[p^2, L_x] = -2i\hbar[p_y, p_z]$

(b) Thus also $[p^2, L_y] = -2i\hbar[p_z, p_x]$

(c) also $[p_y, L_y] = [p_y, zp_x - xp_z] = 0$ (all are diff. variables)

Q3(b) Given $\psi(x,y,z) = \frac{1}{4\sqrt{\pi}}\frac{2z^2 - x^2 - y^2}{r^2} - \sqrt{\frac{3}{\pi}}\frac{xz}{r^2}$

1st step ~ remove $\frac{1}{4\sqrt{\pi}}$, $\sqrt{\frac{3}{\pi}}$, etc ~ we don't need! ~ take A & B ... (ALWAYS remember in such question DONT WORRY ABOUT NORMALIZATION) take A & B ★

Now $\frac{3z^2 - r^2}{r^2} = 3\cos^2\theta - 1$ and $\frac{xz}{r} = \cos\theta\sin\theta\sin\phi$

(use $z = r\cos\theta$
$x = r\sin\theta\cos\phi$

Now $Y_2^0 = A(3\cos^2\theta - 1)$ $\quad Y_2^{\pm1} = Be^{\pm i\phi}\sin\theta\cos\theta$

Thus $\psi = AY_2^0 + B(Y_2^{-1} - Y_2^1)$ — to get $\sin\theta\cos\theta\cos\phi$

Now $L^2|l,m\rangle = l(l+1)\hbar^2$

Thus $L^2\psi = AL^2Y_2^0 + B(L^2Y_2^{-1} - L^2Y_2^1)$

$= 6\hbar^2[AY_2^0 + B_1(Y_2^{-1} - Y_2^1)]$

$= 6\hbar^2\psi$ — ψ is eigenfⁿ of L^2

$L_z\psi = -\hbar B(\psi Y_2^{-1} + Y_2^1)$ — Thus ψ not eigenfⁿ of L_z

Now $L_+\psi$ – use $L_+|l,m\rangle = \sqrt{(l-m)(l+m+1)}\,\hbar\,|l,m+1\rangle$

$AL_+|2,0\rangle + B(L_+|2,-1\rangle - L_+|2,1\rangle)$

$= \hbar[\sqrt{6}AY_2^1 + B(\sqrt{6}Y_2^0 - 2Y_2^2)]$

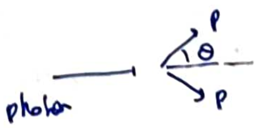
Now $\langle\psi|L_+|\psi\rangle = \langle A\langle 2,0| + B\langle 2,-1| - B\langle 2,1| \, \big(\hbar(\sqrt{6}A|2,1\rangle + \sqrt{6}B|2,0\rangle - 2B|2,2\rangle)\big)\rangle$

$= 0$

Q4(a) Doppler shift $\Delta\nu = \frac{\nu v}{c}$ (using $\nu = \nu_0\sqrt{\frac{1+v/c}{1-v/c}}$)

$= v/\lambda$

Q5(a) To prove pair production cannot occur in free space.



$p_{photon} = \frac{h\nu}{c}$

$p_{product} = 2mv\cos\theta$

Now conserve momentum

$\frac{h\nu}{c} = 2(mc^2)\frac{v}{c} \times \cos\theta$

since $\frac{v}{c} < 1$ always

Thus $h\nu < 2mc^2$ always — but we needed $h\nu = 2(mc^2)$

Thus pair prodⁿ not possible in free space, else violation of conservation of energy

Q5(d) Evidence for Quark Model:

↳ deep inelastic scattering of electrons on Nucleons

↳ Quark model can explain abnormal magnetic moments of Nucleons

↳ SU(3) symmetry can be explained by Quark Model

Q5(b) Always remember: $L = \Delta I = |I_f - I_i|$ — L has multiple values from $I_f - I_i$ to $I_f + I_i$

also: γ-transition → min $L \geq 1$ (since photon spin $= 1$)

Q6(a) splitting of energy levels in shell model due to → centrifugal potential ($\frac{1}{2}m\omega^2 r^2$)
→ spin-orbit coupling

↳ wide gaps occur at certain intervals which are analogous to separate shells

Q6 (c) Tough → refer DIAS soln

$P = \frac{m}{\hbar^2} V_0 ab$

Q7 (a) ISB worked example: to show $E = \frac{\hbar^2 P}{ma^2}$ for $P \ll 1$ (where E is energy of lowest Band)

A →

$\frac{P \sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka = \pm 1$ (since lowest Band corresponds to $ka = \pm \pi/a$)

Thus $P \frac{\sin \alpha a}{\alpha a} = 1 - \cos \alpha a = 2 \sin^2 \left(\frac{\alpha a}{2}\right)$

Thus $\frac{2P}{\alpha a} \sin \frac{\alpha a}{2} \cos \frac{\alpha a}{2} = 2 \sin^2 \left(\frac{\alpha a}{2}\right)$

$\frac{P}{\alpha a} = \tan \left(\frac{\alpha a}{2}\right)$

$\alpha^2 = \frac{2mE}{\hbar^2} = \frac{2P}{a^2}$

Now since $P \ll 1$ ~ $\frac{P}{\alpha a} = \frac{\alpha a}{2}$ → $\alpha^2 = \frac{2P}{a^2}$

Thus $E = \frac{\hbar^2 P}{ma^2}$

Q7 (b) Tough & Random

Q8 (c) Enhancement MOSFET, Depletion MOSFET, JFET working, characteristics & differences by Aniket in DIAS soln