CIVIL SERVICES EXAMINATION (MAINS) 2020
PHYSICS PAPER - II:QUANTUM MECHANICS
TUTORIAL SHEET: 1
Foundations of Quantum Mechanics

1. How Davisson - Germer electron diffraction experiment confirms the de -Broglie hypothesis of matter wave? $\phi = 50^\circ$, $\theta = 65^\circ$ $\lambda = 1.65$ Å **(1995)**

2. Using Heisenberg uncertainty principle, find the ground state energy and Bohr radius of hydrogen atom. ↳ defⁿ it → write expression → Proof → make diagram [$\Delta x \Delta p \sim \hbar$] **(1997)**

3. Using the uncertainty principle $\Delta x \Delta p \sim \frac{\hbar}{2}$, estimate the minimum energy of a particle in a simple harmonic potential $U = \frac{1}{2}kx^2$ → defⁿ → particle as matter wave group $\lambda = \frac{hc}{pc}$ **(1999)**

4. Find the de - Broglie wavelength associated with an electron of energy (i) 10 eV and (ii) 10 MeV

$\lambda = \frac{1.22\ nm}{\sqrt{K.E\ (in\ eV)}}$ & davisson germer proved it. $m_0c^2 \sim 0.51$ MeV ⇒ write nature of wave **(1999)**

5. The momentum of an electron is 600 KeV/c. Determine its de Broglie wavelength and the phase and group velocities of its de Broglie waves. **(2000)**

$\lambda = \frac{hc}{pc} = \frac{hc}{600\ KeV} = 2.06 \times 10^{-12}$ m → defⁿ

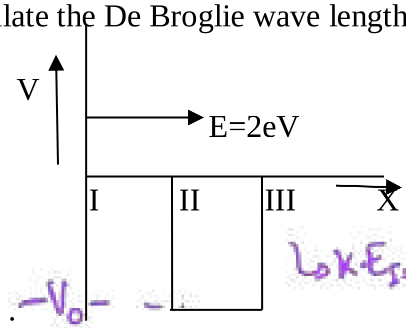
6. A 200 eV increase in the energy of an electron changes its De Broglie wave length by a factor of two. Calculate the initial De Broglie wave length of the electron. **(2002)**

→ defⁿ ↳ applicatⁿ → e⁻ waves, davisson expt. ↳ Fresnel diffractⁿ & specular reflectⁿ of neutral atoms i.e. existence of atomic waves

7. (i) Explain what do you understand by Heisenberg's uncertainty principle. Using this Principle, determine the energy of the ground state of a one dimensional simple harmonic Oscillator. ↳ applicatⁿ & diagram.

(ii) An electron having an energy 2 eV is traveling in the region where V(x) varies as shown below:

Calculate the De Broglie wave length of the electron in region I, II and III. **(2003)**



$K.E_{II} = (2+V_0)$ eV

for non-relativistic case,

↳ $K.E_{I=III} = 2$ eV

$\lambda = \frac{1.22}{\sqrt{K.E (in\ eV)}}$ nm

$U = (A - \langle A \rangle)$ & $V = (B - \langle B \rangle)$

$(\Delta A)^2 = \langle U^2 \rangle$ & $(\Delta B)^2 = \langle V^2 \rangle$

8. (i) If x and P_x are two conjugate variables, show that

$$(\Delta x)(\Delta P_x) \geq |\langle [x, P_x] \rangle / 2i|$$

(ii) An electron has a speed of 4×10^5 m/s, accurate to 0.01%. With what fundamental Accuracy can we locate the position of electron? **(2004)**

$v << c$ ⇒ non-relativistic case

$\Delta p \sim m_e \Delta v$ | $\Delta x \sim \frac{\hbar}{2 m_e \Delta v}$

$\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{\sqrt{(1.51)^2 - (0.51)^2} \text{ MeV}}$

9. Calculate the wave length of neutrons at a temperature of 20º C. **(2004)**

$\lambda = \frac{hc}{pc}$

10. Calculate the de Broglie wavelength of an electron moving with a kinetic energy of 1MeV. **(2005)**

↳ relativistic case

11. Calculate the de Broglie wavelength of a thermal neutron at $27^0 C$ temperature. **(2006)**

↳ non-relativistic as $k_B T < 1$ eV

12. (i) State and explain Heisenberg's uncertainty principle. Experimental data reveal that no electron in an atom has energy greater than 4 MeV. Assuming that the radius of a nucleus is 10^{-14} m, show using Heisenberg's uncertainty principle that an electron cannot exist inside the nucleus.
(ii) Calculate the de Broglie wavelength of thermal neutrons at 300 K. **(2007)**

13. Derive Bohr's angular momentum quantization condition in Bohr's atomic model from the concept of de Broglie waves. **(2010)**

$n\lambda = 2\pi r_n$ | $\lambda = \frac{h}{mv} \Rightarrow \boxed{mvr = \frac{nh}{2\pi}}$

14. Calculate the wavelength of De Broglie waves associated with electrons accelerated through a P.D. of 200 Volts. **(2011)**

def^n → $\lambda = \frac{h}{p}$ → Since energy < 0.51 MeV; K.E. = 200 eV ⇒ $\lambda = \frac{1.22 \text{ nm}}{\sqrt{200}}$

15. Estimate the size of the hydrogen atom and the ground state energy from the uncertainly principle. **(2011)**

def^n uncertainty principle → write expression → write assumptions ↳ as $\Delta p_n \approx h$ ⇒ solve $\frac{\partial E}{\partial r}\Big|_{r=a_0} = 0$; $a_0 = \frac{4\pi\epsilon_0 e^2}{m_e \hbar^2}$

16. Use the uncertainly principle to estimate the ground state energy of a linear harmonic oscillator **(2012)**

17. In a series of experiments on the determination of the mass of a certain elementary particle, the results showed a variation of _+ (plus-minus) 20 m(e), where m(e) is the electron mass. Estimate the lifetime of the particle. **(2013)**

$\{\Delta E \times \tau \approx \hbar/2$ & $\Delta E = 20 m_e c^2\}$

18. Find the deBroglie wave length of (i) a neutron (ii) and electron moving with kinetic energy of 500 eV .(1 eV = 1.602 x 10 ^-19 J) **(2014)**

def^n uncertainty principle $\boxed{\Delta E \Delta t \geq \frac{\hbar}{2}}$

19. The mean life of Lambda particle is 2.6 x 10^-10 s. What will be the uncertainty in the Determination of its mass in eV? **(2014)**

↳ assuming $\Delta t \approx \tau$ & $\Delta E \sim \Delta m c^2$

assuming wave particle duality → nature

20. Find the energy, momentum and wavelength of photon emitted by a hydrogen atom making a direct transition from an excited state with n=10 to the ground state. Also find the recoil speed of the hydrogen atom in this process. **(2016)**

$E_n = \frac{hc}{\lambda} = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ eV | $\frac{h}{\lambda} = m v_{recoil}$ [conservation of momentum]

21. An electron is confined to move between two rigid walls separated by 10^{-9} m. Compute the de Broglie wavelengths representing the first three allowed energy states of the electron and the corresponding energies. **(2016)**

$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$ | $\lambda = \frac{h}{p}$ | $p = \sqrt{2mE}$ ⇒ $p = \frac{n\pi\hbar}{L}$ | λ → range & nature

22. A typical atomic radius is about 5 x 10^{-15} m and the energy of β-particle emitted from a nucleus is at most of the order of 1 MeV. Prove on the basis of uncertainty principle that the electrons are not present in nuclei. **(2016)**

$\Delta p \Delta x \geq \frac{\hbar}{2}$ ⇒ $pc \approx 100$ MeV
↳ β-particle don't come from nuclei

↳ If electrons were to be in nuclei, it would have been difficult to use photoelectric effect which is basis of modern smoke detectors

23. Using uncertainty principle, calculate the size and energy of the ground state hydrogen atom. **(2016)**

$a_0 = \frac{4\pi\epsilon_0 e^2}{m_e \hbar^2} = 0.53$ Å

24. A beam 4.0 keV electrons from a source is incident on a target 50.0 cm away. Find the radius of the electron beamspot due to Heisenberg's uncertainty principle. **(2017)**

25. Estimate the de Broglie wavelength of the electron orbiting in the first excited state of the hydrogen atom. **(2017)**

$\lambda = \frac{h}{p}$ & wave particle duality & davisson-germer

$\lambda = 1.65$ Å

26. Show that the mass and linear momentum of a quantum mechanical particle can be given by $m = h/(\lambda v)$ and $p = h/\lambda$, respectively, where h, λ and v are Planck's constant, wavelength and velocity of the particle, respectively. Comment on the wave-particle duality from these relations. **(2019)**

→ two canonically conjugate variables (whose operators don't commute) can't be simultaneously measured.

27. State and express mathematically the three uncertainty principles of Heisenberg. Highlight the physical significance of these principles in the development of Quantum Mechanics. **(2019)**

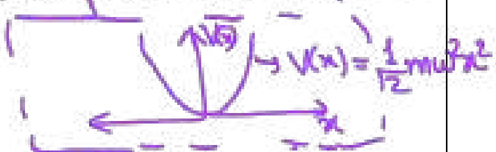
SHO, e^- in nucleus, spin shows uncertainty, tunneling, explains existence of non zero ground energy, fusion, Metastable states

→ If $E < V$ & neighborhood points ⇒ taylor series expansion

28. For a free quantum mechanical particle under the influence of a one-dimensional potential, show that the energy is quantized in discrete fashion. How do these energy values differ from those of a linear harmonic oscillator? $\rightarrow E_n = (\frac{1}{2} + n)\hbar\omega$ $\quad \frac{n^2\pi^2\hbar^2}{2mL^2}$ **(2019)**

29. Using the uncertainty principle $\Delta x \Delta p \geq \hbar/2$, estimate the ground state energy of a harmonic oscillator. (2020)

Harmonic oscillator is ideal conception & simplest oscillator.



for 1-D, $V(x) = \frac{1}{2} m\omega^2 x^2$

Assuming a conservative system, $\frac{\partial E}{\partial x}\Big|_{x_0} = 0 \Rightarrow \boxed{x_0 = \frac{\sqrt{\hbar}}{\sqrt{2m\omega_0}}}$

$E_0 = \frac{\hbar\omega}{2}$ is quantum mechanical result.

TUTORIAL SHEET: 2
Schrodinger's Wave equation and applications

1. Write down Schrodinger's equation in one dimension and explain the significance of the eigen values and eigen functions of this equation. $\to H\psi = i\hbar \frac{\partial \psi}{\partial t} \Rightarrow$ Time independent $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$

   Solve the Schrodinger's equation if the potential function V is given as

   $$V(x) = V_o \text{ (a constant) for } |x| < \frac{a}{2}.$$

   a. $= 0$ for $|x| > \frac{a}{2}$

   $\psi_2(\pm \frac{a}{2}) = \psi_{1,3}(\pm\frac{a}{2})$, $\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_{1,3}}{\partial x}$ at $x = \pm a/2$

   State the boundary conditions you use along with their justification. (In the solution consider particles incident from one side only). If the total energy E < V_o, show that there is an important difference with classical mechanics. Explain, with the help of the uncertainty principle that E < V_o does not mean that kinetic energy is negative. **(1989)**

2. Set up Schrodinger's equation for a one-dimensional harmonic Oscillator. Assuming that the solution is of the form $\psi = Ae^{-\alpha x^2} f(x)$, where f (x) is a polynomial in x, find the energy Eigen values and the ground state Eigen function. Comment on the relation between the ground state energy and Heisenberg's uncertainty principle. **(1990)**

   $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}m\omega^2 x^2 = E\psi$

3. Set up Schrodinger's equation for a free particle confined in a cubical box and find the energy eigen values. What form will Pauli principle take for elections in such a box? Deduce the zero point energy if the length of the box be 10^{-10} metre and there be 10 electrons in it. (The coulomb interaction may be disregarded). **(1991)**

   Assuming time independent one, $V = 0$ in a box | $\psi(x) = A\sin(kx) + B\cos(kx)$, $\psi(x)|_{x=0,L} = 0 \Rightarrow k = \frac{n\pi}{L} \Rightarrow E = \frac{n^2\pi^2\hbar^2}{2mL^2}$ & P_n graph

4. Using Schrodinger's equation show that a free particle in a box can have only discrete energy values. **(1992)**

5. Find the energy Eigen values of a one-dimensional harmonic oscillator whose Hamiltonian is given by

   $$H = \frac{P^2}{2m} + \frac{1}{2}kx^2$$ **(1994)**

   $E_n = (n+\frac{1}{2})\hbar\omega$

6. An election is confined in a one - dimensional box of 1 A° width. Draw the energy level diagram up to three energy states and also draw the corresponding normalized Eigen functions. Derive the expressions for Eigen functions and Eigen values used in the calculations. Show that Eigen functions are orthogonal. **(1995)**

   $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \Rightarrow \int_0^L \psi_m^* \psi_n \, dx = 1 \ (n=m),\ = 0 \ (n \neq m)$ | $\boxed{E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}}$

7. An electron moving with energy E encounters one dimensional potential step as given below:

   V (x) = 0   x < 0
   V (x) = V_0 x > 0

   $\psi_I = Ae^{ik_1x}$, $\psi_R = Be^{-ik_1x}$ | $\psi_T = Ce^{ik_2x}$

   (a) Suppose the election has the energy E > V_0 and is incident from -x direction, find the normalized wave function so it corresponds to unit incident flux.

   for $J_I = 1 \Rightarrow \frac{\hbar k_1}{m}|A|^2 = 1 \Rightarrow \boxed{|A| = \sqrt{\frac{m}{\hbar k_1}}}$ | $\boxed{A = \left(\frac{k_1+k_2}{2k_1}\right)C \ \& \ B = \frac{(k_1-k_2)C}{2k_1}}$

   (b) Solve the above problem for the case E < V_0 and discuss the significance of the result with the help of a suitable example. **(1996)**

   $\psi_T = Ce^{-k_2' x}$, $k_2' = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$ | $T = 0$, $R = |\frac{B}{A}|^2$ (E < V_0)

8. Write down the time independent Schrodinger equation for the x < 0 and x > 0 in case of a step potential $V(x) = 0 \quad x < 0$

$= V_0 \quad x \geq 0$

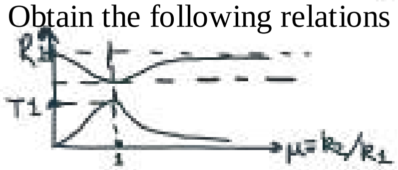
Discuss the solutions thus obtained for the case $E > V_o$. Obtain the following relations

$$|R|^2 = \left(\frac{1-\mu}{1+\mu}\right)^2 \text{ and } \frac{K}{K_0}|T|^2 = \mu\left(\frac{2}{1+\mu}\right)^2$$

Where $\mu = \sqrt{1-\frac{V_0}{E}}$, R and T are reflection and transmission coefficients respectively.

Other symbols have their usual meanings. Plot the behaviorof $|R|^2$ and $\frac{K}{K_0}|T|^2$ with μ **(1997)**

$k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$ $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$



9. An infinite square well potential is

$$V(x) = 0 \qquad -\frac{a}{2} < x < \frac{a}{2}$$

$$= \infty \qquad \frac{a}{2} < x < -\frac{a}{2}$$

$\psi(x) = \frac{2}{a}\cos(kx)$ or $\sin(kx) \Rightarrow \cos(\frac{ka}{2}) = 0 \Rightarrow k_n = \frac{(2m-1)\pi}{a}$ $\frac{\pi}{a}, \frac{3\pi}{a}, \frac{5\pi}{a}$

$\sin(\frac{k'a}{2}) = 0 \Rightarrow k' = \frac{2m\pi}{a}$ $\frac{2\pi}{a}, \frac{4\pi}{a}, \frac{6\pi}{a}$

$\langle x \rangle = 0$, using by parts $\langle p^2 \rangle = 2mE_0$ $\Rightarrow \Delta x \Delta p \geq \hbar/2$

Solve the time independent Schrodinger's equation and find the closed form expressions for the eigen - values and eigen functions of such a potential. Give a schematic representation of the first three energy levels and corresponding wave functions and probability distribution functions. Show that the zero-point energy is in accord with the uncertainty principle. **(1998)**

10. For a particle confined in a one - dimensional potential well of length L the wave function is

$$\psi(x) = C\,\text{Sin}\left(\frac{\pi x}{L}\right) \qquad 0 < x < L \quad \text{and} \quad \psi(x) = 0, \text{ outside}$$

Calculate the expectation values of xand p. **(1999)**

defn expectatn values
→ solve | make graph
→ assumptn

KE incident = E = 1.5 eV, V_0 = 0.5 eV ⇒ $\boxed{R = 0.01 \ \& \ T = 0.99}$

11. A 500 µA beam of electrons of kinetic energy 1.5eV enter a region with a sharply defined boundary in which their energy is reduced to 1.0eV by a difference of potential. Determine the reflected and transmitted currents. Derive the formulae used. **(2000)**

→ Debroglie hypothesis → classical particle as a wave group $y = y_1 + y_2 = a\sin(k_1x - \omega_1 t) + \sin(k_2 x - \omega_2 t) = 2a \sin(\ldots)$

12. Show that the motion of a classical particle is analogous to the motion of a wave packet, which can be constructed by superposition of a large number of plane waves. Construct such wave packet and derive an expression for its group velocity. **(2000)**

↳ envelope of y moves with $v_g = \frac{\Delta\omega}{\Delta k}$

13. A particle is in a quantum system with a parabolic potential well.

(i) Using appropriate method find the ground state energy.

(ii) Obtain an expression for the ground state. **(2001)**

↳ $\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$

14. (i) Write down the matrix representation of the energy operator of a linear oscillator.

(ii) A linear oscillator is prepared in the state given by:

$\psi(x) = 1/\sqrt{5}\,\{\psi_0(x) + \sqrt{2}\,\psi_1(x) + \sqrt{2}\,\psi_2(x)\}$.

Evaluate the energy of the oscillator in this state. **(2001)**

$\hat{A}|\psi\rangle = \sum a_k \hat{A}|\phi_k\rangle = |g\rangle = \sum a'_k |\phi_k\rangle$ in same basis

$|g\rangle = \sum a'_k |\phi_k\rangle$

$\boxed{a'_m = \sum_k a_k \langle \phi_m | \hat{A} | \phi_k \rangle}$ → matrix representation

energy $= \langle \hat{H} \rangle = \frac{1}{5}[E_0 + 2E_1 + 2E_2] = 1.7\hbar\omega$

(since $\psi(x)$ is normalized)

$\boxed{H = \begin{bmatrix} E_0 & & 0 \\ & \ddots & \\ 0 & & E_n \end{bmatrix}}$ in eigenbasis vectors

15. The wave function of a particle is
$\psi(x) = A \exp(-x^2/a^2 + ikx)$.
Find the expectation values of position (x) and momentum (p) for the particle. **(2002)**

16. Solve the one dimensional Schrodinger wave equation with potential :

$$V(x) = \begin{cases} 0 & \text{for } x < -a \\ V & \text{for } -a < x < a \\ 0 & \text{for } x > a \end{cases}$$

(2002)

17. Distinguish between a classical and a quantum mechanical harmonic oscillator. Explain the existence of zero point energy. **(2002)**

18. A particle of mass m, with energy E such that $-V_o < E < 0$, is trapped in a potential well as shown below:
(i) Write time independent Schrodinger equation in regions
(a) $0 < x < a$ and (b) $x > a$
(ii) Obtain an expression from which the energy eigen values can be determined.
(iii) Show that for at least one bound state to exist $a^2 V_o \geq h^2 \pi^2 / 8m$. **(2003)**

19. Consider a particle in a one – dimensional box of length a in a quantum state at t = 0, given by

$$\Psi(q,0) = \begin{cases} \boldsymbol{A; 0 < q, a} \\ \boldsymbol{0; otherwise} \end{cases}$$

Where A = a real constant. Write the expression for $\psi(q,t)$ Obtain the value of A and expansion coefficient of the total wave function. **(2004)**

20. The ground state wave function of a linear simple harmonic oscillator is $\psi = A e^{-\alpha \frac{x^2}{2}}$
Calculate the constant A and the average values of x^2 and x. Given that $\int_0^\infty e^{-x^2} = \frac{\pi}{2}^{1/2}$ **(2005)**

21. Consider a particle of mass m in an infinite one dimensional potential well of width a. the particle is found in the state given by $\psi(x) = c\left[\sin\frac{\pi x}{a} + \frac{1}{2}\sin\frac{2\pi x}{a}\right]$ Calculate c. If a measurement of energy is made, what are the possible results and what are the probabilities for each one of them? **(2005)**

22. (a) Distinguish between potential well and potential barrier. Give their illustrations. Considering one dimensional potential step, prove that sum of the reflection and transmission coefficients is unity.
(b) Prove that $\frac{d}{dt}\langle x\rangle = \frac{1}{m}\langle p_x\rangle$
Define all the terms of this relation and give its physical interpretation. **(2006)**

23. Solve the Schrodinger equation for a linear harmonic oscillator; obtain the eigenvalues and the corresponding eigen functions. **(2007)**

24. A one – dimensional potential barrier is represented by the function $V(x) = 0$ for $x < 0 = V_0$ for $x > 0$ where V_0 is positive. Find the transmission coefficient for particles of mass m incident from the left on the barrier. (Same as below) **(2007)**

25. Show that the probability of transmission across the step barrier represented by the potential $V(x) = \{$ 0 for x < 0is $\boxed{T = \frac{4k_1k_2}{(k_1+k_2)^2}}$ where k_1 and k_2 are wave numbers in regions x < 0 and x >0.

writing time independent schrodinger equat^n, $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$ ① ② (ψ is continuous) **(2008)**

⇒ $A+B=C$, $A-B = C\frac{k_2}{k_1}$ ⇒ $\boxed{T = \frac{J_T}{J_I} = \frac{k_2}{k_1}\left(\frac{C}{A}\right)^2}$ applying boundary condit^n; $\psi_① = \psi_②$, $\psi'_① = \psi'_②$ (from differentiat^n)

26. An electron is moving freely in a one dimensional infinite potential box with walls at x = 0 and $x = a$. If the electron is initially in the ground state of the box and if suddenly the wall at $x = a$ is moved $x = 4a$, calculate the probability of finding the particle in the ground state of the new box. **(2008)**

Sudden change in system does not quantum state, ψ. Under new system, ψ should be combination of eigenstates under modified ϕ_n. Mathematically $\psi_0 = \sum C_n \phi_n$ → for finding particle in ϕ_0, Probability P = $C_0^2 = |\langle\psi|\phi_0\rangle|^2$ $\boxed{P = 0.058}$ Ans

27. The Hamiltonian of a particle moving along the x-axis is given by $\hat{H} = -\alpha\frac{d^2}{dx^2} + 16\alpha\hat{x}^2$,

Where α is a real and positive constant having dimensions of energy.

i. If $\psi(x) = Ae^{-2x^2}$, find the normalization constant A. Check whether ψ is an eigen function of $\hat{H}$. If yes, find the corresponding eigen value.
→ total probability = 1 → $\int \psi^*\psi\, dx = 1 \Rightarrow A = \left(\frac{4}{\pi}\right)^{1/4}$
→ $\hat H\psi = \lambda\psi \Rightarrow \boxed{\lambda = 4\alpha}$ → everytime you measure operator ⇒ 4α eigenvalue is measured.

ii. Calculate the probability of finding the particle anywhere along the negative x-axis.
[P = $\int \psi^*\psi\, dx = 1/2$ (as wavef^n is even → equally likely probability)]

iii. Find the eigen value of $\hat{H}$ corresponding to the eigen function $\phi(x) = x\psi(x)$, where $\psi(x)$ is the same as in part (i).
$\hat H[x\psi(x)] = x\hat H\psi(x) + \psi(x)\hat H x = 12\alpha[x\psi(x)]$

iv. Are the wave functions $\psi(x)$ and $\phi(x)$ orthogonal? **(2008)**
$\int \psi^*\phi\, dx = 0$ means there is zero probability of finding ψ state in ϕ state.

28. Show that the time-dependent part of all the solutions of the Schrodinger equation in one-dimension has the structure $\phi(t) = \exp(-iEt/\hbar)$, provided the potential is not an explicit function of time. **(2009)**
[already done in 35]

29. Consider a positron in a box. If the energy released is 60 eV when it jumps from the third excited state to the ground state, show that the width of the potential is nearly 0.3 nm. **(2009)**
wave potential width/dimension $\left[\Delta E = \frac{(n^2-m^2)\hbar^2\pi^2}{2ma^2} = 60\text{ eV}\right.$ & n=4, m=1 ⇒ $\boxed{a \sim 0.3\text{ nm}}$ Ans

30. Obtain an expression for the probability current for the plane wave $\psi(x,t) = \exp[i(kx - \omega t)]$. Interpret your result. **(2010)**
$\vec J = \frac{\hbar}{2im}[\psi^*\nabla\psi - \psi\nabla\psi^*] \Rightarrow \boxed{\vec J = \frac{\hbar\vec k}{m}}$ → current flux associated with wave, as flux = $\rho\vec v = \vec J = \frac{p}{m} = \frac{\hbar k}{m}$

31. A system is described by the Hamiltonian operator, $H = -\frac{d^2}{dx^2} + x^2$ Show that the function A x $\exp\left(-\frac{x^2}{2}\right)$ is an eigen function of H. Determine the eigen values of H. **(2011)**
eigenf^n & eigenvalues algebra is used to get measurement of a physical quality in a state.
$\left[\hat H\psi = 3\left(Ax\exp\left(-\frac{x^2}{2}\right)\right)\right]$ ⇒ $\boxed{\text{eigenvalue} = 3}$

32. Solve the Schrodinger equation for a particle of mass m in an infinite rectangular well defined by $V_{(x)} = 0$ $0 \le x \le L = \infty$ $x < 0, x > L$ Obtain the normalized eigen functions and the corresponding eigen values. **(2011)**

33. Normalize the wave function $\psi(x) = e^{-|x|} \sin \alpha x$ **(2011)**

$\left[\cos(2\alpha x) = \frac{e^{i2\alpha x} + e^{-i2\alpha x}}{2}\right] \Rightarrow \boxed{A^2 = \frac{2(1+\alpha^2)}{\alpha^2}}$

$\boxed{R = \frac{|B|^2}{|A|^2} = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2}$ & $T = \frac{k_2 |C|^2}{k_1 |A|^2}$ $\quad k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$ & $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$

$\boxed{T = \frac{4k_1 k_2}{(k_1 + k_2)^2}}$

$E \to V_0,\ R = 1$ & $R = 0\ (E \to \infty)$

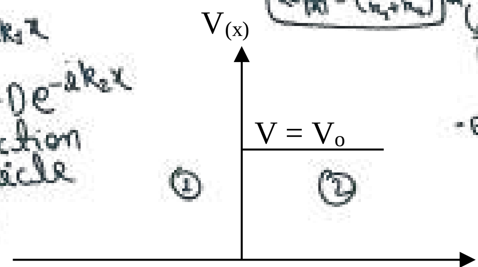
34. (a)

$\psi_1(x) = A e^{ik_1 x} + B e^{-ik_1 x}$

$\psi_2(x) = C e^{ik_2 x} + D e^{-ik_2 x}$

$D = 0$ for no reflection by transmitted particle

$\Rightarrow \boxed{A + B = C\ \&\ (A - B)k_1 = k_2 C}$



Consider a beam of particles incident on one-dimensional step function potential with energy E > V_o as shown in the above figure. Solve the Schrodinger equation and obrain expressions for the reflection and transmission coefficients.

(b) What are the limits of the reflection coefficient for E $\to V_o$ and E $\to \infty$?

(c)Discuss the cases 0 < E < V_o and $\boxed{E < 0}$. → classically finite well **(2013)**

$|\psi|^2 \neq 0$ for ② region

35. Obtain the time independent Schrödinger equation for a particle. Hence deduce the time independent Schrödinger equation. **(2014)**

$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$

$H\Psi = i\hbar\frac{\partial\Psi}{\partial t} \Rightarrow \Psi = \psi(\vec{r})T(t) \Rightarrow -\frac{\hbar^2}{2m\psi}\nabla^2\psi + V = \frac{i\hbar}{T}\frac{\partial T}{\partial t} = E$ → constant

36. Solve the Schrödinger equation for a particle of mass m confined in one dimensional potential well of the form:V=0 , when 0≤x≤L=infinity , when x<0 , x>L. Obtain the discreet energy values and the normalized eigen functions. **(2014)**

37. An electron is moving in a one dimensional box of infinite height and width 1 Å (Angstrom). Find the minimum energy of electron. **(2014)**

→ Introduce → write result → $E_1 = 36.67$ Å

38. Normalized wave function of a particle is given: ψ (x) = A exp (-x ²/ a ² + ikx).Find the expectation value of position. (repeated) **(2015)**

$\left\{T = \frac{k_2}{k_1}\left|\frac{C}{A}\right|^2 \quad R = \left|\frac{B}{A}\right|^2 = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2\right\}$

39. Solve the Schrodinger equation for a <u>step potential</u> and calculate the transmission and reflection coefficient for the case when the kinetic energy of the particle E_0 is greater than the potential energy V (i.e., E_0 > V). **(2016)**

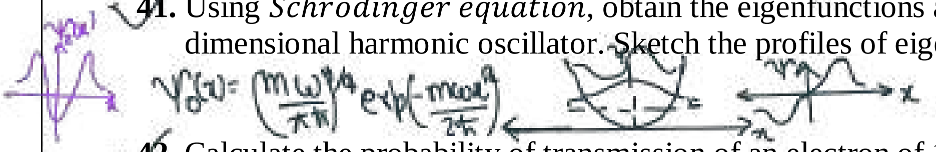
→ write schrödinger equation → $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$ (using boundary conditions)

40. Calculate the lowest energy of an electron confined to move in a 1-dimensional potential well of width 10nm.

$\rightarrow \psi_n(x) = (-1)^n \left(\frac{\sqrt{m\omega/\pi\hbar}}{2^n n!}\right)^{1/2} \exp\left(+\frac{m\omega}{2\hbar}x^2\right) \frac{d^n}{d\eta^n}\exp\left(-\frac{m\omega}{\hbar}x^2\right) \quad \left\{\eta = \sqrt{\frac{m\omega}{\hbar}}\, x\right\}$

41. Using *Schrödinger equation*, obtain the eigenfunctions and eigenvalues of energy for a 1-dimensional harmonic oscillator. Sketch the profiles of eigenfunctions for first three energy states. **(2017)**

$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$ $\quad \psi_2(x) = C(4x^2 - 2)e^{-m\omega x^2/2\hbar}$



$\psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(\frac{2m\omega}{\hbar}\right)^{1/2} x \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$

42. Calculate the probability of transmission of an electron of 1.0 eV energy through a potential barrier of 4.0 eV and 0.1 nm width. **(2017)**

$T = \left(1 + V_0^2 \frac{\sin^2(h k_2 L)}{4E(V_0 - E)}\right)^{-1} \quad \left\{k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}\right\}$

43. The wave function of a particle is given as $\psi(x) = \frac{1}{\sqrt{a}} e^{-|x|/a}$. Find the probability of locating the particle in the range $-a \le x \le a$. **(2018)**

$\left[P = \int_{-a}^{a} |\psi(x)|^2 dx = 2\int_0^a e^{-2x/a} dx = a\int_0^2 e^{-t} dt = a(1 - e^{-2})\right]$

$\left\{\frac{2x}{a} = t \Rightarrow dx = \frac{a}{2}dt\right\}$

44. Calculate the zero-point energy of a system consisting of a mass of $10^{-3}kg$ connected to a fixed point by a spring which is stretched by $10^{-2}m$ by a force of $10^{-1}N$. The system is constrained to move only in one direction. **(2018)**

$L = 10^{-2}m$, $F = 10^{-1}N = kL$ | $k = m\omega^2$, $\boxed{\omega^2 = \frac{k}{m}}$ $\rightarrow \omega^2 = \frac{F}{mL} \Rightarrow \boxed{E_0 = \frac{\hbar\omega}{2}}$

45. The general wave function of harmonic oscillator (one-dimensional) are of the form

$$u_n(x) = \sum_{k=0}^{\infty} a_k y^k e^{-y^2/2}$$

$a_{k+2} = \frac{\left(2k+1 - \frac{2E}{\hbar\omega}\right)}{(k+1)(k+2)}$ → for $k=n$, $\boxed{\frac{2E}{\hbar\omega} = 2n+1}$

With $y = \sqrt{\frac{m\omega}{\hbar}}x$, and coefficients a_k are determined by recurrence relations

$$a_{k+2} = \frac{2(k-n)}{(k+1)(k+2)} a_k$$

→ odd (for $x \le 0$, $V \to \infty$ means $\psi(x=0) = 0$)
→ even

Corresponding energy levels are $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$. Discuss the parity of these wave functions. What happens, if the potential for $x \le 0$ is infinite (half harmonic oscillator)? **(2018)**

↳ only odd n → none of even ψ (hermite eigenfn) is zero at $x=0$

$P = |\psi|^2$

46. Which of the following functions is/are acceptable solution(s) of the *Schrödinger equation*?

Born approximat^n, ψ should be continuous, square integrable

(i) $\psi(x) = A\, e^{-ikx} + B\, e^{ikx}$
(ii) $\psi(x) = Ae^{-kx} + B\, e^{kx}$ × $(x \to \infty)$
(iii) $\psi(x) = A\, sin3kx + B\, cos5kx$ $= A\left(\frac{e^{i3kx} - e^{-i3kx}}{2i}\right) + B\frac{(e^{i5kx} + e^{-i5kx})}{2}$
(iv) $\psi(x) = A\, sin3kx + Bsin\, 5kx$
(v) $\psi(x) = A\tan kx$ × $(kx = \frac{n\pi}{2})$

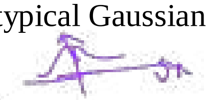
Explain your answer. **(2018)**

47. A beam of particles of energy 9 eV is incident on a potential step 8 eV high from the left. What percentage of particles will reflect back? **(2018)**

$\left\{ R = \left[\frac{k_1 - k_2}{k_1 + k_2}\right]^2 = \frac{1}{289} \right\}$

48. Write down the Hamiltonian operator for a linear harmonic oscillator. Show that the energy eigenvalue of the same can be given by $E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0$ at energy state n with ω_0 being the natural frequency of vibration of the linear oscillator. Prove that $n = 0$ energy state has a wave function of typical Gaussian form. **(2019)**

$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$ | $\eta = \sqrt{\frac{m\omega}{\hbar}}x$ | $\Rightarrow \frac{\partial^2\psi}{\partial\eta^2} + \left(\frac{2E}{\hbar\omega} - \eta^2\right)\psi = 0$ → $\eta^2 \gg \frac{2E}{\hbar\omega}$, $\psi = ue^{-\eta^2/2}$
↳ $\frac{d^2u}{d\eta^2} - 2\eta\frac{du}{d\eta} + \left(\frac{2E}{\hbar\omega} - 1\right)u = 0$
↳ $(E = \frac{\hbar\omega}{2}$ ... $)$



49. Estimate the size of hydrogen atom and the ground state energy from the uncertainty principle. **(2019)**

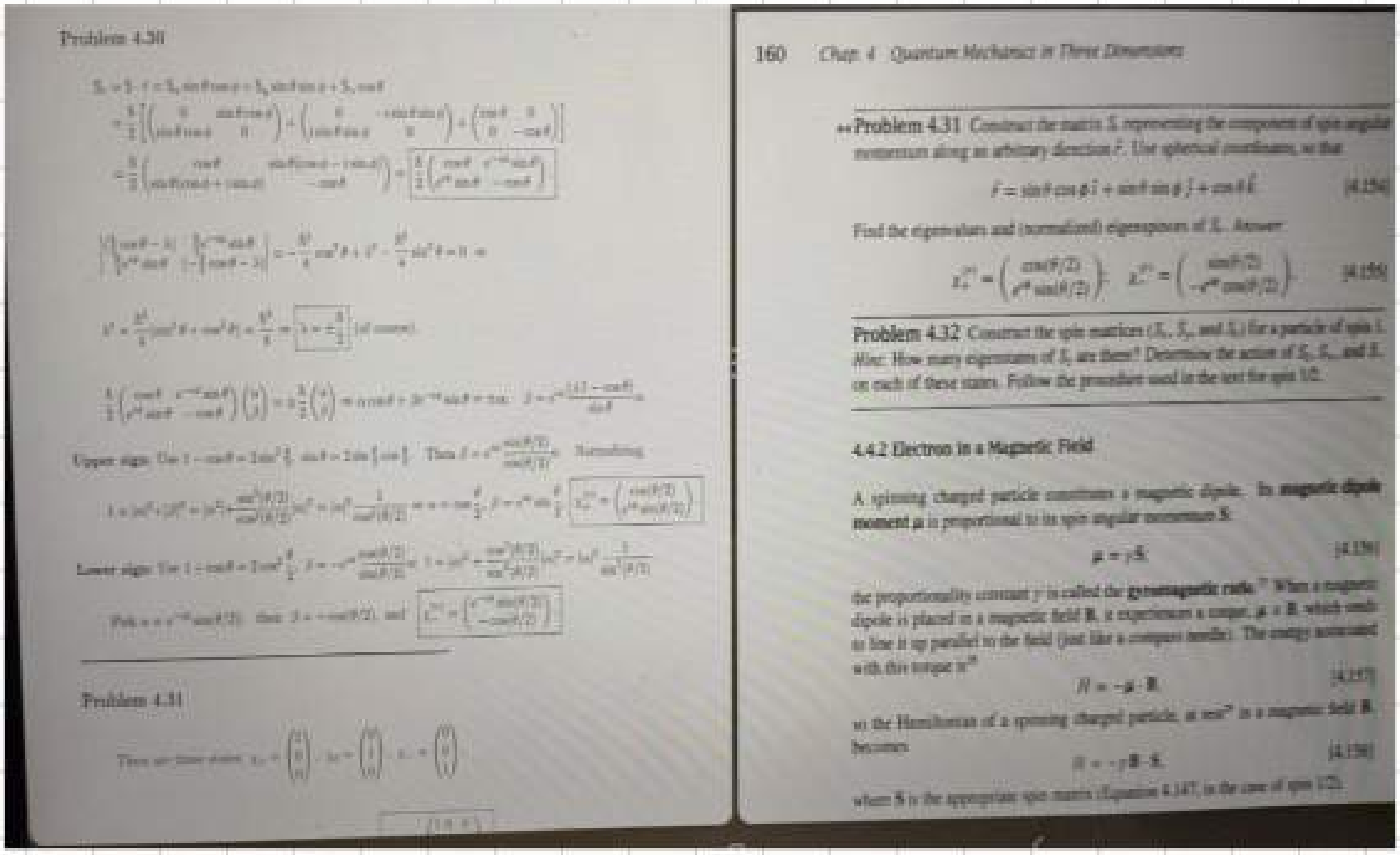
↳ defn → compute & solve → significance

↳ uncertainity means spreading position & momenta in a way, ↳ $E \neq 0$

2020
• $A^3 = I$
$A|\psi\rangle = \alpha|\psi\rangle \Rightarrow A^3|\psi\rangle = \alpha^3|\psi\rangle$
$\Rightarrow \alpha = 1, e^{i2\pi/3}, e^{i4\pi/3}$ ↳ Hermitian $\Rightarrow \boxed{\alpha = 1}$ $\boxed{A|\psi\rangle = |\psi\rangle}$

$|\psi\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

If the z-component of an electron spin is $+\frac{\hbar}{2}$, what is the probability that its component along a direction z' (forming an angle θ with z-axis) is $\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$? What is the average value of spin along z'?


Problem 4.30

Problem 4.31

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Problem 4.31

Problem 4.32

4.4.2 Electron in a Magnetic Field

$S_r = \vec{S}\cdot\hat{r} = (S_x\hat{i} + S_y\hat{j} + S_z\hat{k})\cdot(\sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k})$

or, $S_r = \frac{\hbar}{2}\begin{pmatrix}\cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta\end{pmatrix}$

$\hookrightarrow$ eigen f^n $\rightarrow$ $\frac{\hbar}{2}\begin{pmatrix}\cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \pm\frac{\hbar}{2}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$

- For $\lambda = \frac{\hbar}{2}$; $\beta = \alpha\, e^{i\phi}\frac{\sin(\theta/2)}{\cos(\theta/2)}$ & $|\alpha|^2 + |\beta|^2 = 1$

$\hookrightarrow$ $\alpha = \cos\theta/2$, $\beta = e^{i\phi}\sin(\theta/2)$

$\chi_+ = \begin{bmatrix}\cos\theta/2 \\ e^{i\phi}\sin(\theta/2)\end{bmatrix}$

$$\boxed{|\chi\rangle = \begin{bmatrix}1 \\ 0\end{bmatrix}}$$

- for $\lambda = -\hbar/2$; $\chi'_- = \begin{bmatrix}e^{-i\phi}\sin(\theta/2) \\ -\cos(\theta/2)\end{bmatrix}$

$P_{-\frac{\hbar}{2}} = |\langle\chi'_-|\chi\rangle|^2 = \left|\begin{bmatrix}e^{-i\phi}\sin\theta/2 & -\cos\theta/2\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}\right|^2 = \sin^2\theta/2$ Ans

$P_{+\hbar/2} = |\langle\chi'_+|\chi\rangle|^2 = \cos^2\theta/2$. Ans

$$\boxed{\langle S_r\rangle = \frac{\hbar}{2}\begin{bmatrix}1 & 0\end{bmatrix}_{1\times2}\begin{bmatrix}\cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}_{2\times1} = \frac{\hbar}{2}\cos\theta}$$

Ans

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TUTORIAL SHEET: 3
Quantum Mechanics-II

assuming $\Psi = \Phi\Theta \Rightarrow$ $\{L_z\Phi = -i\hbar\frac{\partial\Phi}{\partial\phi} = m\hbar\Phi\}$ $\Rightarrow \boxed{\Phi = A e^{im\phi}}$

orienting over z axis in a way ϕ is angle by projection of L on to x-y plane & ϕ is angle with x-axis. $\Phi(\phi+2\pi) = \Phi(\phi)$ ⟶ m an integral.

1. (i) Find the eigen states of angular moments vector's component L_z of a spherically symmetric system.
(ii) Write down any two properties of Pauli spin matrices, after defining them through suitable expression. **(2001)**
⟶ trace = 0 & $\boxed{\{\sigma_x, \sigma_y\} = 0}$

2. Write down the eigen value and spin state of an electron for the spin operator S. ⟶ $m_s = -s$ to $+s$
(i) Show that the spin states of electron are orthogonal to each other. $S_z|+\rangle = \frac{\hbar}{2}|+\rangle$, $\langle -|S_z|+\rangle = -\frac{\hbar}{2}\langle -|+\rangle$ & $\langle -|S_z|+\rangle = \frac{\hbar}{2}\langle -|+\rangle$ $\Rightarrow \boxed{\langle -|+\rangle = 0}$
(ii) State the spin angular momentum commutation relations ⟶ $[S_x, S_y] = i\hbar S_z$
(iii) Give the explicit forms of all the Pauli spin matrices. **(2001)**
⟶ $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

3. (i) Express the Cartesian components of the angular momentum L in operator form. Show that $[L^2, L_z]=0$. What is the significance of this commutation relation?
(ii) Show that the Pauli matrices anti-commute. **(2003)**
⟶ means measurableness in same state
$\sigma_x\sigma_y + \sigma_y\sigma_x = 0$

4. Show that the radial probability density for the ground state of the hydrogen atom has a maximum at $r = a$. The ground state wave function of the hydrogen atom is given by $\Psi(r) = 1/\sqrt{\pi}\ a^{3/2}\ e^{-r/a}$, where the Bohr radius. **(2003)**

$\rightarrow dN = \frac{4\pi n^2 dn \times 2}{8} \Rightarrow g(\epsilon) = \frac{dN}{d\epsilon} = \frac{4\pi V}{h^3}(2m)^{3/2}(\epsilon)^{1/2}$

5. N number of non-interacting electrons are confined in a cube of volume L^3. Obtain an expression for the Fermi energy. **(2003)**
$dN_{total} = f(\epsilon)g(\epsilon)d\epsilon \Rightarrow N_{total} = \int_0^{\epsilon_f} f(\epsilon)g(\epsilon)d\epsilon$ ⟶ $\boxed{E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}}$

A hydrogen atom is in the following state $\Psi_{nlm}(r,0) = \frac{1}{\sqrt{14}}\ [2\,\Psi_{100}(r) - 3\Psi_{200}(r) + \Psi_{322}(r)]$
(i) What is the probability of finding the system in the state 200? $\rightarrow P = |\langle\Psi_{200}|\Psi_{nlm}\rangle|^2$ ⟶ $\boxed{P = \frac{9}{14}}$
(ii) What are H and L_z? $\{\langle H\rangle = \frac{4}{14}E_1 + \frac{9}{14}E_2 + \frac{1}{14}E_3\}$ & $\langle L_z\rangle = 0\times\frac{2}{14} + 0\times\frac{9}{14} + 1\times\frac{2\hbar}{14}$ $\boxed{\langle L_z\rangle = \frac{\hbar}{7}}$ **(2004)**

6. Show that for the ground state of hydrogen atom the mean value of r is 3/2 a_o, where a_o is Bohr radius. **(2004)**
$\Rightarrow P = r^2R^2$  ⟶ $\langle r\rangle > a_0$ i.e. most probable radius is due to tail of probability curve

7. Solve the eigen value equation $L^2 Y(\theta,\Phi) = \lambda h^2 Y(\theta,\Phi)$ And obtain the eigen value of L^2. **(2004)**
$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$ & $Y = P(\phi)T(\theta)$ & $\boxed{x = \cos\theta} \Rightarrow (1-x^2)\frac{\partial^2 T}{\partial x^2} - 2x\frac{\partial T}{\partial x} + \left(\lambda - \frac{m^2}{1-x^2}\right)T = 0 \rightarrow T_0^0 = 1,\ T_1^0 = \cos\theta,\ T_1^1 = \sin\theta$, $T_2^0 = (3\cos^2\theta - 1)$

8. Write the commutation relations for the position variable x and the momentum component p_x, p_y and p_z. Explain the physical significance of these relations. $[x, p_x] = i\hbar$, $[x, p_y] = 0$, $[x, p_z] = 0$ **(2005)**

9. The wave function of a particle confined in a cube of volume L^3 is given by

$$\psi(x,y,z) = \left(\frac{2}{L}\right)^{3/2} \sin\frac{\pi x}{L} \sin\frac{\pi x}{L} \sin\frac{\pi z}{L}$$

$\langle p_x\rangle = \frac{\int \psi^*\left(-i\hbar\frac{\partial\psi}{\partial x}\right)}{\int\psi^*\psi\,dx} = \frac{\pi\hbar}{L}$, $\langle p_x^2\rangle = \frac{\pi^2\hbar^2}{L^2}$

Calculate the average values of p_x and p^2_x in the region $0<x<L$. **(2005)**
$\Rightarrow \langle p_x\rangle = \pi\hbar/L$

10. Set up the time-independent Schrodinger equation for an electron moving in a Coulomb field, $V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r}$, in polar coordinates. Solve the radial equation to get the energy eigen values. **(2005)**
$\rho = \frac{2r}{an}$, $a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2}$, $\boxed{r_n = a_0 n^2}$

11. Explain how the problem of the hydrogen atom could be solved using Schrodinger equation. Also derive an expression for its energy eigenvalue and discuss the associated bound states of this case. **(2006)**

12. (i) Define angular momentum. Express it in operator form and show that $\vec{L}\times\vec{L}=i\hbar\vec{L}$ Explain the physical significance of this relation. **(2007)**

→ $[\vec{L}\times\vec{L}]$ is not zero as components of $\vec{L}$ don't commute i.e. not measurable simultaneously.

(ii) Let Ylm be an eigenstate of L^2 and L_z with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$, respectively. Show that $\phi=(L_x+iL_y)Y_{lm}$ is likewise an eigenstate of L^2 and L_z and determine the eigenvalues.

$[L^2, L_\pm]=[L^2, L_z]=0$ & $[L_z, L_+]=\hbar L_+$ & $[L_z, L_-]=-\hbar L_-$ ⇒ $\boxed{L_+Y_{lm}=C_+|l, m+1\rangle}$ — $\boxed{|C_+|^2=[l(l+1)-m^2-m]\hbar^2}$ ↑ $L_-L_+=L^2-L_z^2+i(L_xL_y-L_yL_x)$ ↑ $[L_x, L_y]=i\hbar L_z$

13. Determine the discrete energy levels and the corresponding eigenfunctions for a particle in an infinitely deep potential well inside a cube of dimension L, i.e. assume $V(x,y,z)=0$ for 0< x< L; 0< y < L; 0< z< L = ∞ elsewhere (Repetitive) **(2007)**

14. In the free electron theory of metals, a conductor is regarded as consisting of free electrons in a three dimensional box. Using the results of Q(14), obtain an expression for the density of states. **(2007)**

→ $\chi^\dagger\chi=A^*A(3i \; 4)^*(3i \; 4)=1$ ⇒ $\boxed{A=\frac{1}{5}}$

15. An electron is in the spin state $\chi=A\begin{pmatrix}3i\\4\end{pmatrix}$ Determine the normalization constant A. Find the expectation value of the spin operator $\hat{S}_x$ and also the uncertainty in the value of S_x in this state.

$\langle S_x\rangle=\chi^\dagger S_x\chi=\frac{\hbar}{2}\chi^\dagger\sigma_x\chi=0$; $\langle S_x^2\rangle=\frac{\hbar^2}{4}$, $\Delta S_x=\sqrt{\langle S_x^2\rangle-\langle S_x\rangle^2}=\frac{\hbar}{2}$ **(2008)**

16. Write (do not derive) the formula for the energy levels of a particle in a three dimensional cubical boxof side L. How many electrons can occupy the level having energy $66\hbar^2/8mL^2$? **(2008)**

$E_n=\frac{h^2}{8mL^2}(n_x^2+n_y^2+n_z^2)=\frac{66\hbar^2}{8mL^2}$ ⇒ $(n_x,n_y,n_z)\equiv\{\underbrace{(8,1,1)}_{3}\,|\,\underbrace{(7,4,1)}_{6}\,|\,\underbrace{(5,5,4)}_{3}$ ⇒ 12 states → 24 e^-s

17. Write the commutation relations of angular momentum operator $\hat{L}_x, \hat{L}_y$ and $\hat{L}_z$ and calculate the commutators $\left[\hat{L}_+, \hat{L}_z\right]$ and $\left[\hat{L}_-, \hat{L}_z\right]$ .Show that $\hat{L}_+|l,m\rangle=\sqrt{l(l+1)-m(m+1)}\,|l,m+1\rangle$, Where $|l,m\rangle$ is the state with definite values for L^2 and L_z. **(2008)**

Same as 12 (ii)

→ explains state associated ⇒ $\int_0^\infty P(r)=1$ & take $\boxed{t=2br}$ ⇒ $\boxed{N\propto b^3}$

18. (i) The quantum mechanical probability distribution function of an electron in the ground state of the hydrogen atom is $P(r)=Nr^2\exp(-2br)$. Using the result $\int_0^\infty P(r)dr=1$, deduce that N is proportional to b^3.

(ii) Prove that the value of 40 k_BT at T = 300 K is nearly 1 eV. Hence determine the Fermi temperature of metal whose Fermi energy is 9.4 eV. $\{\boxed{k_BT_F=9.4\text{ eV} \;\&\; \frac{1}{2}mv_f^2=E_f}$

(iii) Show that the Fermi velocity is related to the Fermi energy of electrons through the Relation

$$\frac{v_F}{C}=1\cdot98\left(\frac{E_F}{1MeV}\right)^{1/2}$$

$\left\{\frac{v_f}{c}=\sqrt{\frac{2E_f}{mc^2}}=\sqrt{\frac{2E_f}{0.51\text{MeV}}}=1.98\sqrt{\frac{E_f}{1\text{MeV}}}\right\}$ **(2009)**

19. Prove that the most probable distance of an electron from the proton (in the hydrogen atom) is the Bohr radius of the hydrogen atom. Consider only the ground state. **(2009)**

Start with ψ_{1s} & their graph $\psi_{1s}=\frac{1}{(\pi a_0^3)^{1/2}}\exp\left(-\frac{r}{a_0}\right)$ | radial probability $(P)=4\pi r^2 dr\times|\psi|^2$; $\frac{dP}{dr}\Big|_{r=r_0}=0$ & $\frac{d^2P}{dr^2}<0$ ⇒ $\boxed{r_0=a_0}$

$\left[E = \frac{\pi^2\hbar^2}{2mL^2}(n_x^2+n_y^2+n_z^2)\right]$ & assuming sphere with radius b/w n & n+dn since n_x, n_y & n_z are +ve, $\boxed{dN = \frac{4\pi n^2 dn}{8}}$

20. (i) Consider a particle in a three-dimensional box. Derive an expression for g(E), the density of states.

n_x n_z n_y

(ii) Show that $\frac{g(p)}{g(E)} = \frac{dE}{dp}$, where g(p) is the density of states in the momentum space. Deduce that g(p) is proportional to p^2 for a free non-relativistic particle. $\{g(p)dp = g(E)dE\}$ **(2009)**

→ no. of states with p & p+dp
→ " " with corresponding E & E+dE.

21. Show that the Pauli Spin Matrices obey the following relation:

i. $Tr(\sigma_x) = Tr(\sigma_y)$

ii. $\det(\sigma_y) = \det(\sigma_z)$

If $|\chi_1\rangle$ & $|\chi_2\rangle$ are two eigenstates for half spin fermion operator $\hat{A}$ in matrices form, $\hat{A} = \boxed{\begin{matrix} \langle\chi_1|\hat{A}|\chi_1\rangle & \langle\chi_1|\hat{A}|\chi_2\rangle \\ \langle\chi_2|\hat{A}|\chi_1\rangle & \langle\chi_2|\hat{A}|\chi_2\rangle \end{matrix}}$

iii. The eigenvalues of σ_z and σ_x are the same.

iv. Write down the y-component of the spin angular momentum matrix corresponding to an antineutrino. **(2009)**

→ fermion & lepton ⇒ half spin

22. Show that the Pauli spin matrices satisfy the following:

$$\left.\begin{aligned} &\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1 \\ &\sigma_x\sigma_y = -\sigma_y\sigma_x = i\sigma_z \\ &\sigma_y\sigma_z = -\sigma_z\sigma_y = i\sigma_x \\ &\sigma_z\sigma_x = -\sigma_x\sigma_z = i\sigma_y \end{aligned}\right\}$$

def^n ⇒ solve ⇒ applicat^n/significance of half spin

(2010)

23. The normalized wave function for the electron in hydrogen atom for the ground state is

$\psi(r) = \left(\pi a_0^3\right)^{-1/2} \exp\left(-\frac{r}{a_0}\right)$ Where a_0 is the radius of the first Bohr orbit. Show that the most probable position of the electron is a_0 **(2010)**

→ def^n for half spin

24. Let $\vec{\sigma}$ be the vector operator with component equal to pauli spin matrices $\sigma_x, \sigma_y, \sigma_z$ if $\vec{a}, \vec{b}$ are vectors in 3D space, prove the identity $(\vec{\sigma}.\vec{a})(\vec{\sigma}.\vec{b}) = \vec{a}.\vec{b} + i\vec{\sigma}.(\vec{a}\times\vec{b})$. **(2011)**

$(\vec{\sigma}.\vec{a})(\vec{\sigma}.\vec{b}) = \vec{a}.\vec{b} + i\vec{\sigma}.(\vec{a}\times\vec{b})$ (using $\sigma_x\sigma_y = i\sigma_z$) → fermions statistics → Stern-Gerlach → explains stability of nuclei through exchange energy concept

$\{(\vec{\sigma}.\hat{n})(\vec{\sigma}.\hat{n}) = 1$ & $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1\}$ ⇒ applicat^n

25. For hydrogen atom, ground state is $\psi_{(x)} = \frac{1}{\left([\pi a_0^2)\right]^{\frac{1}{2}}} e^{-\frac{r}{a_0}}$ Calculate $\langle r \rangle$ and $\langle\frac{1}{r}\rangle$ $\left\{\langle\frac{1}{r}\rangle = \frac{1}{a_0}\right\}$ **(2011)**

def^n Pauli spin matrices

26. Show (i) Sin $(\sigma_x \emptyset) = \sigma_x \sin \emptyset$ ⇒ $\left\{\sin(\sigma_x\phi) = \sum_{K=0}^{\infty}\frac{(-1)^K(\sigma_x\phi)^{2K+1}}{(2K+1)!} = \sigma_x \sin\phi\right\}$ **(2011)**

(ii) Cos $(\sigma_z \emptyset)$ = cos $\emptyset$ ⇒ $\left\{\cos(\sigma_z\phi) = \sum_{K=0}^{\infty}\frac{(-1)^K(\sigma_z\phi)^{2K}}{(2K)!} = \cos\phi\right\}\{(\cos(\sigma_z)^{2K} = 1)\}$ **(2012)**

write conclusion

27. What is the degree of degeneracy of the energy – eigen Values. What happened if the spin of the electron is taken into account? ⇒ $g_{total} = g_{spin} \times g_{space}$ (Pauli's exclusion rule) **(2012)**
→ for fermion

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\int_0^\infty t^n e^{-t}\,dt = \Gamma(n+1) = n\Gamma n = n!$ & $\Gamma\frac{1}{2} = \sqrt{\pi}$

28. Solve the radial part of the time independent Schrodinger Equation for a hydrogen atom. Obtain its expression for energy Eigen values. **(2012)**

29. Using the definition $L = \vec{r} \times \vec{p}$ of the orbital angular momentum operator, evaluate [L(x) , L(y)] **(2013)**

30. The normalized wave function for the electron for the electron in the ground state of the hydrogen atom is given by

$\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$. Find $\langle r\rangle$ and $\langle r^2\rangle$. **(2013)**

Let's take $\{r = \frac{a_0}{2} t\}$ for integration

$\langle r\rangle = \frac{\int \psi^* r \psi\, d\tau}{\int \psi^* \psi\, d\tau} = \frac{3a_0}{2}$ & $\{d\tau = 4\pi r^2 dr\}$ $\langle r^2\rangle = 3a_0^2$ ⇒ non-commutation ⇒ non-measurable on same state

31. If x and p are the position and momentum operators, prove the commutation relation $[p^2, x] = -2i\hbar p$ **(2014)**

$[x, p] = i\hbar$ & $[A^2, B] = A[A,B] + [A,B]A \Rightarrow [p^2, x] = -2i\hbar p$

32. Write down Pauli spin matrices. Express J(x) , J(y) and J(z) in terms of Pauli spin matrices. **(2014)**

↳ Half spin $[J(x) = \frac{\hbar}{2}\sigma_x,\ J(y) = \frac{\hbar}{2}\sigma_y,\ J(z) = \frac{\hbar}{2}\sigma_z]$

33. Using the commutation relations
[x, p(x)] = [y, p(y)] = [z,p(z)]=ih
Deduce the commutation relation between the components of angular momentum operator L.
[L(x), L(y)]=ihL(z)
[L(y), L(z)]= ihL(x) and
[L(z),L(x)]=ihL(y) **(2014)**

$L_x = y\hat{p}_z - z\hat{p}_y$
$L_y = z\hat{p}_x - x\hat{p}_z$
$L_z = x\hat{p}_y - y\hat{p}_x$

$L_x \neq L_y \neq L_z$ - non degenerate
degenerate means different eigenstate with same eigenvalue

34. Solve the Schrodinger equation for a particle in a three dimensional rectangular potential barrier. Explain the terms degenerate and non-degenerate states in this context. **(2015)**

write time independent schrodinger equation & $\psi = XYZ \Rightarrow \frac{\partial^2 X}{X\partial x^2} + \frac{\partial^2 Y}{Y\partial y^2} + \frac{\partial^2 Z}{Z\partial z^2} = -\frac{2mE}{\hbar^2}$ & $\{$put $E = \frac{\hbar^2(k_x^2 + k_y^2 + k_z^2)}{2m}\}$ ⇒ $\{k_x = \frac{n_x\pi}{L_x}$ using $X|_{x=0} = 0\}$

35. A particle trapped in an infinitely deep square well of width a has a wave function

$\psi = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \sin\left(\frac{\pi x}{a}\right)$. The walls are suddenly separated by infinite distance. Find the probability of the particle having momentum between p and $p + dp$ **(2015)**

$\psi = \sqrt{\frac{2}{\pi}} \times \frac{1}{2i}\left(e^{i\pi x/a} - e^{-i\pi x/a}\right)$

$\psi = \int C_p \exp\left(\frac{ipx}{\hbar}\right)dp \Rightarrow \boxed{C_p = \frac{1}{2\pi}\int \psi \exp\left(-\frac{ipx}{\hbar}\right)dx}$

$\boxed{\text{Probability} = |C_p|^2 dp}$

36. Write down the matrix representation of the three Pauli matrices σ_x, σ_y and σ_z. Prove that these matrices satisfy the following identities:

- **i.** $[\sigma_x, \sigma_y] = 2i\sigma_z$
- **ii.** $[\sigma^2 . \sigma_x] = 0$
- **iii.** $(\sigma .A)(\sigma .B) = A .B + i\sigma .(A \times B)$
  If A and B commute with Pauli matrices. **(2016)**

Repeated

37. Calculate the density of states for an electron moving freely inside a metal with the help of quantum mechanical Schrodinger's equation for free particle in a box. **(2016)**

38. Evaluate the most probable distance of the electron from nucleus of a hydrogen atom in its $2p$ state. What is the probability of finding the electron at this distance? **(2017)**

$\psi_{21m} = \frac{1}{4\sqrt{2\pi}(a_0)^{3/2}}\left(\frac{r}{a_0}\right)e^{-r/2a_0} f(\theta,\phi)$

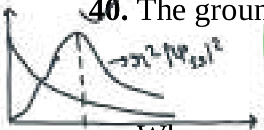
$P(r,\theta,\phi)\,d\tau = 4\pi r^2 dr\,|\psi|^2 = A\,e^{-r/a_0} r^4\,dr$

$\frac{dP(r)}{dr} = \frac{d(r^4 e^{-r/a_0})}{dr}\Big|_{r=r_{mp}} = 0 \Rightarrow \boxed{r_{mp} = 4a_0}$ Ans.

39. Explain why the square of the angular momentum (L^2) and only one of the components (L_x, L_y, L_z) of L are regarded as constants of motion. $\frac{d\langle L^2\rangle}{dt} = \frac{\langle[L^2,H]\rangle}{i\hbar} + \langle\frac{\partial L^2}{\partial t}\rangle$; $\frac{d\langle L^2\rangle}{dt}=0 \Rightarrow \langle L^2\rangle =$ constant **(2017)**

& $\frac{d\langle L_z\rangle}{dt} = 0 \Rightarrow \langle L_z\rangle =$ constant; $[L^2, L_z] = 0 \Rightarrow L^2$ & one of component can be measured simultaneously

40. The ground state wave function for hydrogen atom is



$P(r) = 1 - \int_0^{a_0} 4\pi r^2 |\psi|^2 dr = e^{-2}$

$$\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

Where a_0 is the Bohr radius. Sketch the wave function and the probability density as a function of the separation distance r. Calculate the probability that the electron in the ground state is found beyond the Bohr radius. **(2018)**

41. Prove the following identities:

(i) $[\hat{p}_x, \hat{L}_y] = i\,\hbar\,\hat{p}_z$

(ii) $e^{i(\hat{\sigma}.\hat{n})\theta} = \cos\theta + i(\hat{\sigma}.\hat{n})\sin\theta$ **(2018)**

$e^{i(\hat\sigma.\hat n)\theta} = \sum_{k=0} \frac{(i(\hat\sigma.\hat n)\theta)^k}{k!}$ → odd + even term

$[i(\hat\sigma.\hat n)\sin\theta]$ $(\cos\theta)$

42. Show that for free electron gas, the density of states in three dimensions (3D) varies as $E^{1/2}$, and this dependence changes to E^0 for 2D (quantum well), $E^{-1/2}$ for 1D (quantum wire) and δ function for 0D (quantum dot). **(2018)**

For free e^- gas, free e^- can be assumed to have been present in 3-D box of length L; 3D → $g(E)dE = \frac{4\pi p^2 dp \times L^3}{h^3} \times 2$ & 2D → $g(E) = \frac{dN}{dE} = \frac{2\pi n}{4}\frac{dn}{dE}$ = constant

$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$; 0-D → $N = \delta(E - E_n)$ & $g(E) = \delta(E-E_n)$; 1D → $g(E) = \frac{dN}{dE} \propto \frac{1}{\sqrt{E}}$

43. How do you define density of states? Show that the density of states with wave vector less than $\vec{k}$ in a three-dimensional cubic box of volume V can be given by

$g(\omega)d\omega = \frac{V}{2\pi^2}k^2\left(\frac{dk}{d\omega}\right)$

$$D(\omega) = \frac{V}{2\pi^2}k^2\left(\frac{dk}{d\omega}\right)$$

density of states is no. of available states per unit energy interval; $g(E)dE = g(k)dk$ = no. of available states b/w k & k+dk; $g(k)dk = \frac{4\pi k^2 dk}{(2\pi/L)^3}$; counting states b/w k=0 to k=k and volume of unit state in (3D) in k-space

in the frequency spectrum between ω and $\omega + d\omega$. Here, assume that the number of modes per unit range of k is $L/(2\pi)$, L being the length of each side of the cubic box. **(2019)**

44. Define Pauli spin matrices σ_x , σ_y and σ_z. Using these definitions, prove the following:

(i) $\sigma_x^2 = \sigma_4^2 = \sigma_z^2 = 1$

(ii) $\sigma_x\sigma_y = i\sigma_z$; $\sigma_z\sigma_x = i\sigma_y$; $\sigma_y\sigma_z = i\sigma_x$ **(2019)**

45. Define angular momentum of a particle and find out the three components of the angular momentum operator $\hat{L}$ in Cartesian coordinates. Show that

$$\hat{L}^2 = -\hbar^2\left[r^2\nabla^2 - \frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\right]$$

Prove that the operator $\hat{L}^2$ can also be expressed as

$$\hat{L}^2 = -\hbar^2\left[\frac{1}{Sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{Sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

in spherical polar coordinates $(r, \theta, \emptyset)$. **(2019)**

(1) (i)

$$\vec{L} = -i\hbar\, \vec{r}\times\vec{\nabla} = -i\hbar\,\vec{r}\times\left(\hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial\theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial\phi}\right)$$

$$\boxed{\vec{L} = -i\hbar\left(\hat{\phi}\frac{\partial}{\partial\theta} - \frac{\hat{\theta}}{\sin\theta}\frac{\partial}{\partial\phi}\right)}$$

$$L^2 = -\hbar^2\left(\hat{\phi}\frac{\partial}{\partial\theta} - \frac{\hat{\theta}}{\sin\theta}\frac{\partial}{\partial\phi}\right)\cdot\left(\hat{\phi}\frac{\partial}{\partial\theta} - \frac{\hat{\theta}}{\sin\theta}\frac{\partial}{\partial\phi}\right)$$

$$\left\{\begin{array}{l}\hat{r} = \cos\theta\,\hat{z} + \sin\theta(\cos\phi\,\hat{x} + \sin\phi\,\hat{y}) \\ \hat{\theta} = -\sin\theta\,\hat{z} + \cos\theta(\cos\phi\,\hat{x} + \sin\phi\,\hat{y})\end{array}\;\middle|\;\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}\right\}$$

$$L^2 = -\hbar^2\left[\frac{\partial^2}{\partial\theta^2} + \hat{\phi}\cdot\frac{\partial\hat{\phi}}{\partial\theta}\frac{\partial}{\partial\theta} - \hat{\phi}\cdot\frac{\partial\hat{\theta}}{\partial\theta}\times\frac{1}{\sin\theta}\frac{\partial}{\partial\phi} - \frac{\hat{\theta}}{\sin\theta}\cdot\frac{\partial\hat{\phi}}{\partial\phi}\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + \frac{\hat{\theta}}{\sin\theta}\cdot\frac{\partial\hat{\theta}}{\partial\phi}\times\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right]$$

$$\left\{\hat{\phi}\cdot\frac{\partial\hat{\phi}}{\partial\theta} = 0 = \hat{\phi}\cdot\frac{\partial\hat{\theta}}{\partial\theta} \;\&\; \hat{\theta}\cdot\frac{\partial\hat{\phi}}{\partial\phi} = -\cos\theta \;\&\; \hat{\theta}\cdot\frac{\partial\hat{\theta}}{\partial\phi} = 0\right\}$$

$$\Rightarrow \boxed{L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]} \rightarrow \text{required expression}$$

$$\left[\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{L^2}{\hbar^2 r^2}\right]$$

Hydrogen Atom

$$\frac{-\hbar^2}{2\mu}\left[\frac{\Theta\Phi}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{\Phi R}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right) + \frac{R\Theta}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}\right] = (E-V)R\Theta\Phi$$

$$\left\{\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2} = -m^2 \Rightarrow \Phi(\phi) = e^{im\phi}\right\};\; \boxed{\frac{-\hbar^2}{2\mu R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + (V-E)r^2 = \frac{m^2}{\sin^2\theta} - \frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)}$$

$f(r) = f(\theta)$
$= l(l+1)$

for $R = \frac{u}{r}$ $\Rightarrow$ $\boxed{\frac{\partial^2 u}{\partial r^2} + \frac{2\mu u}{\hbar}(E - V_{eff}) = 0}$ where $\boxed{V_{eff} = V + \frac{l(l+1)\hbar^2}{2\mu r^2}}$

for $\rho = \sqrt{\frac{-2\mu E}{\hbar^2}}\,r$ (as $E<0$) $\Rightarrow$ $\boxed{\frac{d^2u}{d\rho^2} = \left(1 - \frac{\rho_0}{\rho} + \frac{l(l+1)}{\rho^2}\right)u}$

$\left\{\text{where } \frac{V}{E} = \frac{\rho_0}{\rho} = \frac{-Ze^2}{4\pi\epsilon_0 r E}\right\}$

$$\left\{\begin{array}{l}\psi_{1s} = \frac{1}{(\pi a^3)^{1/2}}e^{-r/a} \;\middle|\; \psi_{2s} = \frac{1}{4\sqrt{2\pi}\,a^{3/2}}\left(2-\frac{r}{a}\right)e^{-r/2a} \\ \psi_{2p}^{m=0} = \frac{1}{4\sqrt{2\pi}\,a^{3/2}}\frac{r}{a}e^{-r/2a}\cos\theta \\ \psi_{2p}^{m=\pm1} = \frac{1}{8\sqrt{\pi}\,a^{3/2}}\frac{r}{a}e^{-r/2a}\sin\theta\, e^{\pm i\phi}\end{array}\right.$$

for $\rho\to 0$ $\Rightarrow$ $\boxed{u = D\rho^{l+1}}$ ✓, ρ^{-l} (x) $\Big|$ for $\rho\to\infty \Rightarrow \frac{d^2u}{d\rho^2} = \boxed{u \Rightarrow u = C\exp(-\rho)}$

$\left\{\frac{d^2u}{d\rho^2} \sim \frac{l(l+1)}{\rho^2}u\right\}$

Writing complete solutⁿ by peeling off $\rightarrow$ $\boxed{u = A\rho^{l+1}e^{-\rho}v(\rho)}$ $\Rightarrow$ Using Power series solutⁿ, $\boxed{v(\rho) = \sum a_j\rho^j}$

$R_{nl}(r) = \frac{\rho^{l+1}e^{-\rho}}{r}v_n(\rho) \rightarrow$ Laguerre polynomial

as $v(\rho)$ has to be finite;
$$\left[a_{j+1} = \left(\frac{2(j+l+1)-\rho_0}{(j+1)(j+2l+2)}\right)a_j\right] \Rightarrow \boxed{\rho_0 = 2n}$$
$n = j_{max} + l + 1$

for, $\frac{m^2}{\sin^2\theta} - \frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right) = l(l+1)$ & $x = \cos\theta$

$$\boxed{(1-x^2)\frac{\partial^2\Theta}{\partial x^2} - 2x\frac{\partial\Theta}{\partial x} + \left(\lambda - \frac{m^2}{(1-x^2)}\right)\Theta = 0} \Rightarrow \lambda = l(l+1) \text{ from } |m|<l \;\&\; \underbrace{\Theta(\theta) = BP_l^m(\cos\theta)}_{\text{associated Legendre polynomial}}$$

$$\boxed{a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}}$$

In Stern-Gerlach experiment, magnetic dipole moment experience a force proportional to field gradient since two poles subjected to different fields.

→ j-j coupling is dominant after Z=57

$6s^2 \rightarrow l_1=0,\ s_1=\frac{1}{2},\ j_1=\frac{1}{2}$ | $l_2=0,\ s_2=\frac{1}{2},\ j_2=\frac{1}{2}$; $J=1,0$ — 7th Ques. if j-j coupling is used

ATOMIC PHYSICS
TUTORIAL SHEET: 4

1. What is Lamb shift? Explain it by illustrating through a suitable energy level diagram. Has Lamb shift been observed in any atom other than hydrogen? (done in 47) **(2001)**

→ Stern Gerlach | Pauli's concept | Anomalous Zeeman effect

2. Give briefly an account on the important historical developments to establish the concept of electron spin. **(2001)**

$\vec{J}=\vec{L}+\vec{S}$ (Light) Russel Saunders L-S | J-J (Heavier) $(j_1 j_2)$ (Russel Saunders)

3. On the basis of vector atom model, discuss briefly the L-S and J-J coupling schemes. **(2001)**

Conclude with — explains Zeeman effect; coupling breaks under strong B (Paschen-Back)

4. Write down expressions for spin angular momentum, orbital angular momentum and total angular momentum under L-S coupling scheme. (done) **(2001)**

→ g factor comes due to quantization of coupled angular momenta, $g = 1+\frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}$ ⇒ $L=3$

5. Write the spectral symbol of the term with
   S = 1/2 ,J = 5/2 and g = 6/7 **(2002)**

→ term symbol is crucial for representing angular momenta in succinct form and useful tool in molecular physics

defn of spin-orbit interaction in weak interaction of spin with orbital B. — factor Thomas precession

6. Derive an expression for the spin-orbit interaction energy in one electron system. Calculate the energy separation between the levels $^2P_{1/2}$ and $^2P_{3/2}$. **(2002)**

7. Write the electronic configuration of mercury (Z=80) .Obtain the spectral term for the normal and the first excited configuration on the atom. $Z=80 \equiv 6s^2 \rightarrow j=0, s=0, l=0 \Rightarrow \boxed{^1S_0}$ **(2002)**

$\hookrightarrow 6s^1 6p^1$ — Russel Saunders — Conclude succinctness

(done in 43) $\vec{\mu} = -\frac{e}{2m}(\vec{L}+2\vec{S})$

8. Calculate the Larmor frequency of a spin ½ particle in a magnetic field B. **(2003)**

→ measure ratio of μ & angular momenta | Expression for g-factor → $\vec{\mu}_J = -\frac{e}{2m} g_J \vec{J}$

9. Define the gyromagnetic ratio and obtain an expression for the Lande 'g' factor. What is the importance of 'g' in spectroscopy? → Importance — Zeeman shift | g-factor; Zeeman components determine | Larmor precession **(2003)**

Intro → defn, write 3 types, selectn rule

10. What is Zeeman Effect? How can it be understood on a quantum mechanical basis? Obtain an expression for Zeeman shift. **(2004)**

Calculatn of $\vec{\mu}$ ⇒ $\Delta E = \vec{\mu}_J \cdot \vec{B}$; $\vec{\mu} = -\frac{e}{2m}\vec{L} + (\frac{e}{m})2\vec{S}$; $\Delta E = \mu_B g_J m_J$ ⇒ Zeeman effect $(2L+1)(2S+1)$ states under strong B (coupling breaks)

11. Calculate the Zeeman shift observed in the normal Zeeman Effect when a spectral line a wave length 5000 A° is subjected to the magnetic field of 0.4 wb/m^2. (∵ $g_J = +1$ for normal as $S=0$) **(2004)**

Introduce with Zeeman shift → write $\boxed{\Delta E = \mu_B B g_J}$ ⇒ $\Delta E = \mu_B B$ → Zeeman shift for $S \neq 0$ is corrected by g-Lande factor

12. What are the term symbols for atoms with the following S & L quantum numbers. S=1/2 and L=3? **(2004)**

Term symbols $^{2S+1}L_J$ | $|L-S| \le J \le |L+S|$; $^2F_{5/2}$ & $^2F_{7/2}$

(2J+1) lines & Zeeman effect

13. How many lines occur in the multiplet arising from $^2P_{3/2} \rightarrow {}^2S_{1/2}$ and $^2P_{1/2} \rightarrow {}^2S_{1/2}$ transitions of alkali metal atoms placed in a weak magnetic fields why? **(2005)**

→ components → mathematically $F_{mag} = (\vec{\mu}\cdot\nabla)\vec{B}$ ⇒ $F_{mag} = \mu_z \frac{\partial B_z}{\partial z}\hat{k}$; $\mu_z = -g_J \mu_B m_J$

14. Describe Stern-Gerlach experiment and discuss its implications. **(2005)**

Implications → existence of spin, space quantization, confirms spin's vector model | Confirmed hypothesis by Uhlenbeck & Goudsmit in 1925 | spatial orientation of angular momenta is quantized | using spin to explain Zeeman, LS

15. What is Lamb shift? Has it been observed in any atom other than hydrogen? **(2007)**

(already in 47)

16. Describe Stern- Gerlach experiment. Explain how it demonstrates the discrete nature of the magnetic moment of an atom. (Same as 14th) **(2007)**

R is limiting value of the highest wavenumber (1/lambda) of photon that can be emitted from an atom

(done) (done)

17. Explain normal and anomalous Zeeman Effect. Obtain expression for Zeeman splitting of an alkali metal spectral line, and illustrate with an example. {done in 32,33} **(2008)**

→ $n=\infty$ to $n=1$ ⇒ $\lambda = \frac{1240}{13.6}$ nm = 91.2 nm

18. For transition to the ground state what is the longest wavelength that can be emitted by hydrogen **(2008)**

19. What was the aim of Stern – Gerlach experiment? Why were silver atoms chosen instead of electrons in the experiment? What was the outcome of the experiment? (same as 14) **(2009)**

20. $2^2S_{1/2}$ level in H atom is 1058 MHz above the $2^2P_{1/2}$ level

(i) What is this known as? Lamb shift violating dirac's formula

(ii) Express the above frequency in cm^{-1}. $\bar{\nu} = \frac{1}{\lambda} = 0.035\ cm^{-1}$

(iii) Calculate the energy difference between the above two levels in eV. **(2009)**

$\Delta E = 4.4\ \mu eV$

21. (i) Establish that $hc = 1240 eV.nm$ = 1240 MeV. Fm

$E_n = -\frac{13.6 eV}{n^2}$

(ii) The energy levels of a hydrogen atom are given by $E_n = (-1/n^2).Ryd$.

Where1 Ryd = hcR. Show that $R = 1\cdot097\times10^7\ m^{-1}$ What exactly is R? **(2009)**

22. (i) Find the ground state total orbital (L) and total spin (S) quantum numbers for nitrogen. → $2p^3$

(ii) Find the L and S values of the two "first excited states" of helium atom. $1s^2 2s^2 \rightarrow {}^1S_0$

(b). (i) Explain spin-orbit coupling of an atomic electron.

(ii) Show that the 2p state in the H atom splits up into two substates due to spin-orbit coupling.

(iii) Calculate the energy of separation in eV, resulting from the spin-orbit coupling when the magnetic field experienced by the electron is 0.4 T. **(2009)**

23. What is Zeeman Effect? How it can be understood on quantum mechanical basis? Obtain an expression for Zeeman splitting of atomic energy levels in a magnetic field B. Explain the magnetic splitting of sodium D-lines. (done) **(2010)**

24. What is spin-orbit interaction? Calculate the energy shift due to spin-orbit interaction term in H-like system. Discuss the significance of this shift in relation to the fine structure of hydrogen spectral lines (done) → Lamb shift **(2010)**

→ amount of splitting characterised by fine structure constant $[\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}]$

→ draw diagram → spin-orbit → Lamb shift

25. Discuss the fine structure of hydrogen atom spectrum. Draw the compound doublet spectrum arising as a result between 2 p and 2d levels. **(2011)**

$\Delta l = \pm 1$, $\Delta j = 0, \pm 1$

→ Energy levels of e^- in atoms → mathematically ${}^{2S+1}L_j$

26. What do you mean by term symbols? Obtain term symbols for the following sets of values of S andL: → succinctness & use in Russell's scheme

(i) S= $\frac{1}{2}, L = 2$ (ii) $S = 1, L = 1$ (iii) $S = \frac{3}{2}, L = 1$ **(2011)**

27. Show that ${}^2S_{\frac{1}{2}}$, ${}^2P_{\frac{1}{2}}$, ${}^2P_{\frac{3}{2}}$ levels of sodium spectrum are split in the ratio of 3 : 1 : 2 due to anomalous Zeeman Effect. **(2011)**

↳ def^n it → write formula → ratio split calculation → conclude → weak $\vec{B}$

↳ g, values ↳ g, comes due to precession of angular momenta under $\vec{B}$

$[(g_j)_{{}^2P_{3/2}} = 4/3 \mid (g_j)_{{}^2P_{1/2}} = 2/3 \mid (g_j)_{{}^2S_{1/2}} = 2]$

28. Sodium doublets are produced by transitions $3^2P_{\frac{1}{2}} \rightarrow 3^2S_{\frac{1}{2}}\ (D_1)$ and $3^2P_{\frac{3}{2}} \rightarrow 3^2S_{\frac{1}{2}}\ (D_2.)$ Calculate the Lande's g- factors for these levels. (calculated in ③)
(2012)

$\rightarrow |l-s| \le j \le |l+s|$

29. Discuss the fine structure of sodium D line. Draw D_1 and D_2 lines due to the transitions between ^2P and ^2S levels. $\{\lambda_{D_1} = 5896\,Å,\ \lambda_{D_2} = 5890\,Å\}$ **(2013)**

30. A sample of certain element is placed in a magnetic field of flux density 0.3 tesla. How far apart is the Zeemen component of a spectral line of wavelength 4500 A?
Given: $e/m = 1.76 \times 10^{11}$ c/kg, $c = 3.0 \times 10^8 ms^{-1}$. $\{\Delta E = hc\,\Delta\bar{\omega}\}$ **(2014)**

$\Delta\lambda = \lambda^2\,\Delta\bar{\omega}$ explain zeeman effect $\rightarrow \Delta E = \mu_B B$ (for normal s=0)

31. Obtain an expression for the normal Zeeman shift. Illustrate the Zeeman splitting of spectral lines of H atom and the allowed transitions for the l = 1 and l = 2 states. $\Delta E = \pm\mu_B B$ **(2014)**

32. What is Zeeman Effect? How can it be understood on the basis of quantum mechanics? (done) **(2015)**

33. Obtain Zeeman splitting for sodium D-lines. **(2015)**

↳ Define → types + selection rule

34. Find the magnetic moment of an atom in $3P_2$ state, assuming that LS coupling holds for this case.
$\mu_J = -\frac{e}{2m} g_J \vec{J}$ (assuming LS coupling) ; solve it → conclude strong B breaks coupling **(2015)**

35. In the Stern-Gerlach experiment using Ag atoms, the oven temperature is 1000 K, $l \approx 25$ cm and $\frac{\partial B_z}{\partial z} \simeq 10^{+3}$ Tesla/m. Calculate the separation of the two components. **(2016)**

Draw figure ↳ write formula

36. Describe Stern-Gerlach experiment. Discuss how it has explained space quantization and electron spin. Find the value of angle between the spin angular momentum S and its z-component of an electron moving along the external magnetic field B. $\boxed{\cos(\theta_S) = \frac{m_s}{\sqrt{s(s+1)}}}$ **(2016)**

37. The series limit wavelength of Balmer series in hydrogen spectrum is experimentally found to be 3646 A. Find the wavelength of the first line of this series. [Series limit → shortest wavelength] **(2016)**

$[\lambda_1 = \frac{36}{5R} = 6562.8\,Å]$ $\frac{1}{\lambda_\infty} = R\left(\frac{1}{4} - \frac{1}{\infty}\right) \Rightarrow \lambda_\infty = 4/R$

38. Compute the allowed spectral terms for two non-equivalent p-electrons of the basis of Pauli's exclusion principle. **(2016)**

39. Explain in detail L-S coupling and j-j coupling schemes. (already done) **(2016)**

40. What is Lamb shift? What is its significance in determining the fine structure of H_α Balmer line in hydrogen atom? (explained below) **(2016)**

41. Calculate the radius of electron orbit for Li^{++} in ground state. **(2018)**

$\boxed{r_n = a_0\frac{n^2}{Z}}$

42. Describe the importance of L-S and J-J coupling in atomic spectroscopy. What are experimental evidences of their existence? **(2018)**

43. What is Zeeman effect? Discuss the factors on which Larmor frequency is dependent. **(2018)**

precession of J under influence of B about B direction
↳ Larmor precession

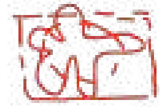
$\boxed{\omega_p = \frac{e}{2m} g_J B}$

44. Discuss the fine structure of hydrogen spectrum. How is it of importance in the astronomical observations? **(2018)**

$\left[a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_0 e^2} \right]$

45. Define mathematically the Bohr radius of a hydrogen atom and who that the binding energy at state n of this atom can be given by

defn Bohr Model → nearly circular model + $L = nh/2\pi$ → e^- motion in stable orbit



→ Conclude fine structure spin effect, but explained ground state

$\left[\frac{mv^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}\right]$ & $\left[m v r_n = \frac{nh}{2\pi}\right]$

putting v in form of r_n $\Rightarrow \left[r_n = \frac{a_0 n^2}{Z}\right]$

$$E_n = -\frac{1}{2}\frac{Ze^2}{(a/Z)}\frac{1}{4n^2\pi\varepsilon_0}$$

$E_n = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\varepsilon_0 r_n}$ $\Rightarrow E_n = -\frac{Ze^2}{8\pi\varepsilon_0 r_n}$

where Z is the atomic number of H atom. Calculate the numerical values of a and E_1 of H atom. **(2019)**

S=0 singlet (only in 2n(d))

normal, transverse, longitudinal (S≠1)

π-components $\Delta m_j = 0, \pm 1$ → transverse

σ-components $\Delta m_j = \pm 1$ → longitudinal

3 component double (m, ±1)

$\boxed{\Delta E = \mu_B g_j B}$ → zeeman shift

46. Describe normal and anomalous Zeeman effect. Explain how it lifts the degeneracy in hydrogen atom. **(2019)**

Zeeman effect is splitting of splitting line into several components (2J+1) under weak B

$g_j = \left[1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}\right]$ (2S+1)(2J+1) components

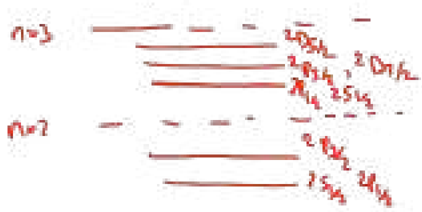
↳ Conclude with coupling breaks under strong B (Paschen Back)

47. What is lamb shift? Discuss its significance in determining the fine structure of H_α Balmer line in hydrogen atom. **(2019)**

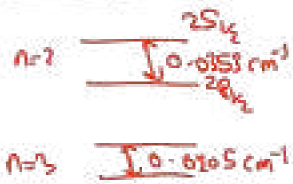
$\Delta S = 0$, $\Delta j = 0, \pm 1$, $\Delta l = \pm 1$

↳ 5 lines of H_α

as per dirac's equatn



Lamb shift



⇒ 7 lines

7 lines

→ shaky e^- motion due to interaction of e^- with emf

↳ for Deuterium at 1058 MHz & He at 14 GHz

• $\Delta E = hc\Delta\bar{\omega}$ } → zeeman shift

{ $\bar{\omega}\lambda = 1$ $\Rightarrow$ $\left[|\Delta\lambda| = \lambda^2 \Delta\bar{\omega}_0\right]$

$\boxed{\Delta\bar{\omega} = \frac{\left(\frac{e}{m}\right)B}{4\pi c}}$ for normal zeeman effect

↳ in terms of Lorentz unit

Example: p^3 configuration [Main] [Q-22 Tut-4 (2009)]

$m_l =$	1	0	−1	1	0	−1
$m_s =$	$1/2$	$1/2$	$1/2$	$-1/2$	$-1/2$	$-1/2$
	(a)	(b)	(c)	(d)	(e)	(f)

→ of these, three electrons are taken for combination, ${}^6C_3 = 20$ such combination

	(abc)	(abd*)	(abe*)	(abf*)	(acd**)	(ace**)	(acf*)	(ade*)	(adf*)	(aef*)
$m_L =$	$\boxed{0}$	2	1	0	1	0	−1	2	1	0
$m_S =$	$\boxed{3/2}$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$-1/2$	$-1/2$	$-1/2$

	(bcd)	(bce**)	(bcf*)	(bde**)	(bdf**)	(bef*)	(cde)	(cdf**)	(cef*)	(def)
$m_L =$	$\boxed{0}$	−1	−2	1	0	−1	$\boxed{0}$	−1	−2	$\boxed{0}$
$m_S =$	$\boxed{1/2}$	$1/2$	$1/2$	$-1/2$	$-1/2$	$-1/2$	$\boxed{-1/2}$	$-1/2$	$-1/2$	$\boxed{-3/2}$

- highest value of m_L as 2 indicate a D term with $m_S = 1/2, -1/2 \Rightarrow S = 1/2$
$\Rightarrow {}^2D$ (* marked terms)

Of remaining highest m_L value is 1, with $m_S = 1/2, -1/2 \Rightarrow$ $\boxed{{}^2P\ (L = 1, S = 1/2)\ \text{terms}}$ ← include ** terms (also incl., $m_L = 0, -1$ each with $m_S = 1/2, -1/2$ include term)

(remaining terms in box corresponds to $S = 3/2$) $\Rightarrow$ $\boxed{{}^4S\ \text{term}}$

$\boxed{{}^2P^o,\ {}^2D^o,\ {}^4S^o}$ → ${}^4S$ is ground state

Example: (Main) two equivalent electrons & non-equivalent electrons

$4p^2 5s \Rightarrow 4p^2 + 5s \Rightarrow$

$4p^2 \to \boxed{{}^1S,\ {}^1D,\ {}^3P}$

$$\left\{\begin{array}{l} \underset{(L=0,\,S=0)}{{}^1S} + \underset{(L=0,\,S=1/2)}{s} = \underset{(L=0,\,S=1/2)}{{}^2S} \\ {}^1D + s = {}^2D \end{array}\right\}$$

PHYSICS PAPER- II: MOLECULAR PHYSICS
TUTORIAL SHEET: 5

1. i. Discuss qualitatively the occurrence of rotational energy levels of diatomic molecule. Write down the selection rule

ii. Is it possible to obtain pure rotational spectra of H2, HF, O2 and No molecules? Can one obtain pure rotational spectra in emission? Comment. (On both of the above mentioned queries).

iii. In CO molecule J=0 $\rightarrow$J=1 line occurs at frequency 1.153× 101^{11} Hz. Calculate the moment of inertia of CO molecule.

i. What is Raman Effect? How does it differ from Rayleigh scattering? Write down its important applications.

ii. Explain Raman Effect with the aid of quantum mechanical theory.

iii. Why is Raman Effect considered complimentary to infrared absorption spectra?

iv. A Raman Stoke's line is observed at 552 nm. When a sample is excited by Hg green line of wave length546 nm. Calculate the wave length of anti-stoke's line **(2001)**

2. i. A sample is irradiated by a 5000 A° radiation to give a Raman line at 5050.5 A°. Calculate the Raman frequency.
ii. Explain, both red and violet degraded bands have been observed in electronic band systems, but rotation-vibration spectra show bands degraded to red only. **(2002)**

3. Calculate the moment of inertia of HCl molecule from the expression= 20.68(J+1) cm where J = 0,1,2,…. **(2002)**

→ def^n rot. spectra → cond^n of permanent $\vec{p}$
→ $\bar{\omega}=2B(J+1)\,cm^{-1} \Rightarrow 2B=20.68 \Rightarrow B=10.34$ $\left\{B=\frac{h}{8\pi^2 I_{com} c}\right\}$ ⇒ Only heteronuclear molecule

4. A diatomic gas is found to have a number of absorption bands in the ultraviolet region. Each band is observed to have a fine structure. How can the observed spectrum be explained and analysed? **(2002)**

→ spin flip transit^n | Hyperfine 21 cm (6×10^{-6} eV)

5. Explain the nature, origin and significance of 21 cm Hydrogen line. **(2002)**

6. Calculate the change in rotational constant Bυ when deuterium is substituted for hydrogen in a hydrogen molecule. **(2002)**

$B_D=\frac{h}{8\pi^2 I_{com} c}$ $\Rightarrow \boxed{B_D=\frac{B_H}{2}}$

7. (i) What are the typical energies (ev) of the radiations, required to excite (i) electronic transitions (ii) vibrational transitions and (iii) rotational transitions in a molecule?
(ii) The rotational spectrum of HI consist of equidistant lines with a separation of 12.8 cm^{-1}. Calculate the (a) M.I. and (b) bond length of HI molecule. **(2003)**

8. Discuss the salient features of O, P,Q,R and S branches of electronic spectrum of diatomic molecules. Which spectrum contains only branches, both branches and bands, branches and progressions? **(2003)**

9. Show that a spinning nucleus processes in a magnetic field. Explain the underlying principle of NMR spectroscopy. The magnetic moment of a proton is 2.793 µn Calculate the radio frequency at which nuclear magnetic resonance occurs in water kept in a magnetic field of 1.5 T. **(2003)**

10. Water vapor shows Infrared absorption bands at 1595 cm^{-1}, at 3652 cm^{1} and at 3756 cm^{-1}. If the Raman spectrum is observed with the Argon ion laser 514.5 nm, at what wavelength would you expect to find the Raman - stokes lines? **(2004)**

11. High resolution laser spectroscopy can resolve lines that cannot be resolved by conventional spectroscopy. Justify this statement. **(2004)**

12. Briefly describe rotational fine structure of electronic spectra of diatomic molecules. **(2004)**

13. What are the selection rules for rotational spectra? Draw a neat diagram to show the allowed rotational transitions. **(2004)**

14. Explain the molecular phenomenon of spontaneous emission between two electronic states of the same multiplicity and differentiate it from that of different multiplicity. **(2005)**

15. The constants B and V_0 for a KCI molecule have values 143×10^{-5} eV and 84×10^{12} s^{-1} respectively. Determine the number of rotational levels between the vibrational levels v =0 and v = 1. **(2005)**

16. How can the pure rotation spectrum of H_2 molecule be observed? If the bond length of H_2 molecule is 0.07417 nm. What would be the spacing of lines in its spectrum? **(2005)**

17. Differentiate between Rayleigh and Raman scatterings. Why is Raman scattering considered to be a breakthrough in molecular spectroscopy? What are the advantages of using laser light in Raman spectroscopy? **(2005)**

18. Obtain an expression for the vibration-rotation energy levels of a diatomic molecule in a given electronic state. The wave numbers of the vibrational transitions occurring in HF, HCI and HI molecules are34143 cm^{-1}, 29889 $cm^{-1}$and 23095 cm^{-1} respectively. Compare the force constants of these three molecules. **(2006)**

19. (i) State and explain Franck – Condon Principle. (done)
(ii) In three separate pairs of electronic states (one ground and another upper state) of diatomic molecules, the average inter nuclear distances are-
(i) equal; (done)
(ii) slightly greater;
(iii) appreciably greater.
In accordance with Franck- Condon principle, obtain most favourable transitions from the ground state vibrational levels in each of the above situations.
Illustrate the transitions by drawing suitable energy – level diagrams. **(2007)**

20. Draw the potential energy of a diatomic molecule as a function of inter atomic distance. Mark the vibrational and rotational energy levels. Explain the selection rule for transitition between vibrational states. **(2008)**

21. Assume a diatomic molecule consisting of two atoms of masses m_1 and m_2, separated by a distance $\vec{r}$. Write down the Hamiltonian operator for the molecule. Determine the rotational energy levels of the molecule, **(2008)**

22. Write full form of acronyms EPR and NMR. Give underlying principle of EPR. **(2008)**

23. Explain fluorescence and phosphorescence in electronically excited molecules. **(2009)**

24. Discuss the vibrational spectra of a diatomic molecule treating it as a harmonic oscillator as well as an anharmonic oscillator and compare them. **(2009)**

25. Explain Born – Oppenheimer approximation. Discuss the intensity distribution in the vibrational electronic spectra of a diatomic molecule on the basis of Franck – Condon principle. done already below **(2009)**

26. Calculate, giving necessary steps, the radio frequency at which nuclear magnetic resonance occurs in water kept in a uniform magnetic field of $2 \cdot 4T$. The magnetic moment of proton is $2 \cdot 793 \mu N$.
(2010)

27. (i) Discuss occurrence of rotational energy levels of a diatomic molecule and show that the pure rotation spectrum of such a molecule consists of a series of equally spaced lines separated by a constant wave number difference 2B. Write down the selection rules.
(ii) Is it possible to obtain pure rotational spectra of H_2, HF, O_2 and NO molecules?
(iii)In CO molecule J = O →J = 1 line occurs at a frequency $1 \cdot 153 \times 10^{11}$ Hz. Calculate the moment of inertia of CO molecule. **(2010)**

28. What is Raman Effect? How does it differ from Rayleigh scattering? Explain Raman Effect on the basis of quantum mechanical theory. How is Raman Effect experimentally studied? What are the advantages of using laser sources in the study of Raman Effect? **(2010)**

most probable transit^n is obtained by joining straight line ... explain →

29. State and explain Franck-Condon principle. Discuss its applications in molecular spectroscopy. **(2010)**

Rule in spectroscopy for intensity distribut^n during vibronic transit^n

30. On the basis of inertia I_A, I_B and I_C each about X, Y and Z axes respectively, how can you classify molecules? **(2011)**

31. (a) (i)Treating a diatomic molecule as a simple oscillator, obtain its energy (vibrational) levels.
(ii)The observed vibrational frequency of the Co molecule is 6.42 $\times$ 10^{13} Hz. What is the effective force constant of the molecule? (C= 12u, O = 16u)
(b) (i) Discuss pure rotational spectra of linear molecules
(ii) What is Lamb shift? **(2011)**

32. Why should rotational Raman spectrum show a separation of the first Raman line from the exciting line as 6B cm^{-1}, While the separation between successive lines is equal to 4B cm^{-1}, when B is the rotational constant? **(2012)**

33. What will happen to the energy level of an unpaired electron when subjected to an external magnetic field? Find the condition for electron spin resonance. **(2013)**

34. (a) With proper selection rules, construct the energy level diagram and allowed transitions for ESR spectrum of hydrogen atom. (b) Why are Raman active vibrations and infrared vibrations in CO_2molecule complementary to each other? (c) In a Raman spectrum of linear triatomic molecule, the first three lines are 4.86, 8.14 and 11.36cm^{-1}. Calculate the rotational constant, B and the moment of inertia of the molecule. (Given h = 6.626 x 10^{-27}J.s, C = 3.0 x 10^{10}cm/sec.) **(2013)**

35. Discuss the vibrational spectra of a diatomic molecule treating it as an anharmonic oscillator. **(2014)**

36. Explain how the nuclear spin I depends on the mass number A and atomic number Z of atoms.**(2014)**

37. (i) Obtain an expression for the resonance condition in NMR.
(ii)Explain the relaxation processes in NMR spectroscopy. **(2014)**

38. Establish that: $hc = 1240\ MeV.\ fm$ **(2015)**

Info → h is planck constant
c is speed of light

$\Rightarrow hc = \frac{6.626 \times 3 \times 10^{-7}}{1.6}$ eV-m
$= 1242$ eV-nm | for 1 nm wavelength 1240 eV is associated.

39. Hydrogen molecule is diatomic. Obtain the rotational energy levels of this molecule. Write down the selection rules. Obtain the smallest energy required to excite the lowest rotational mode. **(2015)**

40. The observed vibrational frequency of CO molecule is $6.42 \times 10^{13}\ Hz$. What is the effective force constant of the molecule? **(2015)**

$\nu_{eff} = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$ $K = 4\pi^2\nu^2 \times \mu_{CO}$ & $\mu_{CO} = \frac{48}{7}$ amu

Assuming SHO motion associated with molecule,

41. Show that the lines in the absorption spectra corresponding to the rotational transitions from two adjacent energy levels of a medium sized molecule at room temperature have comparable intensities. **(2017)**

42. Given the force constant of HC1 molecule = 516 Nm^{-1}, determine the wave number of the fundamental mode of vibration of the molecule. How many transition lines one can expect in the vibration spectra of HC1 molecule at room temperature? **(2017)**

43. Explain Stokes and anti-Stokes Raman scattering with the help of energy level diagram. For a diatomic molecule, obtain expressions for transition energies of its Raman spectra with rotational fine structure and hence the wave numbers of the Stokes lines. **(2017)**

44. Explain why lines in some Raman spectra are found to be plane polarized to different extents even though the exciting radiation is completely unpolarized? **(2017)**

45. State Franck-Condon principle. Define Franck-Condon factors. Using schematic diagram, explain the decay of excited states leading to the phenomena of fluorescence and phosphorescence. **(2017)**

46. Explain the principle of Nuclear Magnetic Resonance (NMR) with the help of an energy level diagram. Give examples of nuclei which exhibit NMR. What major inferences can be drawn from an NMR spectra? **(2017)**

47. In an NMR experiment, hydrogen atoms are subjected to a magnetic field of 5.0 T. Determine the difference in energy (kJ/mol) between two spin states of the nuclei of hydrogen atom and the frequency of radiation required for NMR. **(2017)**

48. What is nuclear precession? How is it used in the principle of working of NMR? **(2018)**

49. Discuss the theory of rotational and vibrational spectra of diatomic molecules. What is the difference between fluorescence and phosphorescence? **(2018)**

change in polarizability ← → change in permanent dipole moment

50. Why are Raman active vibrations and IR vibrations in CO_2 molecule complementary to each other? **(2019)**

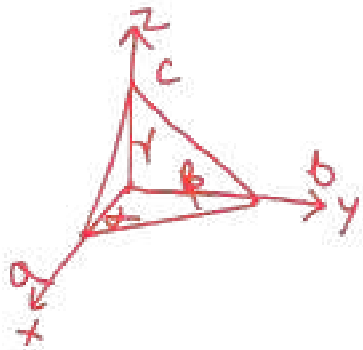
$O=C=O$ → IR inactive, Raman active (symmetric)
$O=C=O$ → Raman inactive, IR active (Unsymmetric)
$O=C=O$ (In phase bending)
$O=C=O$ (out of phase bending) Raman inactive but IR active

CO_2 has inversion centre with centre of symmetry
↳ laws of mutual exclusion → complementarity of IR and Raman of the vibration → centre of symmetry

51. What is Franck-Condon principle? Discuss the intensity distribution in the vibrational electronic spectra of a diatomic molecule on the basis of this principle. (already done) **(2019)**

Born-Oppenheimer Approximation is the assumption that the electronic motion and the nuclear motion in molecules can be separated. It leads to a molecular wave function in terms of electron positions and nuclear positions.

Assumptions: 1. The electronic wavefunction depends upon the nuclear positions but not upon their velocities, i.e., the nuclear motion is so much slower than electron motion that they can be considered to be fixed. 2. The nuclear motion (e.g., rotation, vibration) sees a smeared out potential from the speedy electrons



$h = \frac{a}{A}, \quad k = \frac{b}{B}$

$[h:k:l] = \left[\frac{a}{\alpha}, \frac{b}{\beta}, \frac{c}{\gamma}\right]$

$\alpha = \frac{D}{A} = \frac{a}{h} \qquad \beta = \frac{D}{B} = \frac{b}{k}$

TUTORIAL SHEET: 6
NUCLEAR PHYSICS

[Please check Notebook too]

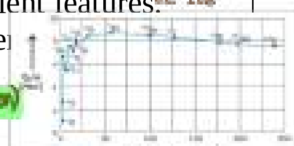
1. Explain why a neutral particle such as neutron possesses a finite value of magnetic moment, and how it is determined? **(2002)**
   using shell model, NMR, quark model (etc) → spin ⇒ π cloud due to nucleon (pions are nucleon) ⇒ -ve μ

2. Write down the configuration of ^7Li, ^{13}C and ^{25}Mg in the ground state of the nuclear shell model. **(2002)**
   defⁿ shell model (magic no., nuclei); odd nucleon gives π, spin; $(1P_{3/2})^1$, $(1P_{1/2})^1$, $(1D_{5/2})^5$; conclude by evidences of nuclear and predict parity & spin

3. Explain how neutrino was discovered from the β -decay of radioactive nuclei. **(2002)**
   $(p \to n + e^- + \bar{\nu}_e$ → to fix spin & Energy); Reines–Cowan method (1956); $(p + \bar{\nu}_e \to e^+ + n)$

4. Explain shell model of the nucleus. Give evidence for nuclear shell structure. What are the limitations of the shell model? **(2002)**
   evidence → High natural abundance | islands of nuclear isomers | lead as product of radioactive series
   limitation → spin; μ doesn't match; doesn't explain stability

5. Sketch carefully the binding energy per nucleon curve for stable nuclei. Explain its salient features. On the basis of this curve explain why fusion is possible only for low mass nuclei, whereas fission takes place in heavy nuclei.
   A = 20–180 (8–8.4 MeV); local peaks due to pairing of spin; ^{56}Fe → most stable; A < 20 (fusion), A > 180 (fission)



6. List some of the important properties of deuteron and show that it is a loosely bound system with only one bound state. **(2003)**
   size = 4.2 fm; spin = 1; μ = 0.8574 μN; E = −2.225 MeV; Q = 0.00282 barn

7. Distinguish between a nuclear reaction and decay. Which conservation laws are obeyed in nuclear reactions? Explain the significance of Q value. Calculate the Q value for the following nuclear reaction $_1{}^2H + {}_1{}^2H = {}_2{}^3He + n$ **(2003)**
   $X + x \to Y + y$; decay involves disintegration of unstable nuclei
   $Q = (m_{2H} + m_{2H} - m_{3He} - m_n)c^2$
   spontaneity & stability, conversion of mass; Q > 0 (exoergic) [$M_{reac} > M_{product}$]; used in nuclear reactor

8. Calculate the fission rate of ^{235}U. required to produce 4 watts and the amount fissioning of 1kg of ^{235}U. (Assuming energy released per fission of ^{235}U is 200 Mev). **(2004)**
   for 4 watts → $\frac{4}{3.2 \times 10^{-11}}$ fission/second
   235 gram → 1 mole; 1 kg of $U^{235} = 2.56 \times 10^{24}$ nuclei | 200 MeV → 1 fission

9. Calculate Binding energy per nucleon of $^4{}_2$He and $^{16}{}_8$O. **(2004)**
   (Draw curve of ⑤) B.E = (mass of nucleon − mass of nuclei) c^2
   energy required to bind constituents into nuclei; B.E is result of mass deficit or energy equivalent

10. Write down the Bethe-Weizsacker formula. Explain various terms of the formula. **(2004)**

11. Explain how energy is released in stars. → explain fusion & write p-p cycle **(2004)**
   Quantum effect → fusion leads to maximum mass → Chandrasekhar limit of new stars; tunneling

12. Describe in detail one experiment for detection of Neutrino. **(2004)**
   (already done in ③)

13. What is the difference between neutrino and antineutrino? How is helicity of neutrino determined? **(2004)**
   (done below)

14. (a) Write the Weizacker mass formula and explain the significance of various terms.
   (b) Determine the amount of $^{210}_{84}P_0$ necessary to provide a source of $^{210}_{84}P_0$ α-particles of strength 5 mC₁. The half-life of is 138 days. **(2005)**
   $\frac{dN}{dt} = 5\ mC = 5 \times 3.7 \times 10^7$; $\lambda = \frac{0.693}{T_{1/2}}$; $\frac{dN}{dt} = -\lambda N$ ⇒ $N = 3.8 \times 10^{15}$ ⇒ $m = \frac{N}{N_A} 210$

15. Describe how parity violation was experimentally detected in $^{60}C_O$ β-decay. How was the observed asymmetry in the distribution of emitted electrons explained?
   Helicity (pseudoscalar)

Single Particle shell Model (asked in Mains) [TS#6 ④]

Assuming QM treatment of nuclei, and assuming unpaired nucleon contribute alone to spin, parity & moments, it assumes nucleon move nearly freely under influence of averaged effect of rest (A-1) nucleons.

We also assume nucleon imagined to move in a shell depending on energy & angular momenta (quantized) analogous to atomic electrons.

Now we use spin-orbit coupling b/w nucleons [l & s combine to give j's & j's couple to give I_{net}].

$$\boxed{W' = \frac{-\lambda}{Mc^2} \frac{1}{r}\left(\frac{\partial U}{\partial r}\right) \vec{l}\cdot\vec{s}} \qquad \left[\vec{l}\cdot\vec{s} = \frac{l^2+s^2-J^2}{2}\right]$$

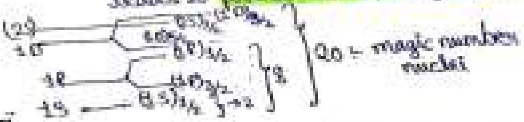
Now putting $U = V_0 + \frac{1}{2} m\omega^2 r^2$;

$$\boxed{E_{nl} = \hbar\omega\left(\Lambda + \frac{3}{2}\right)} \quad (\text{where } \Lambda = 2n + l - 2)$$

for each value of Λ, each shell is occupied by $N_\Lambda = (\Lambda+1)(\Lambda+2)$.

Λ	$N_{max} = \sum N_\Lambda = \sum(\Lambda+1)(\Lambda+2)$
0	2
1	8
2	20
3	40
4	70 → not stable nuclei
5	112

To correct this, one incorporates spin-orbit interacn b/w nucleons and centrifugal term $= \left(\frac{l(l+1)\hbar^2}{2mr^2}\right)$ which leads to prediction of magic numbers.



Significance of Gellmann-Nishijima Relation

$$\boxed{Q = I_z + \frac{1}{2}(B+S)}$$

- useful in explaining charge of Kaon mesons by introducing concept of strangeness ⇒ conserved in production & violated in decay
- explains subparticles with same & isospin ($2I+1$ such sub particles). [postulates analogous to em field, charge gives rise $(2I+1)$ particles like Zeeman effect]

$\therefore Q = aI_z + b \Rightarrow \langle Q\rangle = b$ (as $\langle I_z\rangle = 0$ as I_z goes from $-I$ to $+I$)

($a = e$ & $b = \langle Q\rangle = \frac{eY}{2}$) $\Rightarrow \boxed{Q = e\left(I_z + \frac{Y}{2}\right)}$ → modified relation.

(Since B is always conserved, conservatn of Y tells us about strangeness)

$${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + e^+ + 2\nu$$

$${}_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + 5.5$$

$$2\,{}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1 + 12.8\,MeV$$

$$4\,{}_1H^1 \rightarrow {}_2He^4 + 2e^+ + 24.6$$

Neutron stars are formed when a massive star runs out of fuel and collapses. The very central region of the star – the core – collapses, making every proton & electron into a neutron. (collpase occurs as after iron, sudden stoppage of energy generation causes the core to collapse and the outer layers of the star to fall onto the core.)Gravity constantly works to try and cause the star to collapse. The star's core, however is very hot which creates pressure within the gas. This pressure counteracts the force of gravity, putting the star into what is called hydrostatic equilibrium.When the Sun runs out of hydrogen to fuse, the balance tips in the favor of gravity, and the star starts to collapse. But compacting a star causes it to heat up again and it is able fuse what little hydrogen remains in a shell wrapped around its core.This burning shell of hydrogen expands the outer layers of the star. When this happens, our Sun will become a red giant; it will be so big that Mercury will be completely swallowed. Most neutron stars are observed as pulsars. Pulsars are rotating neutron stars observed to have pulses of radiation at very regular intervals that typically range from milliseconds to seconds. A pulsar (from pulse and -ar as in "quasar")[1] is a highly magnetized rotating compact star (usually neutron stars but also white dwarfs) that emits beams of electromagnetic radiation out of its magnetic poles.

{ ⇉ ⬭ → ⬭ (Ellipsoid) → (Dumbbell) → (critical) → (nuclei) } → fission explained by weizsacker formula

SPUDS IF PUG DFSH

(a) Differentiate between K-capture and inverse β-decay. → used in detectⁿ of neutrino by cowan-reines
(b) Explain what is internal conversion and how it differs from β-decay. **(2005)**

16. Calculate the spin and parities of the ground states of $^{4}_{2}$He and $^{67}_{30}$Zn nuclei. **(2005)**
↳ defⁿ shell model → write spin & parities → success & limitatⁿ → diagram of shell structure)

17. Calculate the Q – value of the reaction : $^{9}_{4}Be\left(^{4}_{2}He, n\right)^{12}_{6}C$

Given

$Q = [(m_{Be} + m_{He}) - (m_C + m_n)]c^2$

Mass $\left(^{9}Be\right) = 9.012183u$

Mass $\left(^{4}He\right) = 4.002603u$

Mass $\left(^{12}C\right) = 12.000u$ **(2007)**

18. Write down the nucleonic configuration of $^{7}Li, ^{12}C, ^{17}O$ and ^{27}Al In the ground state of the nuclear shell model and hence calculate the corresponding ground state angular momenta and parties. How do the observed ground state angular momenta and parities agree with those predicted on the basis of the shell model? [Repeated in Q# 38] **(2007)**

19. (i)Write down the Bethe –Weizsacker semi- empirical mass formula for a nucleus. Explain the significance of each term occurring in it. Discuss the stability of a nucleus against β - decay. What is the effect of pairing term on stability? [M = αA + βZ + γZ² ± δ] $\beta = -(a_a A^{1/3} + a_a)$ & $\gamma = [a_c A^{-1/3} + 4a_a A^{-1}]$ $[Z_0 = -\beta/2\gamma]$ $(Z_0 = 40$ for 93$)$
(ii)What are mirror nuclei? How does the charge independence of nuclear force emerge from this concept? (done already) **(2007)**

inverse β: $p + \bar{\nu}_e \to n + e^+$; $p \to n + e^+ + \nu_e$ ↳ β⁻ decay; emission of beta particle; K-shell capture: $p + e^- \to n + \nu_e$

20. What is β - decay? Give three examples of β - decay of nuclei. Explain how neutrino hypothesis helped in explaining conservation of energy- momentum and angular momentum.What is Kurie plot? How can one use Kurie plot to set limits on mass of the neutrino? **(2008)**
$[n \to p + e^- + \bar{\nu}_e]$

$^{234}_{91}Pa$, $^{109}_{47}Ag$ $(^1F_{5/2} \to ^2D_{3/2})$ $(^1h_{11/2} \to ^3s_{1/2})$

21. What is nuclear isomerism? Give two examples and explain how this can be understood from single particle shell model. ↳ existence of isobaric, isotopic nuclei in diff. energy levels **(2008)**
for nuclei with odd A. (ΔE is small ⇒ long life ⇒ metastable) {ΔE ≠ 0 (forbidden) by odd nucleon} $(1g_{9/2} \to ^2P_{1/2})$
shell is completely filled ⇒ disappearance of isomers

22. Write down the Yukawa potential and derive an expression for the mass of π-meson in terms of range of nuclear force. What part of the Nucleon – Nucleon interaction is explained by this potential? **(2009)**
$V(r) = -\frac{g^2}{r}\exp\left(-\frac{r}{r_0}\right)$ → short range nuclear force → fails to explain spin-dependence, non-central & exchange nature

23. What is the simplest two-nucleon bound system? How does its study help in obtaining information about nuclear forces? Indicate clearly how the non-central forces can be explained by the observed magnetic moment of this system. **(2009)**
$\mu_{deuteron} \neq (\mu_{spin})_{neutron} + (\mu_{spin})_{proton}$ ↳ suggesting mixed state ⇒ $\Delta L \neq 0$ ⇒ non-central force
short range, spin dependence, spin=1 exist, not spin 0

24. Compare the methods of electron scattering and neutron scattering experiments to obtain information about nuclear size. **(2009)**
(K.E. = 200 MeV) more difficult; ↳ electrostatic charge distributⁿ for size of nuclei; ↳ nuclear force based cross section to calculate size; sample expensive

25. Calculate the packing fraction and the binding energy per nucleon for $^{16}_{8}O$ and $^{87}_{38}Sr$ nuclei. **(2010)**

defⁿ B.E. $P.F. = \left(\frac{M}{A} - 1\right)$ where M = mass of nuclei $\boxed{B.E. = [m_p Z + m_n(A-Z)] - M}$
(↓ P.F. ⇒ ↑ stability) ↳ B.E. gives stability of nuclei & mass-energy equivalence

p-p cycle & C-N cycle enable for fusion → star's energy source

26. A star converts all its hydrogen to helium, achieving 100% helium. It then converts the helium to carbon via the reaction ${}_{2}^{4}He + {}_{2}^{4}He + {}_{2}^{4}He \rightarrow {}_{6}^{12}C + 7 \cdot 27 MeV$ The mass of the star is $5 \cdot 0 \times 10^{32}$ kg and it generates energy at the rate of 5×10^{30} kg. How long will it take to convert all Helium to carbon at this rate? **(2010)**

12 amu → 7.27 MeV → [P = ... × 10^43 MeV/second]

t = E/P = ... × 10^43 second → takes years

27. What are magic numbers? Discuss shell structure of a nucleus. How is this model able to explain various properties of nuclei? Discuss the limitations of this model. **(2010)**

(already done) → conclude – Significance, Limitation

→ def^n

28. Predict from the single particle shell model the shell configuration, ground state spin and parity for the following nuclei: ${}_{7}^{13}N$; ${}_{8}^{17}O$ $(1S_{1/2})^2 (1P_{3/2})^4 (1P_{1/2})^1$ ⇒ spin = 5/2 ; Parity = −1 **(2010)**

29. Explain parity violation in β-decay. Describe how parity violation was experimentally detected in the decay of ${}^{60}Co$. (done already) **(2010)**

→ Conservation of energy

30. Find the total kinetic energy of electron and antielectron neutrino emitted in beta decay of free neutron. (The neutron-proton mass difference is 1.30 Mev and mass of electron is 0.51 Mev) **(2010)**

$n \rightarrow p + e^- + \bar{\nu}_e$ $(K.E.)_{e + \bar{\nu}_e} = (M_n - (m_p + m_e)) = 0.79$ MeV Ans

K depends on conc. of nuclei, geometry of nuclei, material of neutrons

(k = multiplication factor)

31. What are chain reactions? What do you mean by critical size of the core in which chain reaction takes place? What is critical mass? **(2010)**

chain reactions ⇒ p-p-cycle ⇒ K = ηεpf

(min. mass for fusion when K = 1) Application: Nuclear reactor, atom bomb, star

32. ^{235}U yields two fragments of A = 95 and A = 140. Obtain the energy distribution of the fission products. Assume that the two fragments are ejected with equal and opposite momentum. (done already) **(2010)**

→ shell model | space quantisation of nuclei | discrete energy level

33. Show that nucleus is a quantum system. **(2011)**

→ follows quantum statistics | nuclear spin, NMR

simplest nucleon model, explains nature of nuclear force

34. (a) (i) What is the importance of study of deuteron? Obtain the solution of Schrodinger equation for ground state of deuteron and show that deuteron is a loosely bound system.

(ii) What do you mean by non- central forces? (Tensor, ΔL ≠ 0)

(b) (i) What are chain reactions? What do you mean by critical size of the core in which chain reactions take place? What is critical mass? (series of self sustaining uncontrolled reaction)

(ii) $U\ 235\ yields$ fragments of A= 95 and A= 140. Obtain the energy distribution of the fission products. Assume that the two fragments are ejected with equal and opposite moments. **(2011)**

[$P_1 = P_2 \Rightarrow \frac{K_2}{K_1} = \frac{A_1}{A_2}$ K.E. ∝ rel. yield

35. (a) Write down the Bethe- Weizsacker Semi-empirical mass formula for a nucleus. Explain the significance of each term. Discuss the stability of a nucleus against $\beta - decay$. What is the effect of pairing term on stability? (done already)

(b) What are mirror nuclei? How does the charge independence of nuclear force emerges from this concept? $(\Delta B.E.) = \Delta E_c$ (liquid drop) ⇒ $\boxed{p - p \equiv n - n}$ → charge symmetry **(2012)**

→ nuclear force is charge independent

36. Calculate the recoil energy of ${}_{26}^{57}Fe$ nucleus when it emits a gamma photon of energy 14 KeV. **(2012)**

$P_{nuclei} = P_\gamma = \frac{E_\gamma}{c}$ ⇒ $E_{recoil} = \frac{E_\gamma^2}{2Mc^2} = 4.89$ meV Ans

37. Calculate the Q-value of the reaction: ${}_{4}^{9}Be\left({}_{2}^{4}He, n\right){}_{6}^{12}C$

Given:

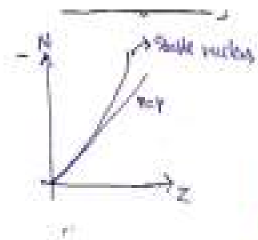
(done already)

Mass $\binom{9}{4}Be$ = 9.01283 u

Mass $\binom{4}{2}He$ = 4.002603 u

Mass $\binom{12}{6}C$ = 12.000 u

Mass (n) = 1.0086652 u



When nucleons ↑, mixing ↑. To keep it stable, one needs stronger binding force and there is also ↑ in coulombic repulsion. So, Neutrons ↑ to offset it. **013)**

38. Write down the nucleonic configuration of, ^{7}Li, ^{12}C, ^{17}O, and ^{27}Al in the ground state of the nuclear shell model and hence calculate the corresponding ground state angular momenta and parities. How do the observed ground state angular momenta and parities agree with those predicted on the basis of shell model? [If high spin comes after low spin shell, higher spin fills for pairing nucleons, before low spin shell can be filled completely.] **(2013)**

39. What is the role of neutrino in the weak interaction of radioactive nuclides? Explain the experimental detection of neutrino. → Cowan-Reines ↳ explained spin, energy problems, explains parity violation **(2013)**

40. Explain parity violation in β-decay. Describe how parity violation was experimentally detected in the decay of ^{60}Co. Mention any other decay process in which the parity violation has been demonstrated.

Wu's expt.

$^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \bar{\nu}_e$

$(1f_{7/2})^{-1} \rightarrow 11$, $(1f_{7/2})^{-2}$, $\boxed{P=1}$

↳ β^+ decay, K-shell capture

• sample is polarized in field and cooled using adiabatic demagnetisation to polarize nucleon spins along B.

41. Explain why stable light nuclei have equal number of protons and neutrons whereas heavy nuclei have excess of neutron. (given above) ↳ explains stable nuclei & predicts product of beta decays **(2014)**

→ Z/A = 20, 28, 50, 82, 126

42. What are the magic numbers in nuclei? List the experimental evidences indicating their existence.

↳ shell model assumes QM treatment of nuclei where unpaired nucleon assumed to move under averaged effect of (A-1) nucleons

↳ Isomers, Doubly magic nuclei, kinks in B.E curve, Q is small **(2014)**

43. It is possible to estimate the nuclear radius from the study of alpha decay? Explain how. **(2014)**

α-particle → 2p + 2n (bound by potential barrier of nuclear force.

↳ range of force can be calculated by tunneling probability helping calculate radius of nuclei

quantum tunnelling of α-particle

44. State the basic assumption of single particle shell model. How do the centrifugal and spin-orbit terms remove the degeneracy of a three-dimensional spherical harmonic oscillator? (previous page) **(2016)**

45. Predict the spin and parity of ground states of the following nuclei on the basis of shell model: (done as above)

(i) $_{8}O^{15}$

(ii) $_{8}O^{16}$

(iii) $_{17}Cl^{33}$ **(2016)**

46. Explain why the deuteron has no excited state. (done already) **(2016)**

47. Estimate the order of nuclear radius of lead (Z= 82) using the large angle (back) scattering of alpha particles of energy 10 MeV incident on a target (lead).

[Given : $(4\pi \in_o)^{-1} = 9 \times 10^9\ Nm^2C^{-2}$] **(2017)**

Assuming back scattering or 180°

$\frac{2Ze^2}{4\pi\epsilon_0 r_{min}} = (K.E)_\alpha$

Z=82 for lead

Conclusion → this gives approximate picture, accurate value can be solved using nuclear force scattering.

48. Distinguish between charge independence and charge symmetry of nuclear force. Give one example of each of these. (Solution in text box) **(2017)**

49. Describe briefly how parity violation in $\beta - decay$ was experimentally observed? What do you understand by the statement, 'neutrinos are left-handed'? **(2017)**

Helicity is pseudoscalar which changes during parity violat". → momentum & spin one in opposite direction

50. Given that the deuteron magnetic moment operator (in units of nuclear magneton) can be expressed as

Deuteron is triplet state $\Rightarrow \vec{\sigma}_n = \vec{\sigma}_p$

$$\vec{\mu}_d = \mu_n \vec{\sigma}_n + \mu_p \vec{\sigma}_p + \frac{1}{2}\vec{l},$$

Where $\vec{l}$ is the relative angular momentum between neutron and proton $\vec{\sigma}_n$ and $\vec{\sigma}_p$ are the Pauli spin operators and μ_n and μ_p are the respective magnetic moments. Find out the D-state probability of deuteron wave function. (done already)

[Given : $\mu_d = 0.857\ \mu_N, \mu_n = -1.913\ \mu_N\ \ and\ \mu_p = 2.793\ \mu_N;\ \mu_N(nuclear\ magneton)$] **(2017)**

$\boxed{M(Z,A) = (m_p Z + (A-Z)m_n) - (B.E)}$

$a_a \to$ excess nucleons

51. Write the semi-empirical mass formula pointing out the role of volum coulomb and symmetry energy correction terms.

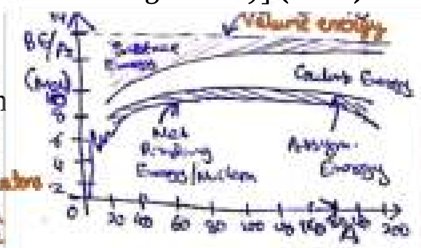
$\boxed{a_c = \frac{3e^2}{20\pi\epsilon_0 R_0}}$

$a_v \to$ attractive interaction

$B.E. = a_v A - a_s A^{2/3} - a_c Z^{-1/3} Z(Z-1) - a_a A^{-1}(A-2Z)^2 - a_p A^{-3/4}$

($a_v = 38.5$ MeV, $a_c = 14$ MeV, $a_s = 13$ MeV, $a_a = 0.6$ MeV, $a_p = 15$ MeV)

$a_s \to$ less neighbours → less stable
$a_c \to$ repulsion b/w positive
$a_p \to$ odd-odd unpaired nucleons → less stable



52. Draw a schematic diagram of the single particle energy levels in a shell model including the effect of spin-orbit coupling. Show how it explains magic numbers in nuclei. Give two examples to show how this scheme predicts the spins and particles of odd A nuclei. **(2017)**

determines order of I.

→ $_8O^{17} \Rightarrow$ spin $= \frac{5}{2}$, Parity $= +1$ & $_7N^{15} \Rightarrow$ Spin $= \frac{1}{2}$ & Parity $= -1$ done

53. Write the semi-empirical mass formula for nuclei and on its basis draw mass parabolas for odd and even isobars. What would be the most stable isobar in each case? (done already) **(2018)**

$[(g_l)_n = 0$ & $(g_s)_n = -3.82]$

Shell model diagram → sketch case

54. Obtain an expression for the magnetic moment of a nucleus having one nucleon outside the closed core. Use this to calculate the magnetic moment of $_8O^{17}$ nucleus. **(2018)**

$\boxed{\langle \mu_z \rangle = g_j \mu_N j}$

→ $\mu_j = \frac{(\vec{\mu} \cdot \vec{J})\vec{J}}{|J|^2}$ ⇒ $\vec{\mu}$ due to neutron $[(1S_{1/2})^2 (1P_{3/2})^4 (1P_{1/2})^2 (1D_{5/2})^1] \Rightarrow [l = 2;\ j = \frac{5}{2};\ s = \frac{1}{2}]$

55. Nuclear forces are mediated by exchange of π-mesons of rest mass 140 MeV. Estimate the range of nuclear forces. **(2018)**

nuclear forces due to exchange of virtual mesons.

$R = \frac{C}{2}\Delta t$ & $\Delta E \sim m_\pi c^2$ & $\Delta E \Delta t \sim \hbar$

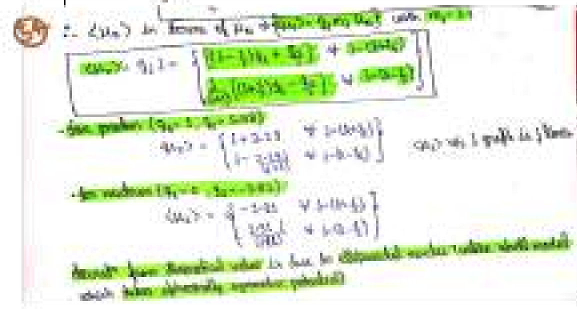
→ $\boxed{R \sim \frac{\hbar}{2 m_\pi c}}$ Ans:

→ $R \sim \frac{1}{4\pi} \times \frac{1240 \text{ eV-nm}}{140 \text{ MeV}}$

$\boxed{R \sim \text{fm}}$ Ans

56. Assuming that the neutron-proton interaction has a square well form

$$V(r) = -V_0\ for\ r \leq b$$
$$= 0\ for\ r > b$$



deuteron nucleus is given as

$$\psi(r) = A sin\, kr\ \ for\ r \leq b$$

Magnetic moment of Deutron

$\left[\vec{\sigma}_n, \vec{\sigma}_p = \text{Pauli spin operators}\right]$

$-\ \boxed{\vec{\mu}_d = \mu_n \sigma_n + \mu_p \vec{\sigma}_p + \vec{L}_p}$ (in terms of $\vec{\mu}_N$) $\left\{ l_p = \frac{L}{2} \text{ in c.o.m. frame} \right\}$

In COM frame

$$\left[\vec{\mu}_d = \left(\frac{\mu_n + \mu_p}{2}\right)(\vec{\sigma}_n + \vec{\sigma}_p) + \left(\frac{\mu_n - \mu_p}{2}\right)(\vec{\sigma}_n - \vec{\sigma}_p) + \frac{\vec{L}}{2} \right]$$ (for triplet state $\vec{\sigma}_n = \vec{\sigma}_p$)

$$\boxed{\therefore \vec{\mu}_d = (\mu_n + \mu_p)\vec{S} + \frac{\vec{L}}{2}} = (\mu_n + \mu_p)\vec{I} - (\mu_n + \mu_p)\vec{L} + \frac{\vec{L}}{2} \quad (\because \vec{L} = \vec{J} + \vec{S})$$

If I is in z-direc^n

$$\langle \mu_d \rangle = (\mu_n + \mu_p)\langle I_z \rangle - (\mu_n + \mu_p - \tfrac{1}{2})\langle L_z \rangle \quad \text{— (i)}$$

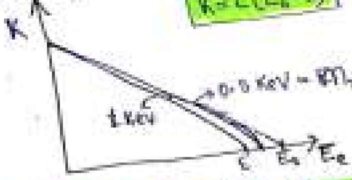
$$\left\{ \boxed{\langle L_z \rangle = \left\langle \frac{(\vec{L}\cdot\vec{J}) J_z}{J^2} \right\rangle} \rightarrow \boxed{\langle L_z \rangle = \frac{L(L+1) + I(I+1) - S(S+1)}{2I(I+1)} \langle I_z \rangle} \right\}$$

$\therefore I = 1, S = I$; $\boxed{\langle L_z \rangle = \frac{\langle L(L+1) \rangle}{4} I_z}$ $(I_z = 1)$ $\left\{ \langle L_z \rangle_S = |C_S|^2 \times 0 = 0 \right\}$

$\left\{ \boxed{|\psi\rangle = C_S |S\rangle + C_D |D\rangle}, L = 2 \text{ for } D \right\} \Rightarrow \boxed{\langle L_z \rangle = \frac{6|C_D|^2}{4} = \frac{3}{2} P_D}$ (as $P_D = |C_D|^2$)

Putting it in (i) gives, $\boxed{\mu_d = (\mu_n + \mu_p) - (\mu_n + \mu_p - \tfrac{1}{2})\frac{3}{2} P_D} \Rightarrow P_D \approx 0.04$

Kurie Plots



$\boxed{K = C(E_0 - E)}$ $\quad K = \sqrt{\frac{P(E_0)}{F(Z,E_0) P_e^2}}$ where $P(E_0)$ is transit^n probability

& $F(Z,E_0)$ is fermi function

& P_e is momenta of e^-

$\{ F(Z,E_0) = 1$ for $Z = 0 \}$

$\therefore E_0 = E_e + E_\nu + m_\nu c^2$ & $P_\nu = \frac{1}{c}[E_\nu(E_\nu + 2m_\nu c^2)]^{1/2}$

$\hookrightarrow \boxed{K = C\{(E_0 - E_e)[(E_0 - E_e)^2 - (m_\nu c^2)^2]^{1/2}\}^{1/2}}$ → for neutrino mass; (above curve $m_\nu = 0$)

m_ν can be calculated from deviation (due to neutrino mass) from straight line near $E_e = E_0$. ⇒ deviat^n shift gives upper limit on m_ν.

$Q_{max} = E_{e^-} + E_{\bar{\nu}_e}$

$\hookrightarrow E_0 \ [p \rightarrow n + e^- + \bar{\nu}_e]$

→ Deutron magnetic moment in $^3S_1$ state

$$\left[\vec{\mu}_J = \left[(g_l \vec{l} + g_s \vec{S}) \cdot \vec{J} \right] \vec{J} \frac{\mu_N}{|J|^2} \right]$$

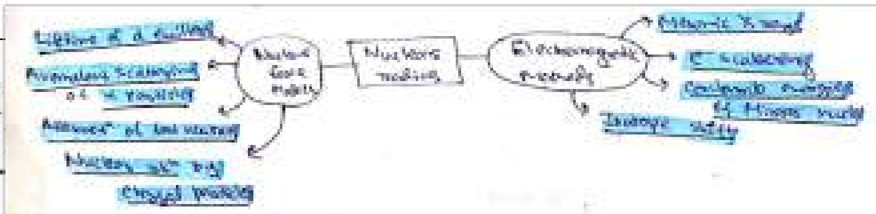
for $l = 0$; $\vec{\mu} = \vec{\mu}_p + \vec{\mu}_{neutron}$

$$\boxed{\vec{\mu}_J = 0.8798\ \mu_N}$$

$$\boxed{\vec{\mu}_{d_{tot}} = P_S \times \vec{\mu}_S + P_D \times \vec{\mu}_D}$$

$$\boxed{\vec{\mu}_{spin} = \left(\frac{g_n + g_p}{2}\right)\vec{S}}$$

in com frame, $$\boxed{\frac{\vec{\mu}}{\mu_N} = \left[\frac{\frac{\vec{L}}{2} \cdot \vec{J} + \frac{1}{2}(g_p + g_n)\vec{S} \cdot \vec{J}}{J(J+1)} \right] \vec{J}}$$



$$= Ce^{-\gamma r} for\ r > b$$

Where k = $\sqrt{\frac{M}{\hbar^2}(V_0 + W)}$ and $\gamma = \sqrt{\frac{MW}{\hbar^2}}$

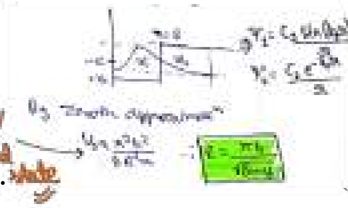
Here M is the nucleon mass, W is the binding energy of deuteron and A and C are constants.

(i) Show that for a just bound state of deuteron

$$V_0 b^2 = \frac{\pi^2 \hbar^2}{4M}$$

[Schrödinger → boundary condition → Calculatⁿ of R]

for excited state, $[x > 3\pi/2] \rightarrow V_0 R^2 \geq \frac{9\pi^2\hbar^2}{8M} \Rightarrow V_0 \sim 276$ MeV ↳ no excited state



(ii) Explain why deuteron is a loosely bound extended structure. (2018)

57. Explain the various methods of finding the size of the nucleus. How will you determine the nuclear radius from the observation of beta rays resulting from nuclear transition when the initial and final nuclei are mirror nuclei? (2019)

$$\Delta E_c = \Delta BE = \frac{a_3'(2Z \pm 1)}{A^{1/3}} = \frac{3e^2}{20\pi\epsilon_0}\frac{(2Z \pm 1)}{R_0 A^{1/3}}$$

$$\Delta B.E. = (\Delta Mass - (m_p - m_n))$$

58. Given that the single particle energy separation between 1 $d_{5/2}$ and 1 $d_{3/2}$ in ^{17}O is $5MeV$. Calculate the strength of spin-orbit interaction. It is observed that 1 $d_{5/2}$ level is lower than 1 $d_{3/2}$ level. (2019)

$[(\Delta E)_{5/2} - (\Delta E)_{3/2}]$

$\alpha = \frac{2}{\hbar^2}$ MeV

spin-orbit interactⁿ defⁿ → $(\Delta E)_{LS} = -\alpha(\vec{L}\cdot\vec{S})$ $\left\{\vec{L}\cdot\vec{S} = \left(\frac{J^2 - L^2 - S^2}{2}\right)\right\}$ | α → strength characteristics of atom.

59. Why is it not possible to detect the parity violation in weak interaction by observing only the beta decay rate? Justify your answer. (2019)

↳ because to detect parity violation, one needs to show direction in which β-decay is more i.e. change in helicity.

↳ this explains existence of neutrino.

[draw diagram mentioned in Q→40]

Q→48

nuclear force has "charge symmetry," which means that we can make an overall switch between protons and neutrons without changing forces among them. For instance, nn & pp scattering are the same (except for the obvious difference due to the electric charge). For example, "mirror nuclei," which are related by switching protons andneutrons, have very similar excitation spectra. Examples include 13C and 13N, 17O and 17F, etc.

A stronger version of the charge symmetry is "charge independence." Not only nn and pp scattering are the same, but also np scattering is also the same under the "same configuration", specified using the concept of isospin.

TUTORIAL SHEET: 7
PARTICLE PHYSICS

1. Illustrate with any five examples that production of strange particles conserves strangeness, parity and isotopic spin while their decay violates all these quantities. **(2001)**

2. Discuss the significance of Gellmann-Nishijima relation in strange particle production and decay. Give-examples in support of your answer. Obtain quark quantum numbers using the modified relation for quarks. (Page 57 of Notes) **(2001)**

3. Which conservation laws are obeyed in nuclear reactions? Explain the significance of Q value of a nuclear reaction. **(2002)**

Dirac experiment

4. Distinguish between particle and antiparticle. How antiparticle were discovered? **(2002)**

same mass, spin & Isospin

5. Mention the conservation laws for strong, electromagnetic and weak interactions. **(2002)**

6. Give the characteristic properties of strange particles which distinguish them from non strange ones. Write the Gellmann – Nishijima relation and show how it is used for the classification of elementary particles. **(2003)**

7. State the conservation law which is violated in each of the following processes:
   (i) $n = p + \bar{e} + r$ (Le, spin violated) (ii) $\pi^- + p = k^+ + k^-$ (B, spin not conserved)
   (iii) $\pi^- + p = \Sigma^+ + k^-$ (iv) $\Sigma^+ + k = p + p$ **(2003)**
   (Iz, spin not conserved) (B, spin, Iz non-conserved)

8. Which of the following reactions are allowed and forbidden under the conservation of strangeness, baryon number and change conservation?
   (i) $\pi^+ + n = \lambda^o + k^+$ (ii) $\pi^+ + n = k^o + k^+$
   (iii) $\pi + p = \lambda^o + k^+$ (iv) $\pi + p = \pi^o + \lambda^o$ **(2004)**

$(\frac{1}{2}, -\frac{1}{2}, 0)$ $(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$ [respectively]

9. State the quantum numbers I_z, Y and S for the u d and s quarks and antiquarks. What combination of these leads to the formation of (i) proton and (ii) neutron? **(2005)**
   $(0, 0, -1)$
   uud udd

10. What are the basic interactions in nature? Give one example for each. Compare their relative strengths and ranges. EM (10^{-2}) | Strong (1) Grav. (10^{-39}) | weak (10^{-13}) **(2005)**

pairs || strong || decay by weak force || need the p, n, π produces

11. Give some characteristic properties of strange particles which distinguish them from non – strange ones. Write the GellMann-Nishijima relation. How is it used for the classification of elementary particles? **(2007)**

$Q = I_z + \frac{Y}{2} = I_z + \left(\frac{B+S}{2}\right) \Rightarrow S = 2(Q - I_z) - B$ ⇒ strange & non-strange
$\Omega^-, \pi^+, \pi^-, K^+, K^0$

12. Explain lepton number conservation and why it is necessary to distinguish between different types of neutrinos? **(2008)**
   Generation of leptons with different masses ($\Sigma L_e = \Sigma L_\mu = \Sigma L_\tau = 0$) to explain why $\pi^- \rightarrow \mu^- + \bar{\nu}_e$ don't exist

13. Give the hypercharge and iso spin of the quarks and antiquarks. What are the quark contents of themesons: $\pi^+, \pi^0, \pi^-, K^+, K^0, K^-$ and η? **(2008)**
   diff. gen.
   $u\bar{d}$ $u\bar{u}/d\bar{d}$ $\bar{u}d$ $u\bar{s}$ $\bar{u}s$ $d\bar{s}$ $u\bar{u}/d\bar{d}$

log E = 2×10^14 GeV

14. At what energies do we anticipate the unification of strong and electroweak interactions? What will happen to coupling constants of these interactions in this situation? How does unification indicate the decay of proton and existence of lepto quark? **(2009)**

15. What are the field quanta of weak and electromagnetic interactions? Show that weak interaction is a short range force while electromagnetic interaction is a long range one. **(2009)**

16. State the quantum number I_z, Y and S for the u d s quarks and anti quarks. Which combination of these leads to the formation of (i) proton and (ii) neutron? [Q#9] **(2010)**

17. Expain conservation of Baryon Number. Comment on the stability of proton. **(2010)**

[$p \rightarrow n + e^+ + \nu_e$] ⇒ possible at electroweak coupling.

18. Which of the following reactions are permitted or forbidden by various conservation laws?

(i) $K^- + P \rightarrow \lambda^0 + \pi^0$ → Strong

(ii) $K^- + n \rightarrow \Sigma^+ + \pi^0$

(iii) $\pi^- + P \rightarrow K^- + \Sigma^+$

(iv) $\pi^+ + n \rightarrow K^- + n^0$

(v) $\pi^- + P \rightarrow \lambda^0 + K^0$

(vi) $\pi^- + P \rightarrow K^- + \Sigma^-$ weak **(2012)**

19. State the quantum I_z, Y and S for the u d s quarks and antiquarks. Which combination of these leads to the formation of (i) proton and (ii) neutron? **(2013)**

20. In the following reaction indicate with an explanation, whether they proceed by strong, electromagnetic or weak interaction or they are forbidden:

(i) $K^- \rightarrow \mu^+ + \nu_\mu$ → Isospin = 0 (weak)

(ii) $p \rightarrow n + e^+ + \nu_e$ → proton is lightest baryon (I_z is not conserved)

(iii) $p + \pi^- \rightarrow K^+ + \Sigma^-$ **(2013)**

↳ (I, B, S) → conserved ⇒ strong interaction.

21. (i) What are salient features of nuclear forces?

(ii) Discuss Yukawa's theory of nuclear forces. **(2014)**

$V = -\frac{g^2}{r} \exp\left(-\frac{mcr}{\hbar}\right)$

↳ due to exchange of mesons

22. (i) How does liquid drop model explain fission?

(ii) What are the limitations of shell model? **(2014)**

ellipsoid (Dumbbell) (fission)

23. Describe grand unification theories (GUT). **(2015)**

24. How many types of neutrinos exist? How do they differ in their masses? **(2015)**

ν_τ, ν_μ, ν_e

25. (i) $\Omega^- \rightarrow \wedge^0 + K^-$ (weak)

(ii) $\wedge^0 \rightarrow p + \pi^-$ (weak)

(iii) $K^- \rightarrow \mu^- + \upsilon_\mu$ (weak) ⇒ Baryon is producing Leptons **(2015)**

26. State the quantum numbers I_z, Y and S for uds quarks and antiquarks. Which combination of these leads to the formation of a (i) proton and (ii) neutron? **(2015)**

based on mass, spin, strangeness, interaction, symmetry

27. What are elementary particles and how are they classified? Describe in brief the different types of interactions that can occur between the elementary particles. **(2016)**

→ internal structure can't be written in form of other particle and basis of classification

→ Strong / EM / weak / Gravitational

28. Explain the various leptonic family members. What is leptonic number conservation? Based on this conservation law, state whether the following reaction are possible or not:

weak | $e^-, \mu, \nu_\mu, \nu_e, \tau^-, \mu^-$ | spin $= 1/2$ | Heavy | $S=0$ $\qquad \Sigma L_e = \Sigma L_\mu = \Sigma L_\tau = 0$

(i) $\pi^- \to \mu^- + \bar{\nu}_\mu$ $\Rightarrow L=0 \Rightarrow L_\mu = 1,\ L_{\bar{\nu}_\tau} = -1$ (not conserved)

(ii) $n \to P^+ + e^- + V_e$ $\Rightarrow L=0 \Rightarrow L_e = 1 + L_{\bar{\nu}_e} = -1$ (conserved) **(2016)**

29. Write down the quark structure of the following hadrons: $\Delta^{++}, \Omega^-, \Sigma^-$ and Λ^0

→ $u_R u_B u_G$ → $s_R s_B s_G$ → dds → uds

Write down the following decays in terms of quarks:

i. $\underset{udd}{n} \to \underset{uud}{P^+} + e^- + V_e$ → quarks are not conserved ⇒ weak decay

ii. $\underset{uud}{\Delta^+} \to \underset{u\bar{d}}{\pi^+} + \underset{udd}{n}$

iii. $\underset{uus}{\Sigma^+} \to \underset{uud}{P^+} + \underset{u\bar{u}/d\bar{d}}{\pi^0}$ ⇒ weak decay **(2016)**

(uud) (udd) π^0

30. Explain unification of electromagnetic and weak interactions. What is Z^o-boson? What is its relevance in electroweak unification? **(2016)**

→ Hadrons → leptons → all charged particle

31. State the three characteristic properties of strong, weak and electromagnetic forces distinguishing one from the other. **(2017)**

($R \approx 10^{-15}$ m) $\tau \approx 10^{-23}$ ($R \approx 10^{-18}$ m) $\tau \approx$ years ($R \approx \infty$) $\tau \approx 10^{-16}$

32. Point out the interactions in which the following conservation laws are obeyed or violated

(a) Isotopic spin (strong) & I_z: strong/EM

(b) Hyper charge strong/EM ($Y = B+S$)

(c) Lepton number (strong, weak, EM)

(d) Charge conjugation (strong/EM) **(2017)**

→ particles to antiparticles

33. Write down the quark constituents of each of the following:

(a) π^+, (b) K^+, (c) Δ^{++}, (d) Σ°, (e) Ω^- **(2017)**

$u\bar{d}$ $\quad u\bar{s}$ $\quad u_R u_B u_G$ $\quad uds$ $\quad s_R s_B s_G$

34. Which of the following elementary particle reactions/decays are allowed under various conservation laws? If allowed , write down the type of interaction and the characteristic time by which it would proceed:

(i) $p + n \to \Lambda^0 + \Sigma^+$

(ii) $\pi^+ + n \to \Lambda^0 + K^+$

(iii) $p + n \to K^+ + \Sigma^+$

(iv) $\pi^0 \to \gamma + \gamma$

(v) $\bar{n} \to \bar{p} + e^+ + v_e$ **(2018)**

TUTORIAL SHEET: 8
Crystal Structure & its Determination

$\left\{\vec{a}=\frac{a}{2}[-\hat{i}+\hat{j}+\hat{k}],\ \vec{b}=\frac{a}{2}[\hat{i}-\hat{j}+\hat{k}],\ \vec{c}=\frac{a}{2}[\hat{i}+\hat{j}-\hat{k}]\right\}\to$ BCC

1. Obtain reciprocal lattice to normal BCC lattice. **(2001)**
2. Determine the reciprocal lattice vectors of an fcc lattice. **(2005)**
3. Show that the reciprocal lattice of a hexagonal lattice is a hexagonal lattice with a rotation of axes **(2009)**

   $\vec{a}=\frac{a}{2}(\sqrt{3}\hat{i}+\hat{j})$; $\vec{b}=\frac{a}{2}(-\sqrt{3}\hat{i}+\hat{j})$, $\vec{c}=c\hat{z}$

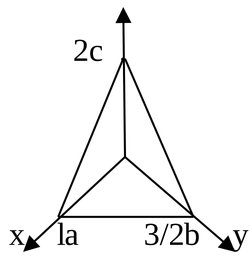
4. Derive Bragg diffraction condition in vectorial form using incident and diffracted wave vectors and reciprocal lattice vector. **(2010)**

   $|\vec{G}|=|\vec{k}'-\vec{k}|=2k\sin\theta \Rightarrow \boxed{2d\sin\theta=n\lambda}$

5. An electron beam of 4 Kev is diffracted through a Bragg angle of 16° for the first maxima. If the energy is increased to 16 keV, find the corresponding Bragg angle for diffraction. **(2010)**

   $\left\{\lambda=\frac{h}{\sqrt{2mE}}\ ,\ 2d\sin\theta=\lambda\right\}$

6.



A crystal plane is shown in the above figure. Find its Miller indices and interplanar spacing. **(2013)**

$\left(\frac{1}{1},\frac{1}{(3/2)},\frac{1}{2}\right)\equiv(6,4,3)$

7. Explain the working of SEM and TEM highlight the major differences in principles. Draw neat schematic diagrams. **(2014)**
8. What is the reciprocal lattice and why is it named so? Derive the relationships for the primitive translation vectors of the reciprocal lattice in terms of those of the direct lattice. **(2014)**

   $\vec{a}_i\cdot\vec{a}_j=2\pi\delta_{ij} \Leftarrow \psi(\vec{r})=\psi(\vec{r}+\vec{T})\Rightarrow\boxed{\exp(i\vec{G}\cdot\vec{T})=1}$

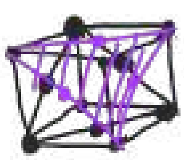
9. How does he energy gap in superconductors differ from the energy gap in insulator? How does it vary with temperature for superconductors? **(2014)**

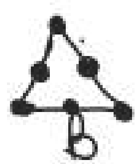
   formed due to cooper pair — due to immobility of e−

10. Obtain Laue's equations for X-ray diffraction by crystals. Show that these are consistent with the Bragg's law. **(2016)**

    Laue's equation: $[\vec{a}\cdot\vec{N}=a\cos\alpha=2a\sin\theta\cos\alpha]$; $\Delta=2n\lambda$; $\vec{N}=(\hat{n}_1-\hat{n}_2)$, $|\vec{N}|=2\sin\theta$; $\boxed{\Delta=\vec{r}\cdot\vec{N}}$; $(\vec{a}\cdot\vec{N}=2a\sin\theta\cos\alpha)$

11. The primitive translation vectors of a two-dimensional lattice are a = 2 i + j , b = 2 j. Determine the primitive translation vectors of its reciprocal lattice. **(2016)**
12. Deduce the Miller indices of the close-packed planes of atoms in the f.c.c. lattice. **(2019)**


(1,1,1)

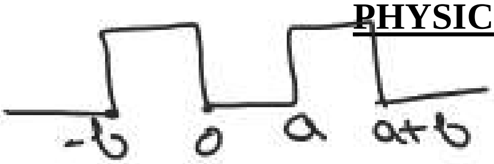

(1,1,1)

$\{2a\cos\alpha:2b\cos\beta:2c\cos\gamma \equiv h:k:l\}$

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ → direction cosines of N is proportional to $\left\{\frac{a}{h},\frac{b}{k},\frac{c}{l}\right\}$

$\Leftarrow$ $\vec{N}$ is ⊥ to plane $[h,k,l]$ $\Rightarrow d=\frac{a}{h}\cos\alpha=\frac{b}{k}\cos\beta=\frac{c}{l}\cos\gamma$

$\boxed{2d\sin\theta=n\lambda}$



PHYSICS PAPER – II : SOLID STATE PHYSICS
TUTORIAL SHEET: 9
Band theory of solids

1. Show that the condition of periodicity of lattice leads to the Bloch's theorem. What is a Bloch function? **(2004)**

$|\psi(x+a)|^2 = |\psi(x)|^2$ & under periodic potential, $V(x+a)=V(x)$ ⇒ $\boxed{\psi(x) = u_k(x)\, e^{ikx}}$ Bloch function

$u_k(x) = u_k(x+a)$

2. By drawing E vs. $\vec{k}$ curve, distinguish between metal, insulator and semiconductor. **(2004)**

3. (i) Discuss briefly the concept of effective mass in semiconductors and explain the significance of negative effective mass. analogous to concept of $E = \frac{\hbar^2 k^2}{2m_e}$ ⇒ $\vec{a}\, m^* = -e\vec{E}$ & $v_g = \frac{dE}{\hbar dk}$

(ii) In isolated atoms, the electrons have discrete and definite energies but in solids they have bands of energies. Explain why. (Kroneig-penny model → Constant Potential) **(2005)**

4. Discuss the motion of an electron in one-dimensional periodic potential and show that it leads to formation of bands of all owed and forbidden states in the electron energy spectrum. How are the Insulators, semiconductors and conductors discriminated on the basis of band structure? **(2005)**

5. Explain the reduced zone scheme of plotting the energy bands in solids. On the basis of E vs k curve distinguish between conductors, semiconductors and insulators. **(2010)**

6. What is nearly free electron approximation? On the basis of this approximation explain the formation of energy bands in solids. **(2010)**

$-1 \le \left(\frac{P\sin(\alpha a)}{(\alpha a)} + \cos(\alpha a)\right) \le 1$ & $\left(\alpha = \sqrt{\frac{2mE}{\hbar^2}}\right)$

$P = \frac{mV_0 ab}{\hbar^2} = \beta^2 ab$

7. Starting with the expression for the density of states for electrons in a band, show that the Fermi energy of an intrinsic semiconductor is at the middle of the band gap. Use these results to estimate the electron density at 300K (Assuming $E_g = 1$ eV and the rest masses of electron and hole as m_e and m_h). **(2013)**

8. Describe the motion of an electron in one dimensional periodic potential and show that it leads to formation of bands of allowed and forbidden states in the electron energy spectrum. How are the conductors, semiconductors and insulators discriminated on the basis of band structure? **(2015)**

9. What is the difference between direct and indirect band gap semiconductors? Which one is suitable for use in solar cells? **(2016)**

10. The Energy (E) and wave vector (k) for a conduction band electron in a semiconductor are related as $E = \alpha \frac{\hbar^2 k^2}{m_0}$ where α is a constant and m_0 is the free electron mass. Calculate the effective mass of the electron. **(2017)**

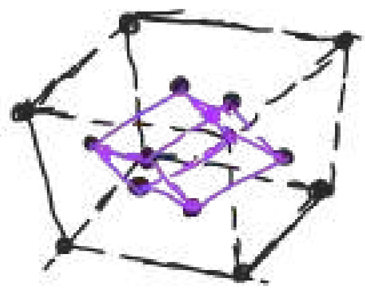
$m^* = \frac{\hbar^2}{\left(\frac{\partial^2 E}{\partial k^2}\right)}$

11. Consider a face-centred cubic lattice of side a. Deduce –

(i) The primitive translation vectors; →

(ii) The volume of the primitive cell;
(iii) The reciprocal primitive translation vectors;
(iv) The volume of the reciprocal lattice **(2019)**

→ Rhombic dodecahedron



$\vec{a} = \frac{a}{2}(\hat{j}+\hat{k})$ {FCC}

$\vec{b} = \frac{a}{2}(\hat{k}+\hat{i})$

$\vec{c} = \frac{a}{2}(\hat{i}+\hat{j})$

TUTORIAL SHEET: 10
Thermal and Magnetic Properties of Solids

1. Distinguish between Einstein and Debye models of specific heats of solids. Obtain an expression for the specific heat of a solid on the basis of Debye model. Discuss the results for low and high temperature ranges. **(2010)**

2. Distinguish between classical and quantum theory of paramagnetism. Explain the paramagnetism of free electrons on the basis of F.D. distribution. **(2010)**

3. Find an expression for lattice specific heat of solids, and its low and high temperature limits. What is Debye temperature? **(2010)**

4. Find an expression for lattice specific heat of solids and its low and high temperature limits. What is Debye's temperature? **(2011)**

5. The velocity of sound in f.c.c. gold and f.c.c. copper is 2100m/s and 3800 m/s respectively. If the Debye temperature of copper is 348 K, then determine the Debye temperature of gold. Take the densities of gold and copper as 1.93 x 10^4kg/m^{3}and 0.89 x 10^4 kg/m^3 respectively. **(2014)**

6. Find an expression for lattice specific heat of a solid, and its low and high temperature limits. What is Debye temperature? **(2015)**

7. Write down the salient features of the Einstein's theory of lattice heat capacity. Further write down the expression for specific heat in Einstein's theory and explain its high and low temperature limits. **(2016)**

8. In a semiconductor, the effective masses of an electron and hole are 0.07 m_0 and 0.4 m_0 respectively, where m_0 is the free electron mass. Assuming that the average relaxation time for the hole is half of that for the electrons, calculate the mobility of the holes when the mobility of the electrons is 0.8 $m^2 volt^{-1} s^{-1}$. $\sigma = \frac{ne^2\tau}{m^*} = ne\mu \Rightarrow \boxed{\mu = \frac{e\tau}{m^*}}$ **(2017)**

9. Derive an expression for lattice specific heat in Debye model. Find its low temperature limit (Debye T^3 law). **(2018)**

(5) $\left[\nu_D^3 = \frac{9N}{4\pi V}\left(\frac{2}{v_t^3} + \frac{1}{v_l^3}\right)^{-1}\right]$; for fcc ⇒ $v_l = v_t$

{ $M_{gold} = 197$ gram & $M_{copper} = 63.55$ gram $\left(\frac{\nu_{D_1}}{\nu_{D_2}}\right)^3 = \left[\left(\frac{\theta_{D_1}}{\theta_{D_2}}\right)^3 = \frac{V_2 v_1^3}{V_1 v_2^3}\right]$

$\rho_{gold} = \frac{M_{gold(s)}}{V_1}$

$\Rightarrow \boxed{\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} \times \frac{M_2}{M_1}}$

$\rho_{copper} = \frac{M_{Cu(s)}}{V_2}$

SOLID STATE PHYSICS
TUTORIAL SHEET: 11
SUPERCONDUCTIVITY

1. Describe Josephson junction and the effect. How is it exploited in the formation of a SQUID? Write down the applications of a SQUID?

→ e^- – phonon – e^- interaction

→ magnetic levitation, Hyperloop, better superconductor.

2. Give a qualitative account of BCS theory of superconductivity. **(2001)**

3. Explain Meissner effect and Josepheson effect. Describe their applications in the field of superconductivity. **(2002)**

H_2S (2015) → T=203K (under high P)
→ Tc is quite high
→ YBCO, TBCO (T_c = 125K)

4. What are high temperature superconductors? Give two examples indicating their transition temperature. Differentiate between the conventional and high temperature superconductivi ty. **(2003)**

T_c ; High T → hole states
Low T → cooper pairs
High T → commercially useful
↳ can't be explained by BCS theory

5. What is a Josephson junction? Discuss DC and AC Josephson effects and obtain the relation $\upsilon = 2eV/h$, where symbols have their usual meanings. **(2003)**

6. What is Josephson Effect? Discuss briefly DC and AC effects. Give some practical applications of Josephson junctions. **(2007)**

measurement of h/e ← → SQUID
$\omega = \frac{2eV}{\hbar}$ → high ω for V_0 = 1 μvolt → Sensitive magnetometer

7. Distinguish between 'soft' and 'hard' superconductors. Explain how penetration depth varies with magnetic field strength and temperature. $\left[\frac{1}{\lambda_d^2} = \frac{1}{\lambda_0^2}\left(1-\left(\frac{T}{T_c}\right)^4\right)\right]$ & $\left[H_c(T) = H_c(0)\left(1-\left(\frac{T}{T_c}\right)^2\right)\right]$ **(2008)**

8. A type-II superconducting material with superconducting transition temperature T_c is placed in a magnetic field. What is the material state (normal or superconducting or mixed) for

(i) $T < T_c; H > H_{C_2}$ (Normal) (ii) $T > T_c; H < H_{C_1}$ (Meissner)
(iii) $T < T_c; H_{C_1} < H < H_{C_2}$ (Mixed) (v) $T > T_c; H > H_{C_2}$ (Normal) **(2009)**

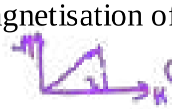
9. Describe the characteristic properties of a superconductor. Derive London equation for a superconductor and hence explain Meissner effect. **(2010)**

10. What is Josephson tunneling? Distinguish between a.c. and d.c. Josephson effects. **(2010)**

constant V_0 ← Quantum tunneling

11. Describe the Characteristics properties of a superconductor. Derive London equation for a super conductor and hence explain Meissner effect. → diamagnetic, zero resistance, meissner effect, cooper pairs, SQUID (quantisation of flux) **(2011)**

12. Explain the variation of Magnetisation of Type I and Type II superconductors as a function of applied magnetic field **(2012)**


(sudden loss)

(gradual loss of M)

13. Cooled to 4 K and then magnetic field is applied. With schematic diagrams, explain path of magnetic field lines in all these situations: **(2013)**


(for T<Tc)

14. Lead in the superconducting state has critical temperature of 6.2 K at zero magnetic field and a critical field of 0.064 $\mathrm{Mam^{-1}}$ at 0 K. Determine the critical field at 4 K. **(2014)**

$T_c = 6.2K$ $H_c = H_c(0)\left(1-\left(\frac{T}{T_c}\right)^2\right)$
$= 0.064 \times \left(1-\left(\frac{4}{6.2}\right)^2\right) = 0.037\ \mathrm{MAm^{-1}}$
Ans

→ cooper pair, meissner effect → e^-, frozen field, entropy, heat capacity

15. Distinguish between a superconductor and perfect conductor. Explain what is a Cooper pair. **(2015)**

e^- - phonon - e^- pair

16. With the help of a schematic diagram, show how entropy and specific heat vary with temperature for a superconductor. $\left\{\vec{\nabla}\times\vec{J} = -\frac{ne^2}{m}\vec{B}\right\}$ → $\left[\frac{\partial \vec{J}}{\partial t} = \frac{\vec{E}}{\mu_0\lambda^2}\right]$ & $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$, $\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = -\nabla^2\vec{B}$ **(2016)**

17. Obtain the expression for penetration depth using London's equation of superconductivity and explain its significance. **(2017)**

→ very small for soft superconductor

→ characteristic λ_L is distance upto which $\vec{B}$ can penetrate → meissner effect

18. What are type I and type II superconductors? Give examples. Discuss and compare Meissner effect and perfect diamagnetic behaviors for type I and type II super conductors. **(2018)**

For lattice constant a,

$$\boxed{a^3 = \frac{n_{eff} \cdot M_{moral}}{N_A\, \rho}}$$

$n_{eff} \rightarrow a^3$

$\frac{n_{eff} M_{moral}}{N_A} \rightarrow a^3$

$\rho = \frac{mass}{volume}$

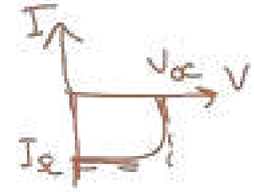
& $\boxed{\text{number density } (n) \times \text{mass} = \text{density } (\rho)}$

$V_{bi} = \frac{K_BT}{e}\ln\left(\frac{n_n}{n_p}\right)$ $\Delta P = \Delta P_0 \exp\left(-\frac{L}{\sqrt{D\tau}}\right)$ $\left[\sigma_{\text{conduction}} = \frac{\epsilon}{\sigma \approx n_i e(\mu_n+\mu_p)}\right]$

DIAS Delhi Institute for Administrative Services

India's Leading Institute for Civil Services Examination

$\{eD = \mu K_BT\}$



Electronics/ TUTORIAL SHEET: 12
Semiconductors/Solar cell

1. Draw the device structure of a-p-n junction solar cell and explain how light energy is converted into electrical energy. Draw and explain its I-V characteristics. **(2015)**

2. A silicon semiconductor sample at T = 300 K having cross-sectional area of 0.5 μm^2 has a pentavalent donor doping profile given by $(x) = 5 \times 10^{16}\, e^{(-x/L_n)}\ cm^{-3}$. Given, the mobility of the electrons in the sample is 1250 $cm^2V^{-1}s^{-1}$ and the diffusion length of the electrons, L_n , is 4 μm. Calculate the diffusion current in the sample at distance $x = 2\ \mu m$. $(V_T = 0.026\ \text{Volt})$ **(2019)**

$D_p = \mu_p V_T$ $[J = eD_n\, dn/dx]$ acceptor

3. A silicon semiconductor sample is doped with $6 \times 10^{16} cm^{-3}$ of aluminium and $7 \times 10^{15} cm^{-3}$ of phosphorus atoms. Given at T = 300 K, the intrinsic carrier concentration, $n_i = 1.5 \times 10^{10} cm^{-3}$; the band gap, $E_g = 1.1\ eV$; the electron mobility, $\mu_n = 1250\ cm^2V^{-1}s^{-1}$ and the hole mobility, $\mu_p = 480\ cm^2V^{-1}s^{-1}$.

Donors

Determine in the sample of the following:

(i) The type of the semiconductor, n or p $\{N_a > N_d\} \Rightarrow$ p-type

(ii) The hole carrier concentration $p \sim (N_a - N_d)$ $\Rightarrow p = 5.3 \times 10^{15}\ cm^{-3}$ as n_i is very small compared to it.

(iii) The electron carrier concentration $\rightarrow n = \frac{n_i^2}{p}$

(iv) The position of the Fermi level in the sample with respect to the bottom of the conduction band

(v) The conductivity of the sample $\sigma \simeq pN_a\mu_p$ $E_f = E_{fi} - K_BT\ln\left(\frac{p}{n_i}\right)$ (E_{fi} is at mid of band gap) **(2019)**

4. A 5 cm^2 Ge solar cell with a dark reverse saturation current of 2 nA has solar radiation incident upon it, producing 4×10^{17} electron-hole pairs per second. The electron and hole diffusion lengths are given to be 5μm and 2μm, respectively. Calculate for the cell of the following:

(i) The short-circuit current

(ii) The open-circuit voltage **(2019)**

$p = n_i \exp\left(-\frac{E_f - E_i}{K_BT}\right)$

[Diffusion length, $l_p = \sqrt{D_p\tau_p}$]

Hall effect is appearance of $\vec{E}$ in conductor due to current & $\vec{B}$ in ⊥ direction

$eE_H = evB$

& $J = n_0 e v$

$R_H = \frac{E_H}{JB} = \frac{1}{n_0 e}$

for BCC, 2 atoms → a^3

$n = \frac{2}{a^3}$

Solar cells are photovoltaic cells converting solar radiation incident into electrical energy

For a solar cell, I-V characteristic equation

$I = -I_{ph} + I_0\left(\exp\left(\frac{eV}{nK_BT}\right) - 1\right)$ (reverse saturation current; photo current)

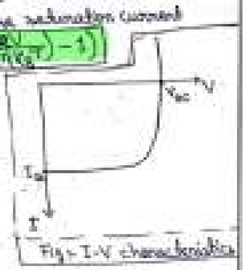


Fig - I-V characteristics

For open circuit, $I = 0$ & assuming $V_{oc} \gg \frac{nK_BT}{e}$,

$V_{oc} = \frac{nK_BT}{e}\ln\left(\frac{I_{ph}}{I_0}\right)$ — ③

Since $I_{ph} = I_{short\ circuit}$ (as $I = I_{ph}|_{V=0}$);

Since $I_{ph} \propto$ light's intensity, thus,

$I_{ph_2} = I_{ph_1} \times 2$ (as light intensity is doubled)

$\Rightarrow \boxed{I_{ph_2} = 300\ mA}$ (as $I_{ph_1} = I_{sc} = 150$ mA (given)) — required expression

Similarly from equation ③,

$V_{oc_2} - V_{oc_1} = nK_BT/e\ \ln\left(\frac{I_{ph_2}}{I_{ph_1}}\right)$

TUTORIAL SHEET: 13

TRANSISTORS (Amplifiars/Oscillators)

1. 1. Circuit as shown in the figure. It is designed to establish the quiescent potential at V_{CE} = 12V, I_C = 1.5 mA and stability factor S $\leq$3. Find the values of resistors R_E, R_1 and R_2 where symbols have their usual meaning. **(2004)**

2. Explain the working of a phase-shift oscillator. Draw the circuit diagram of a phase-shift oscillator using either a transistor or an op-amp. What are the conditions which must be satisfied to achieve stable oscillations?Calculate the frequency of oscillations for $R = 1k\Omega$ and C = 01 μ F. **(2007)**

3. Explain the difference between n-p-n and p-n-p transistors. Give their device structure and biasing circuits. Draw a circuit for a single-state common-emitter amplifier. **(2007)**

4. Draw the circuit diagram of a two-stage RC-coupled common emitter transistor amplifier. Show how the magnitude and phase of voltage gain vary with frequency. Define bandwidth of this amplifier. An amplifier with open loop voltage gain, $A_v = 1000 \pm 100$ is avallable. It is necessary to have an amplifier whose voltage gain varies by no more than $\pm 0 \cdot 1\%$. Find the reverse transmission factor β of the feedback network used and the gain with feedback. **(2008)**

5. A certain colpits oscillator uses a tank circuit with L=20mH; $C_1 = 200$ pf and $C_2 = 200$ pf. What is the frequency of oscillation? **(2008)**

$f = \frac{1}{2\pi\sqrt{L C_{eff}}}$ $C_{eff} = \frac{C_1 C_2}{(C_1+C_2)}$

6. (i) "Transistors are current operated devices, while vacuum tubes are voltage operated" – Explain.
(ii) Why are junction transistors called bipolar devices? **(2008)**

7. Draw the common-base amplifier circuit, using an n-p-n transistor and briefly discuss its working. **(2010)**

8. Draw and explain the collector characteristics of a bipolar junction transistor in common emitter configuration. Using the plot, explain how the transistor can be used as an ON-OFF switch. **(2013)**

9. Explain how the circuit shown above can be a source of oscillations. Use this circuit to construct a transistor oscillator and explain its working. What is the frequency of oscillations of this circuit? **(2013)**

EBC

10. Differentiate between p-n-p and n-p-n transistors. Give their device structure and biasing circuits when used as an amplifier. **(2015)**

CBE

11. Design a transistor based Colpitt oscillator which can oscillate at 9 MHz. Explain how the oscillations are created and sustained. **(2015)**

→ Feedback for active device is taken from Voltage divider with Tank Circuit.

$\boxed{L_{eq} = L_1 + L_2 + 2M}$ $f_{Hartley} = \frac{1}{2\pi\sqrt{L_{eq} C}}$

TUTORIAL SHEET: 14
Digital Electronics/Microprocessors

1. Simplify the equation $Y = [A\bar{B}(C + BD) + \overline{AB}C]$ Using Boolean algebra. Give the logic circuit for the equation before and after the simplification. **(2007)**

2. Give the Boolean expression and the truth table for an XOR gate. Why are NAND gates called universal gates? Realise OR and XOR gates employing only NAND gates. **(2007)**

3. How can the NAND-gates be combined to perform the OR – operation? **(2008)**

$Y = A+B = \overline{\bar{A}\cdot\bar{B}}$ 

4. Simplify the logical expression AB + AB + ABC using a karnaugh map. **(2010)**

↳  $Y = A\bar{B}C + ABC = AB$

5. Simplify the logical expression (A + B) (B+C) (A + C) and draw the logical circuit to implement it. **(2010)**

$Y = (B + AC)(A + C) = AB + BC + AC$.

6. Construct a digital circuit to add three bits A, B and C and provide their sum and carry as-outputs. Show appropriate Boolean expressions and truth table to justify the outputs. **(2013)**

→ Full Adder

Carry = $((A \oplus B)\cdot C_{in} + AB)$

7. (a) Give the two forms of the De Morgan's theorem.

→ $\overline{AB} = \bar{A} + \bar{B}$ & $\overline{(A+B)} = \bar{A}\,\bar{B}$

(b) What is the principle used in 'half –adder'? Give the truth table for it. **(2014)**

↳ $S = A \oplus B$ & $C = AB$

8. Simplify the logical expression [AB (C + BD) + A B] C. **(2015)**

9. Why are NAND and NOR gates called universal gates? Give the logic diagram, Boolean equation and the truth table of NAND gate. **(2016)**

$$\chi_{diamagnetic} = -\frac{N\mu_0 Ze^2}{6m}\langle r^2\rangle$$

Don't forget to see examples of Puri & Babbar

Classical Zeeman effect (Mains 2020)

$\vec{F}_c \longleftrightarrow \vec{F}_B$ | $\vec{F}_c \updownarrow \vec{F}_B$

$$m r\omega^2 \pm evB = mr(\omega+d\omega)^2$$

$$m r\omega^2 \pm evB = 2mr\omega\, d\omega + mr\omega^2$$

(neglecting $(d\omega)^2$)

$$\Rightarrow \boxed{2mr\,d\omega = \pm evB}$$

$\{v = r\omega\}$

$$\Rightarrow \boxed{d\omega = \pm \frac{eB}{2m}}$$

$$\left\{ \omega = \frac{2\pi c}{\lambda} \Rightarrow d\omega = -\frac{2\pi c}{\lambda^2} d\lambda \right\}$$

$$\Rightarrow -\frac{2\pi c}{\lambda^2} d\lambda = \pm \frac{eB}{2m}$$

$$\Rightarrow \boxed{\frac{e}{m} = \pm \frac{4\pi c}{B\lambda^2} d\lambda}$$

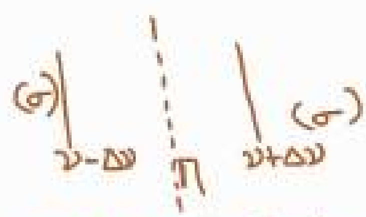
$$\boxed{d\lambda = \pm \frac{Be\lambda^2}{4\pi mc}}$$

$$\left\{ \nu = \frac{c}{\lambda} \Rightarrow \boxed{d\nu = -\frac{c}{\lambda^2} d\lambda} \right\}$$

or,

$$\boxed{d\nu = \pm \frac{eB}{4\pi m}}$$ Ans

→ required expression



→ If we observe spectra parallel to applied $\vec{B}$ → $\boxed{\text{Longitudinal}}$ → only 2 lines

$\boxed{\text{Transverse}}$ → 3rd line.