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PAPER I - Tut sheets

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Thermodynamics

Tutsheet : 20 : Basic Concepts

(Q1) Adiabatic Process is defines as the process for which no heat is exchanged by the system. For such a process,

$$PV^{\gamma} = \text{Const} = \alpha \quad \text{—} \ (1)$$

where P : Pressure of system
V : Volume of system
γ : ratio of specific heat at const. Pressure and volume

specific heat ratio

$$\text{Work Done in Adiabatic Process} = \left(\frac{P_1V_1 - P_2V_2}{\gamma - 1}\right) \quad \text{—} \ (2)$$

As given

$$V_2 = \frac{1}{2}V_1 \quad \text{—} \ (3)$$

⇒ ~~from ideal gas equation~~

$$\text{Volume of gas } V_1 = \left(\frac{10^{-3}}{1.293}\right) m^3$$

$$= 7.73 \times 10^{-4} \ m^3$$

Pressure $P_1 = 10^5$ Pascal

From (1) and (3)

$$P_1 V_1^{\gamma} = P_2 \left(\frac{V_1}{2}\right)^{\gamma}$$

$$\Rightarrow P_2 = P_1 \cdot 2^{\gamma} = \underline{\underline{2.63 \times 10^5 \ Pa}}$$

From (2),

$$\text{Work Done} = \left[10^5 * 7.73 \times 10^{-4} - 2.63 \times 10^{5} \times \frac{7.73 \times 10^{-4}}{2}\right] \Big/ 0.4$$

$$= \underline{\underline{-60.87 \ J}}$$

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We can note that since the gas is compressed, Work done by the system is negative.

External Work Done $= -dW = \underline{\underline{60.87 J}}$

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Q2) For an adiabatic process, heat exchanged is zero

$$\Rightarrow \quad dQ = dU + PdV = 0$$

(from 1st law of thermodynamics)

$$\Rightarrow \quad dU = -PdV$$

For an ideal gas, $dU = C_v dT$ as interaction between molecules are neglected.

$$\Rightarrow \quad C_v dT = -PdV \quad — \text{(1)}$$

Also from ideal gas equation, $PV = RT$, we have

$$P = \frac{RT}{V} \quad — \text{(2)}$$

Using (2) in (1)

$$\Rightarrow \quad C_v dT = -\frac{RT}{V} dV$$

$$\Rightarrow \quad \frac{C_v}{R}\left(\frac{dT}{T}\right) = -\left(\frac{dV}{V}\right) \quad — \text{(3)}$$

Now, we know for an ideal gas,

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$C_p - C_v = R \quad \& \quad \frac{C_p}{C_v} = \gamma$$

$$\Rightarrow (\gamma - 1) = \frac{R}{C_v} \quad — \ (4)$$

Using in (4)

$$\left(\frac{dT}{T}\right) = -\left(\frac{R}{C_v}\right)\frac{dv}{V} = -(\gamma-1)\left(\frac{dv}{V}\right)$$

Integrating, we have

$$\ln T = -(\gamma-1)\ln V + A$$

$$\Rightarrow T V^{\gamma-1} = e^{A} = \text{const}$$

$$\Rightarrow \boxed{T V^{\gamma-1} = \text{const}}$$

(Q3) For an isothermal gas, temperature remains const. ∴ for an ideal gas internal energy does not change

$\Rightarrow$ From 1^{st} law of thermodynamics

$$dQ = \cancel{dU}^{\,0} + P\,dV = P\,dV$$

Hence energy required is equal to work done by the system.

$$dWork_{isothermal} = P\,dV$$
$$= \frac{nRT}{V}\,dV$$

$$\Rightarrow W = \int dw = nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$= \frac{1}{2} \times 8.314 * 300 \ln\left(\frac{25}{100}\right)$$

$$= \frac{-13457.69}{2} \ J$$

$$= -1728.8 \ J$$

Hence energy has to be removed instead.

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Q4) For a reversible expansion of an ideal gas from volume V_1 to V_2, the gas is maintained at equilibrium with the surroundings at all intermediate points. This can be done by placing very small weights on the top of piston and slowly removing them one by one, maintaining equilibrium at all points.

However for an irreversible process of expansion, all weights are removed at once such that gas expands at once from V_1 to V_2 against const. external pressure P_2

The work done by gas in reversible process is $$W_R = \int_{V_1}^{V_2} P\,dV$$

In irreversible process, work done is
$$W_{ir} = P_2 (V_2 - V_1)$$

The difference in the two values of works is seen in following PV curve.

W_R = area (ABCD)
W_{ir} = area (EBCD)

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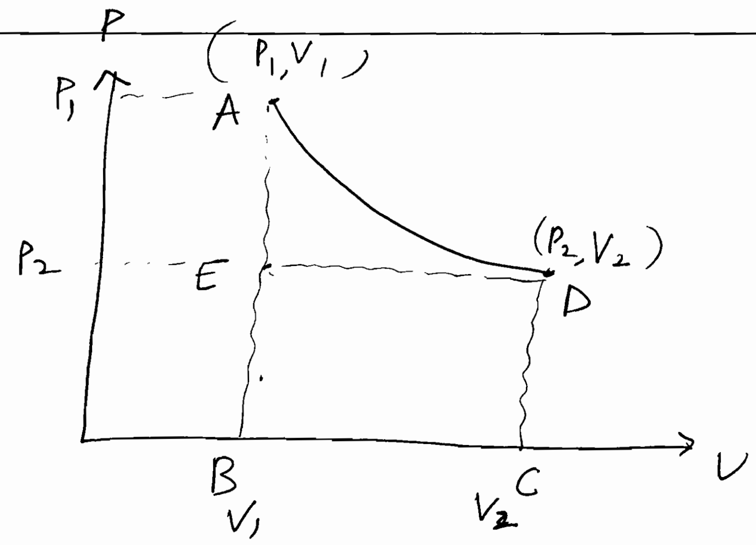

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Clearly $W_R > W_{ir}$

Q5) Any system which Constitutes matter is made up of a large number of molecules which are in constant state of motion and hence posses internal kinetic energy.

Due to interactions of molecules and intermolecular attraction, they also posses an internal potential energy. The sum total of the two energies of all molecules of the body is called $\boxed{\text{internal energy}}$ of the system

First law of thermodynamics states that if an amount of heat dQ is supplied to a system, it can be spent in two ways :

(i) increase the internal energy of system, dU

(ii) Work done by the system, dW

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i.e. $dQ = dU + dW$

The law is simply another form of Conservation of energy theorem i.e. energy can neither be destroyed nor be Created but converted from one form to another form.

<Q3>

Q6) <Q1>

3 engineering example of adiabatic process include :

(i) Heat Engine : Adiabatic ~~Compression~~ Expansion stroke does useful work.

(ii) Referigerator

(iii) Adiabatic Demagnetization

(iv) Liquification of gases via ~~Adiabatic~~ adiabatic expansion of gases which causes cooling

In engineering uses Adiabatic Process is performed by either

(i) thermally insulating the system from surroundings

OR

(ii) performing the process very rapidly so that there is no enough time for heat transfer to take place.

Q7) Process 1

Its an isothermal Process.

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From 1^{st} law of thermodynamics

$$dQ = \cancel{dU}^{0} + p\,dV \qquad \text{(isothermal)}$$

$$\Rightarrow dQ = p\,dV \quad \text{—— (1)}$$

$$= \frac{RT}{V}\,dV \qquad \text{(from ideal gas eqn)}$$

$$\Rightarrow Q = \int dQ = RT\int \frac{dV}{V} = RT\ln\left(\frac{V_2}{V_1}\right) \quad \text{—— (2)}$$

Hence from (1) and (2), Heat absorbed by the gas is equal to work done by the system equal to $RT\ln\left(\frac{V_2}{V_1}\right)$.

Process 2

Adiabatic expansion

For adiabatic expansion, $TV^{\gamma-1} = \text{const}$.

For diatomic molecule, $\gamma = (7/5)$

$$\Rightarrow T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$\Rightarrow T_2 \cdot (2)^{\frac{2}{5}} = T_1$$

$$\Rightarrow T_2 = \frac{T}{(2)^{2/5}} = \underline{0.75\,T}$$

(Q8) Second law of thermodynamics can be written in two forms:

(1) Kelvin - Planck Statement

It is impossible to ~~device~~ construct a device whose sole job is to take heat from a source at higher Temperature and convert

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it into equivalent amount of work without producing any changes in other parts of the system.

(2) <u>Claussius – Ostwald statement</u>

It is impossible to construct a device whose ~~sole~~ job is to take heat from lower temperature and release it to higher temperature.

In terms of entropy, the second law of thermodynamics can be states as "For an isolated system, entropy cannot decrease"

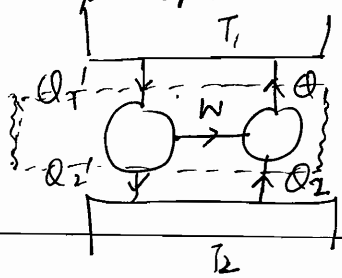
In order to prove the Carnot theorem, let us use method of contradiction and assume that efficiency of an arbitrary engine is more than efficiency of Carnot engine.

i.e. $\eta' > \eta_c \Rightarrow \eta'_1 = \frac{W'}{Q'_1}$ & $\eta_c = \frac{W}{Q_1}$

as $\eta_1 > \eta_c \Rightarrow \boxed{Q_1' < Q_1}$ Also, $Q_1' - Q_2' = W = Q_1 - Q_2$ $\Rightarrow \boxed{Q_2' < Q_2}$

Since Carnot engine is based on reversible cycle, it can applied in reverse and can behave as a referigerator or heat pump.

We assume that engines are constructed to give the same amount of work W.



Better to give calculation before & show coupling afterwards

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Let us couple the two engine as shown in adjacent figure.

Since we have assumed $\eta' > \eta_c$

$$\Rightarrow 1 - \frac{Q_2'}{Q_1'} > 1 - \frac{Q_2}{Q_1}$$

$$\Rightarrow \frac{W}{Q_1'} > \frac{W}{Q_1}$$

$$\Rightarrow Q_1 > Q_1'$$

Hence $(Q_1 - Q_1') > 0$

$\Rightarrow$ Net heat is being supplied to a system at higher temperature; but this violates 2^{nd} law (Claussius-Ostwald statement).

Hence our assumption is wrong. No engine operating between 2 temp. can be more efficient than Carnot engine.

Q 9) As given, $P = \alpha V$

Work done in the process $dW = PdV$

$$\Rightarrow dW = \alpha V dV$$

$$\Rightarrow W = \int dW = \alpha \int_V^{mV} V dV$$

$$= \alpha (m^2 - 1) \frac{V^2}{2}$$

$$= \frac{\alpha (m^2-1)}{2} V^2$$

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From ideal gas equation, we know

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{\alpha V_1^2}{T_1} = \frac{\alpha V_2^2}{T_2} = \frac{\alpha m_1^2 V_1^2}{T_2}$$

$$\Rightarrow \boxed{T_2 = m_1^2 T_1}$$

For an ideal, change in internal energy, dU is given as

$$dU = C_v\, dT$$

$$\Rightarrow \Delta U = C_v (T_2 - T_1)$$

$$\boxed{\Delta U = C_v (m^2 - 1)\ T_1}$$

From 1st law of thermodynamics, heat supplied dQ,

$$dQ = dU + P\,dV$$

$$\Rightarrow \Delta Q = \Delta U + W$$

$$= C_v T_1 (m^2-1) + \frac{\alpha}{2}(m^2-1)\cancel{V}^2$$

$$= C_v . \frac{P_1V_1}{R}(m^2-1) + \frac{\alpha}{2}(m^2-1)V_1^2$$

$$= \frac{C_v . \alpha (m^2-1) V^2}{R} + \frac{\alpha}{2}(m^2-1)V^2$$

$$= V^2 \alpha (m^2-1)\left[\frac{1}{2} + \frac{C_v}{R}\right]$$

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Now, ~~specific~~ heat capacity for the process C' is given as

$$C' = \left(\frac{\Delta Q}{\Delta T}\right)$$

$$= \frac{V^2 \alpha (m^2-1)\left[\frac{1}{2} + \frac{C_v}{R}\right]}{(m^2-1).\ T}$$

$$= \frac{V^2\alpha}{\frac{\alpha V^2}{R}}\left[\frac{1}{2} + \frac{C_v}{R}\right]$$

$$= \left[\frac{R}{2} + C_v\right]$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q1) A Carnot engine use a Carnot Cycle which is shown below.

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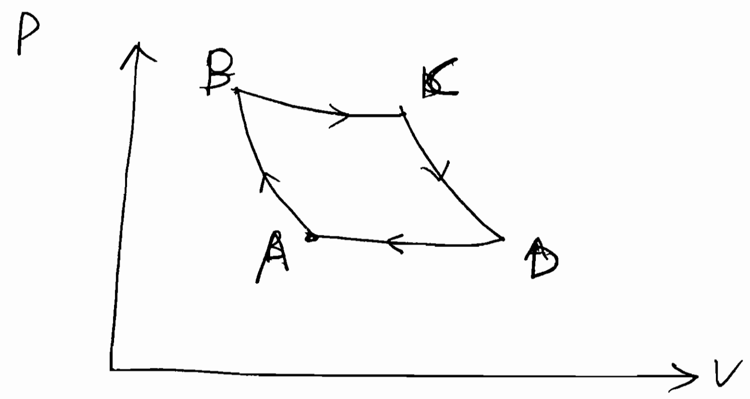

Carnot Cycle consists of Adiabatic Compression (AB) followed by isothermal expansion (BC), then adiabatic expansion (CD), finally followed by isothermal compression (DA).

For an isothermal process,

$$dQ = dU + PdV \quad (1^{st}\ \text{law})$$
$$= PdV \quad (dU = 0) \quad \because dT = 0 \text{ for isothermal}$$
$$= \frac{RT}{V}\, dV \quad (\text{for ideal gas})$$

$$Q = \int dQ = RT \ln\left(\frac{V_2}{V_1}\right)$$

Now $Q_{AB} = 0$

$Q_{BC} = RT_B \ln\left(\frac{V_C}{V_B}\right)$

$Q_{CD} = 0$

$Q_{DA} = -RT_D \ln\left(\frac{V_D}{V_A}\right)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \text{Efficiency} = \frac{W}{Q_{in}} = \frac{Q_1+Q_2+Q_3+Q_4}{Q_{BC}}$$

$$= 1 - \frac{T_D \ln\left(\frac{V_D}{V_A}\right)}{T_B \ln\left(\frac{V_C}{V_B}\right)}$$

Also for an adiabatic process,

$$TV^{\gamma-1} = \text{Const}$$

$$\Rightarrow T_A \cdot V_A^{\gamma-1} = T_B V_B^{\gamma-1}$$

$$T_C V_C^{\gamma-1} = T_D V_D^{\gamma-1}$$

$$\Rightarrow \frac{\cancel{T_C} V_C^{\gamma-1}}{\cancel{T_B} V_B^{\gamma-1}} = \frac{\cancel{T_D} V_D^{\gamma-1}}{\cancel{T_A} V_A^{\gamma-1}} \qquad \left(\begin{matrix} T_C = T_B \\ T_D = T_A \end{matrix}\right)$$

$$\Rightarrow \left(\frac{V_C}{V_B}\right) = \left(\frac{V_D}{V_A}\right)$$

$$\Rightarrow \text{Efficiency} = \eta = 1 - \left(\frac{T_D}{T_B}\right)$$

$$= 1 - \left(\frac{T_2}{T_1}\right)$$

T_2 : Temperature of sink
T_1 : Temperature of source

For given Carnot Engine $\eta = 1 - \frac{73}{273} = 73.2\%$

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Q2)

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For an isothermal process,

$$dQ = \cancel{dU} + P\,dV \qquad (1^{st}\text{ law})$$

$$= P\,dV \qquad (\text{as } T = \text{const.})$$

$\Rightarrow$ dS = Change in entropy

$$= \frac{dQ}{T}$$

$$= \frac{P\,dV}{T}$$

For an ideal gas $\dfrac{P}{T} = \dfrac{R}{V}$

$$\Rightarrow dS = \frac{R\,dV}{V}$$

$$\Rightarrow \Delta S = \int dS = R\int \frac{dV}{V} = R\ln\left(\frac{V_2}{V_1}\right)$$

$$= R\ln\left(\frac{4V}{V}\right)$$

$$= \boxed{R\ln 4}$$

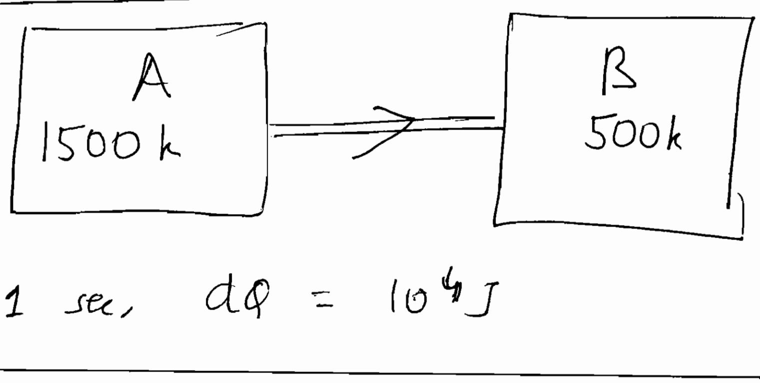
Q3)

L for condensation $= 3.36 \times 10^{+5}$ J/kg

$$\Rightarrow dQ = mL = 3.36\times10^{5}\text{ J}$$

$$dS = \frac{dQ}{T} = \frac{mL}{T} = \frac{3.36\times10^{5}}{273} = \underline{\underline{1230.76\text{ J/K}}}$$

Q4)



In 1 sec, $dQ = 10^{4}$ J

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$$dS_A = -\frac{dQ}{T_A}$$ (negative as heat flows from heat Temp to lower temp)

$$dS_B = \frac{dQ}{T_B}$$

$$\Rightarrow dS_{Total} = dS_A + dS_B$$

$$= dQ\left[\frac{1}{T_B} - \frac{1}{T_A}\right]$$

$$= 10^4\left[\frac{1}{500} - \frac{1}{1500}\right]$$

$$= 10^4 \cdot \frac{1000}{500 * 1500}$$

$$= 13.33 \text{ J/k}$$

$\Rightarrow$ Rate of entropy change $= 13.33$ J/k/s

(Q5) Entropy is a measure of the disorder of the system. Greater the ~~entropy~~ disorder, more is the entropy. Entropy (S) is a state variable and thus for a process between initial and final states, Entropy change (dS) is independent of path (or the process)

If the system is at temperature T, and an amount dQ_{rev} of heat is added reversibly to the system, entropy change dS is defined as

$$dS = (dQ_{rev}/T)$$

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From 1^{st} law,

$$dQ = dU + P\,dV$$
$$= C_v\,dT + P\,dV \quad (\text{for ideal gas})$$

$$\Rightarrow dS = \frac{dQ}{T} = C_v \frac{dT}{T} + \frac{P}{T} dV$$

$$dS = C_v \frac{dT}{T} + \frac{R}{V} dV \quad \text{——} \quad (1)$$

$$\Rightarrow \boxed{\Delta S = \int dS = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)}$$

For isothermal change $T_1 = T_2$

$$\Rightarrow \Delta S = R \ln\left(\frac{V_2}{V_1}\right)$$

For isochoric change $V_2 = V_1$

$$\Rightarrow \Delta S = C_v \ln\left(\frac{T_2}{T_1}\right)$$

To calculate isobaric change, we know

$$PV = RT$$
$$\Rightarrow P\,dV + V\,dP = R\,dT$$
$$\Rightarrow dV = \frac{R\,dT}{P} - \frac{V\,dP}{P}$$

Using in (1),

$$dS = C_v \frac{dT}{T} + \frac{R}{P}\frac{R}{V}.dT - \frac{R}{V}\frac{V}{P}dP$$

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$$\Rightarrow dS = C_v \frac{dT}{T} + \frac{R}{T}.dT - \frac{R}{P}.dP$$

$$= (C_v + R)\frac{dT}{T} - \frac{R}{P}dP$$

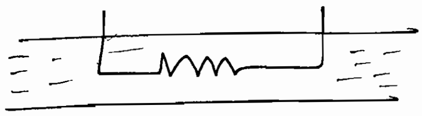
$$= C_p \frac{dT}{T} - \frac{R}{P}dP$$

$$\Rightarrow \boxed{\Delta S = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)}$$

For <u>isobaric process</u> $P_1 = P_2$

$$\Rightarrow \Delta S = C_p \ln\left(\frac{T_2}{T_1}\right)$$

Q 6)



Give Resistance $R = 10\Omega$

Current $I = 3A$

Temperature $T = 300k$

The energy obtained from the circuit (the source of EMF) is dissipated in water. For resistor, there is no net loss or gain of heat dQ

i.e. $dQ_R = 0$

$$\Rightarrow dS_R = \frac{dQ_R}{T} = 0$$

Since the ~~the resistor~~ Surrounding gains heat dQ at temperature 300k, change in entropy dS

In the process, state of resistor remains ~~Consta~~ unchanged, as it is maintained at Temp 300 k $\therefore$ entropy being a state function, change in entropy $= 0$

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is given as

$$dS_{sur} = \frac{dQ}{T} = \frac{I^2R\,dt}{T}$$

$$\Rightarrow \left(\frac{dS}{dt}\right)_{surr} = \frac{I^2R}{T} = \frac{9\times 10}{300} = 0.3\ J/Ks$$

Hence change in entropy per second is $0.3\ J/K$

Since $\Delta S_{univ} = \Delta S_{resistor}^{\,0} + \Delta S_{surrounding}$

$= \Delta S_{surrounding}$

$\Rightarrow \Delta S_{universe}$ per second $= 0.3\ J/k$

Q7) <Q5><Q5>

Q8) <Q5>

For a process, specific heat is C_p given as

$$C_p = a + bT + cT^2$$

Now $dQ = mC_p\,dT$

where m is mass of the system

Now, dS (change in entropy) is given as $dS = \frac{dQ}{T} = \frac{mC_p}{T}dT$

$$= \left(\frac{ma}{T} + mb + mcT\right)dT$$

$$\Delta S = \int_{T_1}^{T_2} dS = ma\int_{T_1}^{T_2}\frac{dT}{T} + mb\int_{T_1}^{T_2} dT + mc\int_{T_1}^{T_2} T\,dT$$

$$= m\left[a\ln\left(\frac{T_2}{T_1}\right) + b\,(T_2 - T_1) + c\,(T_2^2 - T_1^2)\right]$$

Q9) Carnot theorem states that all reversible engines working between same heat reservoirs are equally efficient. Thus, any reversible heat engine operating between temperature T_1 and T_2 must have same efficiency. i.e. efficiency is a function of only temperatures

$$\eta = 1 - \frac{Q_C}{Q_H} \Rightarrow \frac{q_C}{q_H} = f(T_C, T_H)$$

Let $T_3 > T_2 > T_1$

$$f(T_3, T_1) = \frac{q_1}{q_3} = \left(\frac{q_1}{q_2}\right)\left(\frac{q_2}{q_3}\right)$$

$$= f(T_1, T_2)\; f(T_2, T_3)$$

But left hand side is not a function of T_2.

$\Rightarrow$ in order to achieve this

$f(T_1, T_2)$ should be of form

$$\frac{g(T_1)}{g(T_2)}$$, such that

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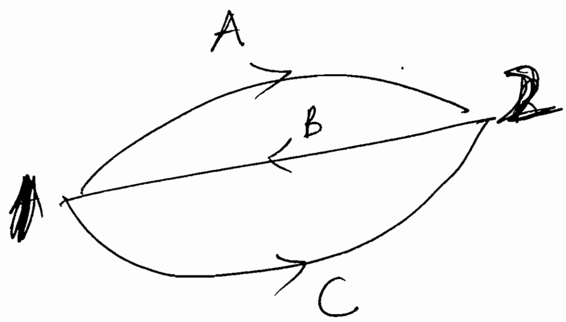
$$f(T_1,T_2)\, f(T_2,T_3) = \frac{g(T_1)}{g(T_2)} \frac{g(T_2)}{g(T_3)}$$

$$= \frac{g(T_1)}{g(T_3)} = f(T_1,T_3)$$

As a matter of convenience, we choose $g(T) = T$. Choosing one temperature as reference, we establish Thermodynamic Temperature Scale. We choose Triple point of water as 273·16 k and Absolute Zero as 0 k.

Q 10) Let us consider a cyclic path AB and other ~~BB~~. Let both be reversible.



From Clausius Inequality,

$$\oint_{rev} \frac{dQ}{T} = 0$$

$$\Rightarrow \oint_{AB} \frac{dQ}{T} = 0 \Rightarrow \int_A \frac{dQ}{T} + \int_B \frac{dQ}{T} = 0 \quad \text{—— (1)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Also, $\oint_{CB} \frac{dq}{T} = 0$

$\Rightarrow \int_C \frac{dq}{T} + \int_B \frac{dq}{T} = 0$

$\Rightarrow \int_C \frac{dq}{T} + \left(-\int_A \frac{dq}{T}\right) = 0$ — (from (I))

$\Rightarrow \boxed{\int_C \frac{dq}{T} = \int_A \frac{dq}{T}}$

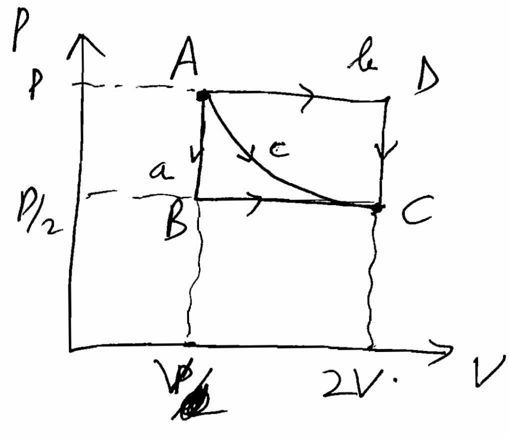
Hence path independent.

Entropy function (S) is defines a state function such that change in entropy dS for a reversible process is given as

$$\boxed{dS = \frac{dq_{rev}}{T}}$$

$$\Delta S = \int dS = \int_{rev} \frac{dq}{T}$$

For path a



$dS = dS_1 + dS_2$

$dS_1 = \frac{dq}{T}$

$= C_v \frac{dT}{T}$

$= C_v \ln(T_B/T_A)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B} \Rightarrow \frac{PV}{T_A} = \frac{P}{2}\frac{V}{T_B}$$

$$\Rightarrow T_B = \left(\frac{T_A}{2}\right)$$

$$\Rightarrow \Delta S_1 = -C_v \ln 2$$

$$dS_2 = C_p \frac{dT}{T} \Rightarrow \Delta S = C_p \ln\left(\frac{T_E}{T_B}\right)$$

Also $\frac{P_B V_B}{T_B} = \frac{P_C V_C}{T_C}$

$$\Rightarrow \frac{1}{T_B} = \frac{2}{T_C} \Rightarrow T_C = 2T_B = T_A$$

$$\Rightarrow \Delta S_2 = C_p \ln 2$$

$$\Rightarrow \boxed{\Delta S_a = (C_p - C_v)\ln 2 = R \ln 2}$$

$\boxed{\text{For path c}}$ (Isothermal path)

$$dS = \frac{dq}{T} = \frac{P\,dV}{T} = \frac{R}{V}\,dV$$

$$\Rightarrow \Delta S = R \ln\left(\frac{V_C}{V_A}\right) = \underline{R \ln 2}$$

Similarly for Path b $\boxed{\Delta S = R \ln 2}$

Q11)

$$\frac{1}{4} = 1 - \frac{T}{(267+273)}$$

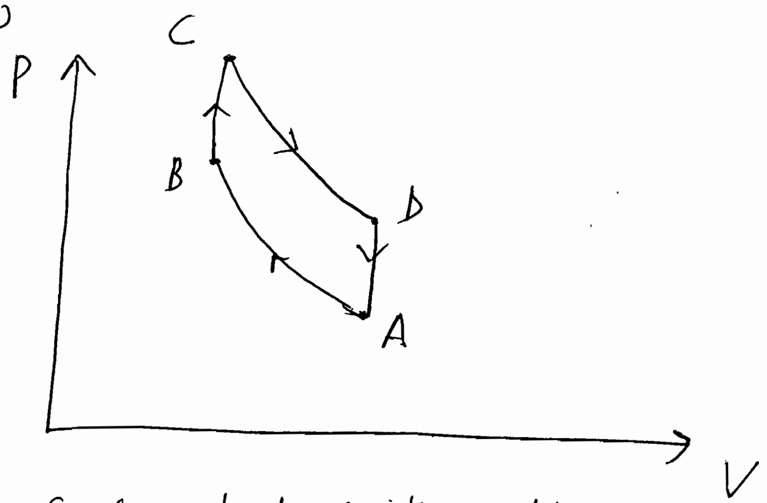
$$\Rightarrow T = \frac{3}{4} * 540 = 405\,k = \underline{132^\circ C}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q(12) Q(13) (14)

Q – 3
Q – 1

Otto Cycle is a cycle used in Otto Engines that are used in Cars all over the world. The cycle is shown below



Otto Cycle starts with adiabatic compression (AB) followed by isochoric heating (BC), then adiabatic expansion (CD) [here useful work is done) and finally isochoric cooling (DA)

Efficiency of ~~Carnot~~ Otto Cycle

AB : $dQ = 0$

BC : $dQ = dW = C_v (T_C - T_B)$

CD : $dQ = 0$

DA : $dQ = -C_v (T_D - T_A)$

$$\Rightarrow \eta = 1 + \frac{1}{Q_{BC}/Q_{DA}} = 1 - \frac{1}{\left(\frac{T_C - T_B}{T_D - T_A}\right)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Also,

$V_B = V_C$

$V_D = V_A$

$T_C V_C^{\gamma-1} = T_D V_D^{\gamma-1}$

$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}$

$$\Rightarrow \frac{T_B V_B^{\gamma-1}}{T_C V_C^{\gamma-1}} = \frac{T_A V_A^{\gamma-1}}{T_D V_D^{\gamma-1}}$$

$$\Rightarrow \frac{T_B}{T_C} = \frac{T_A}{T_D}$$

$$\Rightarrow \eta = 1 - \frac{T_C\left(1 - \frac{T_B}{T_C}\right)}{T_D\left(1 - \frac{T_A}{T_D}\right)} = 1 - \frac{1}{\left(\frac{T_C}{T_D}\right)} = 1 - \left(\frac{T_D}{T_C}\right)$$

$$= 1 - \left(\frac{V_C}{V_D}\right)^{\gamma-1}$$

$$= 1 - \left(\frac{V_B}{V_A}\right)^{\gamma-1}$$

$$\boxed{\eta = 1 - \left(\frac{1}{\rho}\right)^{\gamma-1}}$$

ρ : Compression ratio $= \left(\frac{V_A}{V_B}\right)$

In otto cycle, highest temperature is for T_C and lowest for T_A

(Obtained by comparing isothermal process and adiabatic process slopes)

$$\Rightarrow \eta_{Carnot} = 1 - \frac{T_A}{T_B}.$$

Now $T_D > T_A$

$$\Rightarrow \frac{T_D}{T_C} > \frac{T_A}{T_B}$$

$$\Rightarrow \left(1 - \frac{T_D}{T_C}\right) < \left(1 - \frac{T_A}{T_B}\right)$$

$$\Rightarrow \boxed{\eta_{otto} < \eta_{carnot}}$$

(Q15) For isothermal process,

(i) $dU = 0$

(ii) $dS = \frac{dQ}{T} = \frac{P\,dV}{T} = \frac{R}{V}\,dV =$

$$\Rightarrow \Delta S = \int dS = R \ln\left(\frac{V_2}{V_1}\right)$$

(iii) Change in enthalpy $= dH$

$$= dU + P\,dV + V\,dP$$

$$= T\,dS + V\,dP$$

$$= \frac{TR}{V}\,dV + \frac{RT}{P}\,dP$$

$$\Rightarrow \Delta H = \int dH = TR \ln\left(\frac{V_2}{V_1}\right) + RT \ln\left(\frac{P_2}{P_1}\right)$$

$$= RT \ln\left(\frac{V_2}{V_1} \cdot \frac{P_2}{P_1}\right)$$

$$= RT \ln 1$$

$$= 0$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For an adiabatic process,

$$dQ = 0$$

$$dU = -P\,dV$$

$$= -\frac{\alpha}{V^{\gamma}}\,dV \qquad (PV^{\gamma} = \alpha)$$

$$\Rightarrow \Delta U = \int dU = -\alpha \int_i^f \frac{dV}{V^{\gamma}}$$

$$= -\alpha \cdot \left(V_2^{-\gamma+1} - V_1^{-\gamma+1}\right) / (-\gamma+1)$$

$$= -\left[\frac{\alpha V_1^{-\gamma+1} - \alpha V_2^{-\gamma+1}}{\gamma - 1}\right]$$

$$= -\left[\frac{P_1V_1^{\gamma} \cdot V_1^{-\gamma+1} - P_2V_2^{\gamma}\, V_2^{-\gamma+1}}{\gamma-1}\right]$$

$$= -\left[\frac{P_1V_1 - P_2V_2}{\gamma-1}\right]$$

Use

$$\Delta H = \Delta U + \Delta(PV)$$

$$= \Delta U + [P_2V_2 - P_1V_1]$$

$$dS = \frac{dQ}{T} = 0 \qquad (\text{as } dQ = 0)$$

$$dH = T\,dS + V\,dP$$

$$= V\,dP$$

$$= \beta \frac{dP}{P^{\frac{1}{\gamma}}} \qquad [P^{\frac{1}{\gamma}} \cdot V = \beta]$$

$$\Rightarrow \Delta H = \int dH = \beta \int_i^f \frac{dP}{P^{\left(\frac{1}{\gamma}\right)}}$$

★ Remember only for ideal gas $U = f(T)$
otherwise $U = f(T, V)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$= \beta \frac{\left(P_2^{-\frac{1}{\gamma}+1} - P_1^{-\frac{1}{\gamma}+1}\right)}{\left[-\frac{1}{\gamma}+1\right]}$$

$$= \frac{P_2^{\frac{1}{\gamma}} . V_2 \; P_2^{-\frac{1}{\gamma}+1} - P_1^{\frac{1}{\gamma}} V_1 P_1^{-\frac{1}{\gamma}+1}}{-\frac{1}{\gamma}+1}$$

$$= \gamma \frac{[P_2V_2 - P_1V_1]}{(\gamma - 1)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q16) From 1^{st} law of thermodynamics,

$$dq = dU + PdV$$
$$= C_v dT + PdV \quad (\text{for ideal gas}) \quad —— (1)$$

Now change in entropy dS is defined as $dS = \left[\frac{dQ_{rev}}{T}\right]$ —— (2)

Let us consider a mole of ideal gas undergoing a reversible process s.t.

$$dS = \frac{dq}{T} = \frac{C_v dT}{T} + \frac{P}{T} dV$$
$$= C_V \left(\frac{dT}{T}\right) + \frac{R}{V} dV \quad —— (3)$$

⇒ Integrating

~~$S = C_v \ln T + R \ln V + S_0$~~

Now, for an ideal gas $PdV + VdP = RdT$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Using in ③,

$$dS = \frac{C_v}{T}\frac{(PdV + VdP)}{R} + \frac{R}{V}dV$$

$$= dV\left[\frac{C_v}{V} + \frac{R}{V}\right] + \frac{C_v V}{RT}.dP$$

$$= C_p\frac{dV}{V} + C_v\frac{dP}{P}$$

⇒ Upon integration, we get

$$\boxed{S = C_p \ln V + C_v \ln P + S_0}$$

Q17) 1^{st} of all we need to find out the final condition of Equilibrium.

Let us suppose whole ice melts and finally water is at $T°C$.

⇒ Heat lost by water $= 10 \times 4.2 \times (30 - T)$
$= 42(30 - T)$ kJ

Heat gained by ice $= 1 \times [336 + 4.2 \times T]$
$= 336 + 4.2T$

for equilibrium

$$42(30 - T) = 336 + 4.2T$$

$$\Rightarrow \boxed{T = 20°C}$$

Since $0 < T < 30$, hence our assumption is correct.

Now,

$$\Delta S_{system} = \Delta S_{water} + \Delta S_{ice}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Delta S_{water} = \int dS = +\int mc\frac{dT}{T} = +mc\ln\left(\frac{T_2}{T_1}\right)$$

$$= 10 * 4.2 * \ln\left(\frac{273+20}{273+30}\right)$$

$$= \underline{\underline{-1.409 \text{ kJ/k}}}$$

$$\Delta S'_{ice} = \int dS_1 + \int dS_2$$

$$= \int \frac{mL}{T} + \int mc\frac{dT}{T}$$

$$= \frac{336}{273} + 1 * 4.2 * \ln\left(\frac{273+20}{273}\right)$$

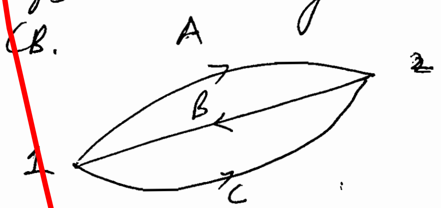
$$= +1.527 \text{ kJ/k}$$

$$\Rightarrow \Delta S_{total} = 0.118 \text{ kJ/k}$$
$$= \underline{\underline{118 \text{ J/k}}}$$

Q 18) <u>Proof of Entropy Change ≥ 0</u>

<u>Case I</u> : $\boxed{\text{Isolated System}}$

Consider any system.
Let the system undergo a cyclic process, AB and CB.



st. A, B are reversibly executed and C is irreversible.

From Claussius Inequality,

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\oint_{rev} \frac{dQ}{T} = 0$$

$$\Rightarrow \int_A \frac{dQ}{T} + \int_B \frac{dQ}{T} = 0$$

But from definition of entropy

$$\int_A \frac{dQ}{T} = S_2 - S_1$$

$$\Rightarrow \int_B \frac{dQ}{T} = -\int \frac{dQ}{T} = S_1 - S_2$$

Also from Claussius Inequality,

$$\oint_{irr.} \frac{dQ}{T} \leq 0$$

$$\Rightarrow \int_C \frac{dQ}{T} + \int_B \frac{dQ}{T} \leq 0$$

$$\Rightarrow \int_C \frac{dQ}{T} \leq -\int_B \frac{dQ}{T} = S_2 - S_1$$

$$\Rightarrow \boxed{S_2 - S_1 \geq \int_C \frac{dQ}{T}}$$ $\leftarrow$ general derivation for any system

Now for isolated system, $dQ = 0$

$$\Rightarrow \boxed{S_2 - S_1 \geq 0}$$ $\leftarrow$ specific for isolated system

Case II : Open System

Let heat flows from body at temp T_1 to

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

another body at temperature T_2

$$dS = \frac{dQ}{T_2} - \frac{dQ}{T_1} = dQ\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

Since heat flows from body at high temperature to body at lower temperature

$\Rightarrow \quad T_1 \geq T_2$

$\Rightarrow \quad \frac{1}{T_1} \leq \frac{1}{T_2} \Rightarrow \frac{1}{T_2} - \frac{1}{T_1} \geq 0$

$\Rightarrow \quad dS \geq 0$

$\Rightarrow \quad \boxed{S_2 - S_1 \geq 0}$

As proved, for a spontaneous system,

$$dS \geq \frac{dQ}{T}$$

$\Rightarrow \quad TdS \geq dQ = dU + PdV$

for constant Pressure $dU + PdV \Leftrightarrow d(U+PV)$

$\Rightarrow \quad TdS \geq d(U+PV) = dH$

$\Rightarrow \quad dH - TdS \leq 0$

For temperature Const, $dH - TdS \Leftrightarrow d(H-TS)$

$\Rightarrow \quad d(H-TS) \leq 0$

$\Rightarrow \quad \boxed{dG \leq 0}$ —— (1)

Now $G = H - TS$

$\Rightarrow \quad dG = VdP - SdT$ (Temp and Pressure being const)

$\Rightarrow \quad \boxed{dG = 0}$ —— (2)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

From ① and ②, we see that for a spontaneous reaction, G can ~~[illegible]~~ only decrease and finally becomes minimum at equilibrium state where Pressure and Temperature become constant.

Q19) Final temperature of the system will become equal to temperature of the reservior

$$\Rightarrow \quad T_2 (\text{system}) = T_2$$
$$T_1 (\text{system}) = T_1$$

$$dS_{system} = \frac{dQ_{gain}}{T} = C_p \cdot \frac{dT}{T}$$

$$\Rightarrow \Delta S_{system} = C_p \ln\left(\frac{T_2}{T_1}\right)$$

$$\Delta S_{reservoir} = \frac{dQ_{lost}}{T_2} = \frac{-C_p (T_2 - T_1)}{T_2}$$

Now $\Delta S_{universe} = \Delta S_{system} + \Delta S_{reservoir}$

$$= C_p \ln\left(\frac{T_2}{T_1}\right) + C_p \left(\frac{T_1}{T_2}\right) - C_p$$

$$= C_p \left[\frac{T_1}{T_2} - \ln\left(\frac{T_1}{T_2}\right)\right] - C_p$$

<u>Claim $\Delta S > 0$</u>

<u>Proof</u> Let $\eta = \frac{T_2}{T_1} > 1$ (as $T_2 > T_1$)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

let $f(\eta) = \ln \eta + \frac{1}{\eta} - 1$

Now $f(1) = 0$

Also $f'(\eta) = \frac{1}{\eta} - \frac{1}{\eta^2} = \left(\frac{\eta - 1}{\eta}\right)$

As $\eta > 1$

$\Rightarrow f'(\eta) > 0$

i.e. $f(\eta)$ is monotonically increasing function of η.

$\Rightarrow f(\eta) > f(1)$

$\Rightarrow f(\eta) > 0$

$\Rightarrow \ln \eta + \frac{1}{\eta} - 1 > 0$

$\Rightarrow \Delta S = C_p f(\eta) > 0$

Hence Proved.

The result is commensurate with the fact that the entropy of the universe always increases in an irreversible process. (The process is irreversible as the difference of temperature across which heat flows is finite, s.t. heat cannot flow in reverse direction)

Q20) Q 8 Tut 20

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q 21) (ii) <Q 5> (iii) Q - 18

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(ii) Boltzmann Relation derivation

The disorder is expressed in terms of "thermodynamic probability" Ω which gives the number of microstates possible for a given configuration of particles. The entropy is, infact, directly related to the disorder, greater the disorder, greater the entropy. To formulate relation between S and Ω, we consider the following:

Considering two independent systems with entropies S_1 and S_2 & thermodynamic probabilities Ω_1 and Ω_2, we see that entropy being an extensive property, the total entropy S is

$$S = S_1 + S_2 \quad \text{— (1)}$$

whereas for each microstate in system 1, Ω_2 microstates are possible in system 2,

$\therefore$ total microstate = total thermodynamic probability $= \Omega = \Omega_1 \Omega_2$

Let $S = f(\Omega)$

$\therefore S_1 = f(\Omega_1) \qquad S_2 = f(\Omega_2)$

$S_f = f(\Omega_1 \Omega_2)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

and $f(\Omega_1) + f(\Omega_2) = f(\Omega_1 \Omega_2)$

taking partial derivative w.r.t. Ω_1 and Ω_2,

$$f'(\Omega_1) = \Omega_2 f'(\Omega_1 \Omega_2)$$

$$f'(\Omega_2) = \Omega_1 f'(\Omega_1 \Omega_2)$$

$$\Rightarrow \Omega_1 f'(\Omega_1) = \Omega_2 f'(\Omega_2) = \Omega_1 \Omega_2 f'(\Omega_1 \Omega_2)$$

$$\Rightarrow \Omega f'(\Omega) = \text{const} = k_B \quad (\text{say})$$

(k_B is called Boltzmann Constant)

$$\Rightarrow df = k_B \frac{d\Omega}{\Omega}$$

$$\Rightarrow f(\Omega) = k_B \ln \Omega$$

$$\Rightarrow \boxed{S = k_B \ln \Omega}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

P V S T

Tut Sheet 22 : Thermodynamic Relationships

Q 1)

Consider a phase change process in which L amount of heat is absorbed, and volume of the phase$_1$ is V_1 and phase$_2$ be V_2.

From Maxwell Relation

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

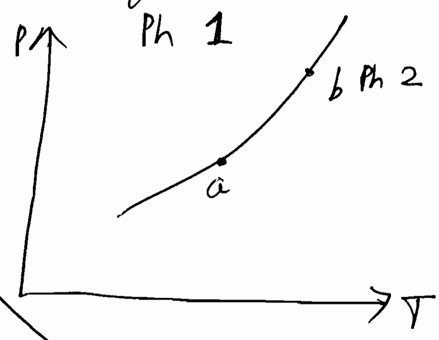
$$\Rightarrow \text{R.H.S.} = \left(\frac{dS}{dV}\right)_T = \left(\frac{T\,dS}{T\,dV}\right)_T$$

$$= \left(\frac{dQ}{T\,dV}\right)_T \approx \frac{L}{T\,\Delta V}$$

$$= \frac{L}{T(V_2 - V_1)}$$

$$\Rightarrow \boxed{\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}}$$

It can also be shown from Gibbs Relation



~~For transition from a to b, if both phases are in equilibrium~~

$\cancel{\Rightarrow dg_1 = dg_2}$

$\cancel{\Rightarrow (V\,dP - S\,dT)_1 = (V\,dP - S\,dT)_2}$

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

~~$\Rightarrow (S_2 - S_1)\, dT = (V_2 - V_1)\, dP$~~

~~$\Rightarrow \dfrac{dP}{dT} = \dfrac{S_2 - S_1}{(V_2 - V_1)} = \dfrac{L}{T(V_2 - V_1)}$~~

~~Hence, Proved.~~

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For two substance in reversible equilibrium at constant temperature at pressure, the gibbs function remains constant.

Hence for a change of phase (phase i to phase f) at T and P, the specific gibbs functions are equal for two states

$$g^{(i)} = g^{(f)}$$

Similarly for a phase change at $T+dT$, $P+dP$

$$g^{(i)} + dg^{(i)} = g^{(f)} + dg^{(f)}$$

Subtracting,

$$dg^{(i)} = dg^{(f)}$$

$$\Rightarrow -s^{(i)} dT + v^{(i)} dP = -s^{(f)} dT + v^{(f)} dP$$

$$\Rightarrow s^{(f)} - s^{(i)}\, dT = v^{(f)} - v^{(i)}\, dP$$

$$\Rightarrow \left(\frac{dP}{dT}\right) = \frac{s^{(f)} - s^{(i)}}{v^{(f)} - v^{(i)}} = \frac{L}{T(v^{(f)} - v^{(i)})}$$

★ Claussius Clapeyron equation relates change in Pressure dP & change in Boiling / Melting Point dT.

Q2)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Let us consider entropy S as a function of Temperature T and volume V

i.e. let $S = f(T, V)$

$$\Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_V . dT + \left(\frac{\partial S}{\partial V}\right)_T . dV$$

Differentiating with Temperature at constant Pressure,

$$\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T \cdot \left(\frac{\partial V}{\partial T}\right)_P$$

$$\Rightarrow \left(\frac{\partial S}{\partial T}\right)_P - \left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \cdot \left(\frac{\partial V}{\partial T}\right)_P$$

Multiplying by Temperature T

$$\Rightarrow T\left(\frac{\partial S}{\partial T}\right)_P - T\left(\frac{\partial S}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial V}\right)_T \cdot \left(\frac{\partial V}{\partial T}\right)_P$$

From Maxwell relation $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

Also using $T\left(\frac{\partial S}{\partial T}\right)_P = C_p$ and

$T\left(\frac{\partial S}{\partial T}\right)_V = C_v$, we get

$$\boxed{C_p - C_v = T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q3) Joule Thomson or Joule-kelvin Coefficient is defined as $\left(\frac{\partial T}{\partial P}\right)_H$ i.e. rate of change of Temperature with change in Pressure at constant enthalpy. Mathematically, expression for Joule kelvin Coefficient comes as

$$\mu = \left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

<derivation>

when C_P : Specific heat capacity at const. Pressure

For an ideal gas,

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}$$

$$\Rightarrow \mu = \frac{1}{C_P}\left[T \cdot \frac{R}{P} - V\right]$$

$$= \frac{1}{C_P}\left[\frac{PV}{P} - V\right] = \frac{1}{C_P}\left[V - V\right] = 0$$

<derivation>

Let enthalpy H be expressed as a function of T and P i.e. $H = f(T, P)$

$$\Rightarrow dH = \left(\frac{\partial H}{\partial P}\right)_T dP + \left(\frac{\partial H}{\partial T}\right)_P \cdot dT$$

For an isoenthalpic process $dH = 0$

$$\Rightarrow \left(\frac{\partial H}{\partial P}\right)_T dP = -\left(\frac{\partial H}{\partial T}\right)_P \cdot dT$$

Joule kelvin effect says tha[t] when a real gas experience[s] change of Pressure at constant enthalpy, it is accompan[ied] by a change i[n] Temperature. Joule-kelvin effect is expressed in terms of Joule-kelvin Coefficient, μ

$$\mu = \left(\frac{\partial T}{\partial P}\right)_H$$

P V S T

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \left(\frac{dT}{dP}\right)_H = -\frac{\left(\frac{\partial H}{\partial P}\right)_T}{\left(\frac{\partial H}{\partial T}\right)_P} \quad \text{--- } (1)$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

we know $H = U + PV$, by definition

$$\Rightarrow dH = TdS + VdP$$

replacing dH in (1), we get

$$\Rightarrow \left(\frac{dP}{dT}\right)_H = -\frac{1}{\left(\frac{TdS + VdP}{dT}\right)_P} \cdot \left(\frac{TdS + VdP}{dP}\right)_T$$

$$= -\frac{1}{\left(\frac{TdS}{dT}\right)_P} \cdot \left[T\left(\frac{\partial S}{\partial P}\right)_T + V\right]$$

$$= -\frac{1}{C_P}\left[T \cdot -\left(\frac{\partial V}{\partial T}\right)_P + V\right]$$

(from Maxwell equation)

$$= \frac{+1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

Q4) From Clausius Clapeyron equation

$$\frac{dP}{dT} = \left(\frac{L}{T(V_2 - V_1)}\right)$$

$$\Rightarrow \frac{dP}{dT} = \frac{L}{\left(\frac{1}{d_2}\right) - \left(\frac{1}{d_1}\right)} \quad \text{(as V is volume per unit mass)}$$

$$\Rightarrow dT = \frac{dP}{L} \cdot \left(\frac{1}{d_2} - \frac{1}{d_1}\right) T$$

$$\left[d_{water} = 1 g/cm^3 = 10^{-3} kg/cm^3 = 1 kg/l = 1000 kg/m^3 \right]$$

$$= -\frac{6}{76} \times \frac{10^5}{2.3 \times 10^6} \left[\frac{1}{0.6} - \frac{1}{1000} \right] \times 373$$

$$= 2.13\ k$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For boiling, temperature of boiling drops when pressure is reduced. Hence difficult to cook at hills as high Temp can't be reached.

Q5) <Refer copy>

Q6)
$$\mu = \left(\frac{dT}{dP}\right)_H = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

<derivation>

$$= \frac{1}{8.21 \times 4.2}\left[\cancel{T} \cdot \frac{V-b}{\cancel{T}}\left(1 + \frac{2a}{RTV}\right) - V\right]$$

$$= \frac{1}{8.21 \times 4.2}\left[\frac{2a}{RT} - b\right]$$

$$= \frac{1}{8.21 \times 4.2}\left[\frac{2 \times 1.39 \times 10^{-6} \times 10^5}{8314 \times 293} - 3.92 \times 10^{-5}\right]$$

$$= \underline{2.172 \times 10^{-6}}\ k/\text{Pascal}$$

<derivation>

To prove $\left(\frac{\partial V}{\partial T}\right)_P$ for real Vander Wall gas $= \frac{V-b}{T}\left[1 + 2a/RTV\right]$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For Vander Walls gas,

$$\left(P+\frac{a}{V^2}\right)(V-b) = RT$$

$$\left(\frac{\partial V}{\partial T}\right)_P = ?$$

$$P dV - \frac{a}{V^2} . dV + \frac{2ab}{V^3} dV = R dT$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P - \frac{a}{V^2} + \frac{2ab}{V^3}}$$

$$= \frac{R}{\left(P+\frac{a}{V^2}\right) - \frac{2a}{V^2} + \frac{2ab}{V^3}}$$

$$= \frac{R}{\frac{RT}{V-b} + \frac{2a}{V^2}\left[\frac{b}{V}-1\right]}$$

$$= \frac{R}{\left(\frac{RT}{V-b}\right)}\left[1 - \frac{\frac{2a}{V^2}\left(\frac{b}{V}-1\right)}{\left(\frac{RT}{V-b}\right)}\right]$$

$$\approx \frac{V-b}{T}\left[1 + \frac{2a}{V^2}\frac{V-b}{RT}\right]$$

$$\approx \frac{V-b}{T}\left[1 + \frac{2a}{RTV}\right]$$

(Q 7) To prove $\left(\frac{\partial U}{\partial V}\right)_T = 0$ for ideal gas

$\neq 0$ for real gas

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

From 1^{st} law

$$dU = dQ - PdV$$

$$= TdS - PdV$$

$$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P$$

$$\underline{\left(\frac{\partial U}{\partial V}\right)_T} = \underline{T\left(\frac{\partial P}{\partial T}\right)_V - P}$$ (from Maxwell Relation)

For ideal gas

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{R}{V}\right)$$

$$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T \cdot \frac{R}{V} - P = P - P = 0$$

Hence, internal energy independent of volume.

For a real gas, eg: VanderWall gas

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{R}{V-b}\right)$$

$$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T \cdot \frac{R}{V-b} - P = P + \frac{a}{V^2} - P$$

$$= \left(\frac{a}{V^2}\right) \neq 0$$

Hence internal energy depends upon volume.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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This is concomitant with the fact that for ideal gas, the molecules are point particles, they do not occupy volume and they do not interact. Their energy solely depends on Temperature while the real gas molecules occupy volume and interact.

(Q8) Physical Significance of drop in Helmholtz free energy and Gibbs Free energy

Decrease in Helmhold Free Energy (F), dF is given as

$F = U - TS$

$\Rightarrow dF = dU - TdS - SdT$

$= -PdV - SdT$

For Constant temperature i.e. isothermal process $\boxed{dF = -PdV}$

= Expansive Work done on the system

$\Rightarrow -(dF) = PdV$

$\therefore$ the drop in Helmholtz free energy is the amount of work done (expansive) by the system.

From application of Clausius Inequality, we know, for any process

$$\boxed{dS \geq \left(\frac{dQ}{T}\right)}$$

This is the relation to get inequalities in thermodynamic systems....

$$\Rightarrow TdS \geq dQ$$

Using 1^{st} law,

$$dW = dQ - dU \leq TdS - dU$$

define $F = U - TS$ [Helmholtz Free Energy]

$\therefore$ at constant temperature, $dF = dU - TdS$

$$\Rightarrow \boxed{dW \leq -dF}$$

$\therefore$ The maximum amount of work that can be done in an isothermal process is equal to decrease in the Helmholtz free energy F.

Let $dW = dA + PdV$.

where dA is the non expansive work.

$$\Rightarrow dA = dQ - dU - PdV$$
$$\leq TdS - dU - PdV$$

define $G = U + PV - TS$

at const. temp, Pressure,

$$dG = dU + PdV - TdS$$

$dQ \leq TdS$

$PdV + dU \leq TdS$

at const. Temp

$PdV + dF \leq 0$

$PdV \leq -dF$

Maximum expansive work

$dQ \leq TdS$

$dU + PdV + dW' \leq TdS$

At const. P

$dH + dW' \leq TdS$

at const T

$dG + dW' \leq 0$

$dW' \leq (-dG)$

Maximum non expansive work

कृपया इस स्थान में कुछ न लिखें।
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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$\Rightarrow dA \leq -dG$

$\therefore$ the maximum amount of non expansive (non PdV) work that can done by the system at constant temp T and pressure P is equal to the decrease in the Gibbs Free Energy, G.

Now F is defined as

$$F = U - TS \quad \text{—— (1)}$$

$$dF = dU - TdS - SdT$$
$$= -PdV - SdT$$

$$\Rightarrow \left(\frac{\partial F}{\partial T}\right)_V = -S \quad \text{—— (2)}$$

Using (2) in (1)

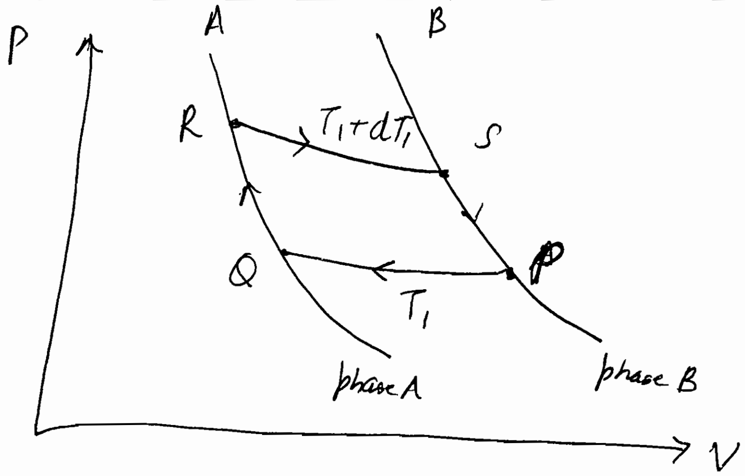
$$F = U + T.\left(\frac{\partial F}{\partial T}\right)_V$$

$$\Rightarrow \boxed{U = F - T\left(\frac{\partial F}{\partial T}\right)_V}$$

Q9) Let the two phases be represented in adjoining figure as isoentropic lines A and B for the two phases A and B.

Let phase change occur at Temp T_1 (isothermal process PQ) and the reverse

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)



Change of phase occur at temperature $T+dT_1$. Let the latent heat for 1st phase change be L_1 and second one be L_1+dL_1. The 4 process form Carnot Engine as can be seen PQRS.

From Carnot Engine,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \frac{L_1+dL_1}{L_1} = \frac{T_1+dT_1}{T_1}$$

$$\Rightarrow \left(\frac{dL_1}{L_1}\right) = \left(\frac{dT_1}{T_1}\right)$$

$$\Rightarrow dL_1 = \frac{L_1}{T_1}\, dT_1$$

Also Work Done $= Q_1 - Q_2 = \Delta P \Delta V$

$$\Rightarrow dL_1 \approx P\,dV$$

$$\Rightarrow dP(\Delta V) = \frac{L_1}{T_1}\, dT_1$$

$$\Rightarrow \frac{dP}{dT} = \frac{L_1}{\Delta V T_1} = \frac{L}{T(V_2-V_1)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

From the given data

$$L = T.dS = 273k * 22.2 \times 10^3 J/k$$
$$= 6.06 \times 10^6 J/k\text{-mole}$$

1 kilo mole of ice = 18 kg

$$\Rightarrow \boxed{L = 336 kJ}$$

Density of water = $\underline{\underline{1000\ kg/m^3}}$

density of ice = $\frac{9}{10} \times 1000 = \underline{\underline{900\ kg/m^3}}$

$$dP = P_2 - P_1 = 2\times10^5 - 1\times10^5 = 10^5 N/m^2$$
$$dT = ?$$

Now, from the desired Clausius Clapeyron equation,

$$\frac{10^5}{dT} = \frac{336\times10^3}{273\left[\frac{1}{1000} - \frac{1}{900}\right]}$$

$$\Rightarrow \boxed{dT = -9.02\times10^{-3}\ k}$$

Q10) Let the entropy of liquid and saturated vapour be S_ℓ and S_g. The change in entropy for the phase change can be written as

$$\Rightarrow S_g - S_\ell = \left(\frac{L}{T}\right)$$

$\Rightarrow$ differentiating wrt. Temp T.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\frac{dS_s}{dT} - \frac{dS_l}{dT} = \frac{1}{T}\left(\frac{dL}{dT}\right) \cancel{+} \left(\frac{L}{T^2}\right)$$

Multiplying by T,

$$\left(\frac{T\,dS_s}{dT}\right) - \left(\frac{T\,dS_l}{dT}\right) = \left(\frac{dL}{dT}\right) - \left(\frac{L}{T}\right)$$

$$\Rightarrow \left(\frac{dQ_s}{dT}\right) - \left(\frac{dQ_l}{dT}\right) = \left(\frac{dL}{dT}\right) - \left(\frac{L}{T}\right)$$

$$\Rightarrow \boxed{C_s - C_l = \left(\frac{dL}{dT}\right) - \left(\frac{L}{T}\right)}$$

$$C_s = C_l + \left(\frac{dL}{dT}\right) - \left(\frac{L}{T}\right)$$

$$= 1.01\ \text{cal/k.g} \cancel{+} 0.64\ \text{cal/k g} - \left(\frac{539}{373}\right)\ \text{cal/k.g}$$

$$= -\underline{\underline{1.075\ \text{cal}\ k^{-1}\ gm^{-1}}}$$

Negative value of C_s implies that heat needs to to removed in order the temperature. This is due to the fact that for increasing the ~~Press~~ Temperature of saturated vapour, it needs to be compressed i.e. Pressure must be increased. If Compression is done adiabatically, the saturated vapour would become super heated vapour. ∴ to convert to saturated vapour at higher temp, some amount of heat needs to be extracted out while compressing $\Rightarrow dQ < 0 \Rightarrow \boxed{C_s < 0}$

Q11)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

<Q2> : $C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$ — (1)

We can mathematically write Volume V as a function of Temp T and Pressure P

$$P = f(T, V)$$

$$dP = \left(\frac{\partial P}{\partial T}\right)_V dT + \left(\frac{\partial P}{\partial V}\right)_T dV$$

For constant volume $dV = 0$

$$\Rightarrow 0 = \left(\frac{\partial P}{\partial T}\right)_V dT + \left(\frac{\partial P}{\partial V}\right)_T dV$$

$$\Rightarrow \left(\frac{\partial P}{\partial V}\right)_T dV = -\left(\frac{\partial P}{\partial T}\right)_V dT$$

$$\Rightarrow \left(\frac{dV}{dT}\right)_P = -\frac{\left(\frac{\partial P}{\partial T}\right)_V}{\left(\frac{\partial P}{\partial V}\right)_T}$$

$$= -\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial P}\right)_T \quad \text{— (2)}$$

Putting (2) in (1)

$$C_p - C_v = \cancel{-T \left(\frac{\partial V}{\partial T}\right)_P \cdot \left(\frac{\partial V}{\partial T}\right)_P \cdot \left(\frac{\partial P}{\partial V}\right)_T}$$

$$\cancel{= -T \left(\frac{\partial V}{\partial T}\right)}$$

$$= -T \left(\frac{\partial p}{\partial T}\right)_V \cdot \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial p}\right)_T$$

$$= -T \cdot \frac{\partial p}{\partial T}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q11)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$C_p - C_v = T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P \quad \text{—①} \quad (Q2)$$

Let us write Volume V as function of Temperature and Pressure

$$V = f(T, P)$$

$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP$$

For constant volume $dV = 0$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_P dT = -\left(\frac{\partial V}{\partial P}\right)_T dP$$

$$\Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = -\left(\frac{\partial V}{\partial T}\right)_P \Big/ \left(\frac{\partial V}{\partial P}\right)_T$$

$$= -\left(\frac{\partial V}{\partial T}\right)_P \cdot \left(\frac{\partial P}{\partial V}\right)_T \quad \text{—②}$$

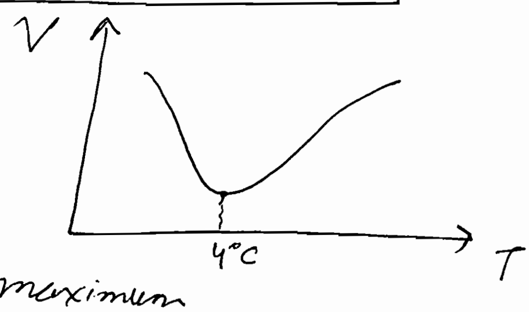
Using ② in ①, we get

$$\Rightarrow C_p - C_v = -T \cdot \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial V}\right)_T \cdot \left(\frac{\partial V}{\partial T}\right)_P$$

$$\boxed{C_p - C_v = -T \cdot \left(\frac{\partial V}{\partial T}\right)_P^2 \cdot \left(\frac{\partial P}{\partial V}\right)_T}$$

At 4°C,

$$\left(\frac{dV}{dT}\right) = 0$$



as the density is maximum

$$\Rightarrow \underline{C_p = C_v}$$

$P dV + V dP = 0$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For an ideal gas

$$C_p - C_v = -T \cdot \left(\frac{\partial V}{\partial T}\right)_P^2 \left(\frac{\partial P}{\partial V}\right)_T$$

$$= +T \cdot \left(\frac{R}{P}\right)^2 \left(\frac{P}{V}\right)$$

$$= \frac{T \cdot R^2}{PV} = \underline{\underline{R}}$$

For a real gas, say Van der Waals' gas

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{V-b}{T} \cdot \left[1 + \frac{2a}{RTV}\right]$$ Q-6

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{R}{V-b}\right)$$

$$\Rightarrow C_p - C_v = \cancel{T} \cdot \frac{R}{\cancel{V-b}} \cdot \frac{\cancel{V-b}}{\cancel{T}} \cdot \left[1 + \frac{2a}{RTV}\right]$$

$$\boxed{C_p - C_v = R + \left(\frac{2a}{VT}\right)}$$

$$R + \frac{2a}{VT} > R$$

$$\Rightarrow (C_p - C_v)_{\text{real gas}} > (C_p - C_v)_{\text{ideal gas}}$$

Q12) Q 3

Q13) From Q 2) $C_p - C_v = \underline{T\left(\frac{\partial p}{\partial T}\right)_V} \left(\frac{\partial V}{\partial T}\right)_P$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

~~Let~~ $\cancel{U = f(T,V)}$

$$\cancel{dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T \cdot dV}$$

$$\cancel{= \left(\frac{dQ - PdV}{dT}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV}$$

$$\cancel{= T\left(\frac{dS}{dT}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV}$$

$$\Rightarrow C_p - C_v = T\left(\frac{\partial S}{\partial V}\right)_T \cdot \left(\frac{\partial V}{\partial T}\right)_P$$

$$= \left(\frac{dU + PdV}{dV}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$$

$$= \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]\left[\frac{\partial V}{\partial T}\right]_P$$

$$R.H.S. = \left(p + \frac{a}{V^2}\right)\cdot\left(\frac{V-b}{T}\right)\left[1 + \frac{2a}{RTV}\right]$$

$$= R\left[1 + \frac{2a}{RTV}\right]$$

$$\Rightarrow C_p = C_v + R + \left(\frac{2a}{TV}\right)$$

Q14) From Clausius Clapeyron equation,

$$\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$$

$$\Rightarrow \frac{dP}{-0.91} = \frac{-80 \times 4.2}{(0.091) \times 10^{-6} \times 273}$$

$$\Rightarrow \boxed{dP = 123 \text{ atm}}$$

★ water → ice
⇒ L negative
$V_2 - V_1 > 0$

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Q1) According to Curie's law for Paramagnetic Materials, the susceptibility χ of a paramagnetic Material is inversely proportional to temperature

i.e. $\chi = \frac{C}{T}$ where C: const.

$\Rightarrow \chi_1 T_1 = \chi_2 T_2$

At very low temperatures, its very difficult to measure Temperature by any Ordinary means. Here this magnetic relation is used to define Temperature

$$\boxed{T_1 = \frac{\chi_{ref} T_{ref}}{\chi_1}}$$

χ_1 is measured through measurement of inductance.

When an external magnetic field is applied, dipoles are aligned in line with it. As soon as demagnetization is done adiabatically, the aligned dipole's Potential energy is lost & dipole are de aligned. The loss of internal energy results in drop of Temperature.

In Order to find Mathematical expression for drop in Temperature, Consider entropy as function of Temperature T and applied Magnetic Field, H

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

i.e. $S = f(T, H)$

$$\Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_H dT + \left(\frac{\partial S}{\partial H}\right)_T dH$$

$$\Rightarrow TdS = \left(\frac{dQ}{dT}\right)_H dT + T\left(\frac{\partial S}{\partial H}\right)_T dH$$

$$= C_H\, dT + T\left(\frac{\partial S}{\partial H}\right)_T dH$$

where C_H is heat capacity at constant external magnetic field.

Also Work done PdV is equivalent to Work Magnetic $-HdM$

$\therefore$ writing equivalent Maxwell equation

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \quad \text{as}$$

$$\left(\frac{\partial S}{\partial H}\right)_T = +\left(\frac{\partial M}{\partial T}\right)_H$$

$\boxed{\text{This is derived from Gibbs Equation}}$

$$\Rightarrow TdS = C_H\, dT + T\left(\frac{\partial M}{\partial T}\right)_H dH$$

From Curies law $M = \frac{C}{T}\cancel{H}$

$$\Rightarrow \left(\frac{\partial M}{\partial T}\right)_H = -\frac{CH}{T^2}$$

$$\Rightarrow \boxed{TdS = C_H\, dT - \frac{CH}{T}\, dH}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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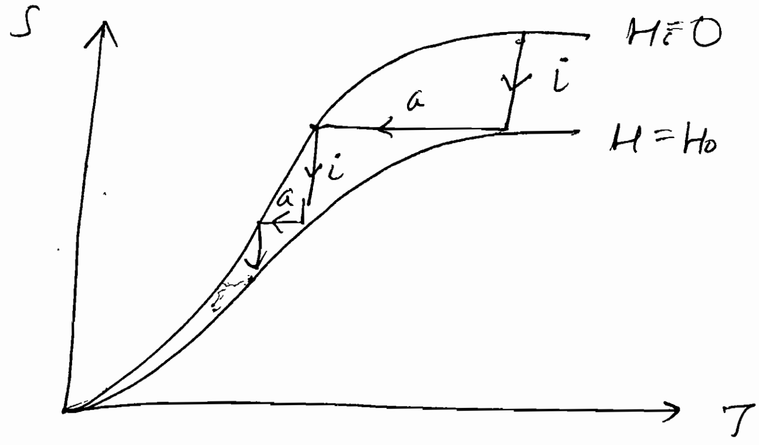

a : adiabatic demagnetization

i : isothermal magnetization

For adiabatic demagnetization, $dS = 0$

$$\Rightarrow C_H\, dT - \frac{CH}{T} dH = 0$$

$$\Rightarrow C_H\, dT = \frac{CH\, dH}{T}$$

$$\Rightarrow T\, dT = \left(\frac{C}{C_H}\right) H\, dH$$

$\Rightarrow$ Integrating both sides

$$\int_{T_1}^{T_2} T\, dT = \frac{C}{C_H} \int_H^0 H\, dH$$

$$\Rightarrow \frac{T_2^2 - T_1^2}{2} = \left(\frac{C}{C_H}\right) \cdot \frac{-H^2}{2}$$

$$\Rightarrow T_2 - T_1 \approx -\frac{C}{C_H} \frac{H^2}{2T} \qquad (T_1 + T_2 \simeq 2T)$$

$$\Rightarrow \boxed{T_2 = T_1 - \frac{C}{C_H} \frac{H^2}{2T}}$$

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At higher Temperature, drop in temperature $\frac{CH^2}{C_H 2T}$ is not significant

Hence this cooling works better for lower temperatures.

(Q2,3,4) Q1

(Q5) Correction for a Vander Waal Gas need to be two fold

① Finite size of molecule

Since volume of real gas particles $\neq 0$

$\Rightarrow$ space for their movement or interaction is reduced

$\Rightarrow V_{real} = (V_{available} - b)$

b: correction for volume of molecules.

② Intermolecular Attraction

Vander Wall found out that $\left(\frac{\partial U}{\partial V}\right)$ for a real gas is not zero,

Rather $\left(\frac{\partial U}{\partial V}\right) \approx \frac{A}{r^6} - \frac{B}{r^{12}}$

He consider $\left(\frac{\partial U}{\partial V}\right) = \frac{a}{V^2}$

Note that $\left(\frac{\partial U}{\partial V}\right) = \frac{\partial U}{A.\partial r} = \frac{F}{A} = P$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$\Rightarrow$ Pressure correction $= P_{observed} + \dfrac{a}{V^2}$

After applying the corrections,

$$\boxed{\left(P + \frac{a}{V^2}\right)(V - b) = RT}$$

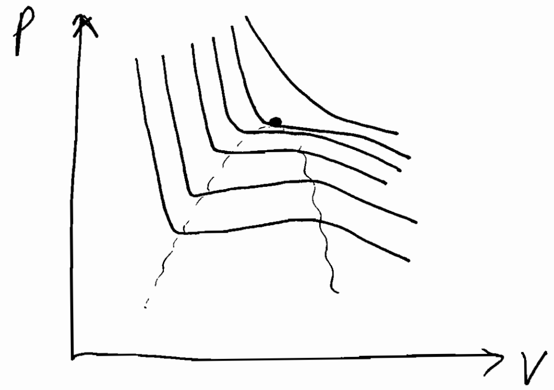
Expanding Vander Wall equation,

$$(PV^2 + a)(V - b) = RTV^2$$

$$PV^3 - (Pb + RT)V^2 + aV - ab = 0$$

$$\Rightarrow V^3 - \left(b + \frac{RT}{P}\right)V^2 + a\frac{V}{P} - \frac{ab}{P} = 0 \quad \text{—— (1)}$$

Also from liquification of gases curve



At critical point all the three roots of Vander Wall gas converge at V_c

$\Rightarrow (V - V_c)^3 = 0$ is equivalent equation of cubic equation (1)

$$\Rightarrow V^3 - V_c^3 - 3VV_c(V - V_c) = 0$$

$$\Rightarrow V^3 - 3V^2V_c + 3V_c^2V - V_c^3 = 0 \quad \text{—— (2)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$V_c^3 = \frac{ab}{P_c} \quad \text{— (3)}$$

$$3V_c^2 = \frac{a}{P_c} \quad \text{— (4)}$$

$$3V_c = b + \frac{RT_c}{P_c} \quad \text{— (5)}$$

$$\Rightarrow \frac{V_c}{3} = b \Rightarrow \boxed{V_c = 3b}$$

$$P_c = \frac{3V_c^2}{a} = \frac{3 \cdot 9b^2}{a} = \frac{27b^2}{a} \Rightarrow \boxed{P_c = \frac{a}{27b^2}}$$

$$T_c = \frac{(3V_c - b)P_c}{R} = \frac{8b}{R} \cdot \frac{a}{27b^2}$$

$$\Rightarrow \boxed{T_c = \frac{8a}{27Rb}}$$

$$\text{Critical Coefficient} = \frac{RT_c}{P_cV_c}$$

$$= \frac{\cancel{R} * \frac{8a}{27\cancel{R}\cancel{b}}}{\frac{a}{\cancel{27}\cancel{b^2}} * 3\cancel{b}}$$

$$= \left(\frac{8}{3}\right)$$

Since its const., its independent of the gas taken !!

Q8)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(a) Q5

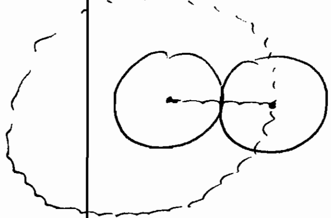
(b) In terms of Critical Parameters,

as $V_c = 3b$ & $P_c = \frac{a}{27b^2}$

$$\Rightarrow \boxed{b = \frac{V_c}{3}} \qquad a = 27 \cdot \frac{V_c^2}{9} \cdot P_c$$

$$\boxed{a = 3V_c^2 P_c}$$

We can also write b as $4Nv$ where v is volume ocupied by 1 molecule N : Avogadro's number.



$$N \frac{V_{2R}}{2}$$

$$\frac{N 8V}{2}$$

$$= 4NV$$

Also $T_c = \frac{8a}{27Rb}$

$$\Rightarrow a = \frac{27Rb\,T_c}{8}$$

$$= \frac{27 R T_c V_c}{3 \times 8}$$

$$\boxed{a = \frac{9RT_cV_c}{8}}$$

(c) From given Parameters,

$$V_c = \frac{3}{8}\frac{RT_c}{P_c} = \frac{3}{8} \times \frac{80 \times 10^{-6} \times 10^5 \times 154.2}{49.7 \times 10^5}$$

$$= 9.3 \times 10^{-5} \text{ m}^3$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow b = \frac{V_c}{3} = 3.1 \times 10^{-5} \ m^3$$

$$a = 3 V_c^2 P_c$$

$$= 3 \times 9.3^2 \times 10^{-15} \times 49.7 \times 10^5$$

$$= 1.28 \times 10^{-6} \ m^6 \ \text{Pascal}$$

Q6 For a closed PVT system, from the combined 1st and 2nd law, we get

$$dU = TdS - PdV \quad —① $$

$$\Rightarrow U \equiv U(S, V)$$

$$\therefore dU = \left(\frac{\partial U}{\partial S}\right)_V . dS + \left(\frac{\partial U}{\partial V}\right)_S . dV \quad —②$$

From ① & ②,

$$\Rightarrow \left(\frac{\partial U}{\partial S}\right)_V = T \qquad \left(\frac{\partial U}{\partial V}\right)_S = -P$$

The internal energy U is an extensive property and is proportional to number of moles included in the system.

When the system is open, so that we can add ~~to~~ remove material, internal energy becomes a function of 'n' along with S and V

$$dU = \left(\frac{\partial U}{\partial S}\right)_{V,n} dS + \left(\frac{\partial U}{\partial V}\right)_{S,n} dV + \left(\frac{\partial U}{\partial n}\right)_{S,V} . dn$$

Tut Sheet 24 : Specific Heat of Solids

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

The coefficient of dn is defined as chemical potential μ,

$$\mu = \left(\frac{\partial U}{\partial n}\right)_{S,V}$$

Thus Chemical Potential is change of internal energy per mole. Hence, it is ~~in~~tensive as it is per mole.

Since the change is at constant S and V, it is dependent only on remaining thermodynamic parameters viz, Pressure P and Temperature T.

You can do this question better by taking G instead of U

$$\Rightarrow \mu = \left(\frac{\partial G}{\partial n}\right)_{P,T}$$

Since μ is defined for a particular value of P and T, Hence it is a function of these two parameters.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q1) Thermal Energy E, is given as

$$E = \int_0^{\nu_m} \frac{h\nu \; g(\nu)\, d\nu}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$= \int_0^{\nu_m} \frac{h\nu \, .}{\left(e^{\frac{h\nu}{k_B T}} - 1\right)} \frac{6Nh^2\nu}{(k_B \theta_D)^2} \, d\nu$$

$$\boxed{\text{Put } \frac{\cancel{h}\nu}{k_B T} = x} \qquad \text{Also } \boxed{\theta_D = \frac{\nu_m h}{k_B}}$$

$$= \frac{6Nh^3}{(k_B \theta_D)^2} \left(\frac{k_B T}{h}\right)^3 \int_0^{\frac{\theta_D}{T}} \frac{x^2 \, dx}{(e^x - 1)}$$

$$= \frac{6N . k_B^3 T^3}{k_B^2 . \theta_D^2} \int_0^{\frac{\theta_D}{T}} \frac{x^2}{e^x - 1} dx$$

$$\boxed{E = 6R . \frac{T^3}{\theta_D^2} \int_0^{\frac{\theta_D}{T}} \frac{x^2}{e^x - 1} dx}$$

for $T << \theta_D$ $\quad \frac{\theta_D}{T} \to \infty$

$$\Rightarrow E \approx 6R \frac{T^3}{\theta_D^2} \int_0^{\infty} \frac{x^2}{e^x - 1} dx$$

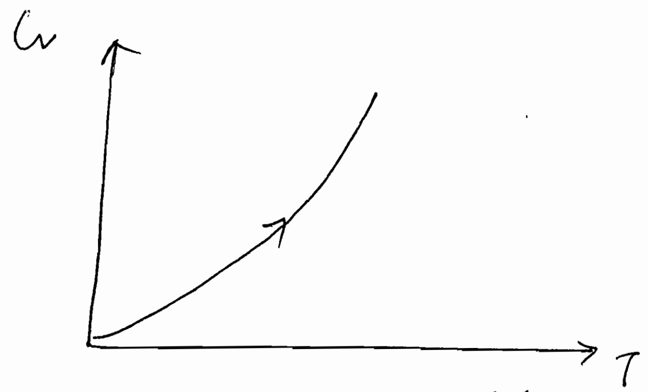
$$= 2.404 * 6 R \left(\frac{T^3}{\theta_D^2}\right)$$

$$\Rightarrow E = \left[\frac{14.424\,R}{\theta_D^3}\right] \cdot T^3$$

Now $C_v = \dfrac{dE}{dT}$

$$= \frac{14.424\,R}{\theta_D^3} \cdot 3T^2$$

$$= \left(\frac{43.272}{\theta_D^3}\right) T^2$$



Parabolic variation. (low temperature)

Q 2) (30 marks)

Einstein's theory of specific heat was a major improvement over Classical theory of Dulong and Petit, as it explained that C_v drops with temperature. Instead of a classical approach, Einstein followed quantum theory to derive specific heat. He assumed :

(1) All molecules are Planck Oscillators vibrating quantum mechanically with

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

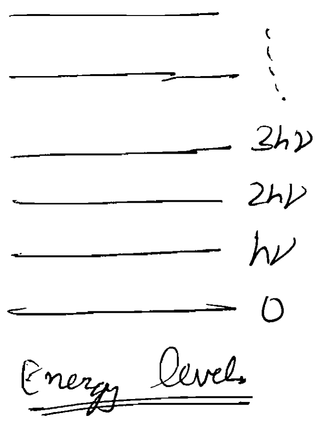
a frequency ν, characteristic of a material.

(2) Average energy per mode of vibration is given as $\left(\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}\right)$

(3) Total number of modes of vibration are 3N. as each molecule has 3 degrees of freedom.

(4) Maxwell distribution of energy levels hold. Energy levels are discrete in units of $nh\nu$.

<u>Claim</u> $\langle E \rangle = \left(\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}\right)$



<u>Proof</u>

According to Plank's theory, energy levels are discrete in unit of $nh\nu$.

Also from ~~Plank's~~ Maxwell distribution, Molecules in n^{th} energy level are ~~[illegible]~~

$$N_n = N_0 e^{-\left(\frac{nh\nu}{kT}\right)}$$

$$\Rightarrow N = N_0 + N_1 + N_2 + \ldots = N_0\left[1 + e^{-\frac{h\nu}{kT}} + e^{-\frac{2h\nu}{kT}} + \ldots\right]$$

$$= \left(\frac{N_0}{1-e^{-\frac{h\nu}{kT}}}\right)$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Also, total energy is

$$E = N_0\epsilon_0 + N_1\epsilon_1 + N_2\epsilon_2 + \ldots$$

$$= N_0 h\nu\, e^{-\frac{h\nu}{kT}} + 2N_0 h\nu\, e^{-\frac{2h\nu}{kT}} + \ldots$$

This is AG series,

$$\Rightarrow E = \frac{N_0 h\nu.\, e^{-\frac{h\nu}{kT}}}{\left(1 - e^{-\frac{h\nu}{kT}}\right)^2}$$

Now the average energy per molecule per degree of freedom is

$$\Rightarrow \langle\epsilon\rangle = \frac{E}{N}$$

$$= \frac{h\nu e^{-\frac{h\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}}$$

$$\boxed{\langle\epsilon\rangle = \left[\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}\right]}$$

Using these assumptions, Einstein proposed total energy of system of molecules E, is given as for 1 gram mole of solid

$$E = 3N * \langle\epsilon\rangle$$

$$= \frac{3N h\nu}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Now specific heat of solid C_v is defined as,

$$C_v = \left(\frac{dE}{dT}\right)$$

$\therefore$ C_v in einstein theory is given as,

$$C_v = + \frac{3Nh\nu}{\left(e^{\frac{h\nu}{kT}} - 1\right)^2} \cdot e^{\frac{h\nu}{kT}} \cdot \frac{h\nu}{kT^2}$$

Put $\frac{h\nu}{k} = \theta_E$: Einstein Temperature

$$\Rightarrow C_v = 3Nk \cdot \left(\frac{h^2\nu^2}{k^2T^2}\right) \cdot \frac{e^{\frac{\theta_E}{T}}}{\left(e^{\frac{\theta_E}{T}} - 1\right)^2}$$

$$\boxed{C_v = 3R \left(\frac{\theta_E}{T}\right)^2 \frac{e^{\frac{\theta_E}{T}}}{\left(e^{\frac{\theta_E}{T}} - 1\right)^2}}$$

For $T >> \theta_E$

$$\frac{\theta_E}{T} << 1 \Rightarrow e^{\frac{\theta_E}{T}} - 1 \approx \left(\frac{\theta_E}{T}\right)$$

$$\Rightarrow C_v = 3R \cdot \left(\frac{\theta_E}{T}\right)^2 \cdot \frac{\left(1 + \frac{\theta_E}{T}\right)}{\left(\frac{\theta_E}{T}\right)^2}$$

$$\boxed{C_v \simeq 3R}$$

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$\therefore$ for higher Temperature, Einstein specific heat is equal to Dulong & Petit specific heat.

At lower temp $T << \theta_E$

$$\frac{\theta_E}{T} >> 1$$

$$\Rightarrow C_v = 3R \cdot \left(\frac{\theta_E}{R}\right)^2 \cdot \frac{e^{\frac{\theta_E}{R}}}{\left(e^{\frac{\theta_E}{T}}\right)^2}$$

$$= 3R \cdot \left(\frac{\theta_E}{R}\right)^2 \frac{1}{1 + \left(\frac{\theta_E}{T}\right) + \left(\frac{\theta_E}{T}\right)^2 + \cdots}$$

$$\approx 0$$

Hence for lower temperature, $C_v \to 0$.

In Einstein theory, C_v falls much faster as compared to observed behaviour.

$\boxed{\text{Shortcomings}}$

(1) Einstein assumed all molecules to have same frequency. This is not a good assumption.

(2) θ_E or Einstein Temperature is difficult to measure.

(3) At low temperature, C_v is found to vary as T^3 with Temperature T. This is not proved by Einstein theory.

Q3)

कृपया इस स्थान में कुछ न लिखें।
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<(Q2)>

$$C_v = 3R\left(\frac{\theta_E}{T}\right)^2 \cdot \frac{e^{\frac{\theta_E}{T}}}{\left(e^{\frac{\theta_E}{T}}-1\right)^2}$$

For $T = T_E/2$ or $(\theta/2)$, we have

$$C_v = 3R\cdot(2)^2\cdot\frac{e^2}{(e^2-1)^2} = \underline{\underline{2.17\,R}}$$

Q4) The expression for specific heat are as:

Einstein: $$C_v = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{e\left(\frac{\theta_E}{T}\right)}{\left[e\left(\frac{\theta_E}{T}\right)-1\right]^2}$$

Debye: $$C_v = 9R\left[4\left(\frac{T}{\theta_D}\right)^3\int_0^{\frac{\theta_D}{T}}\frac{x^3}{e^x-1}dx - \left(\frac{\theta_D}{T}\right)\frac{1}{e^{\theta_D/T}-1}\right]$$

1st expression सब बड़ा बड़ा
4
Power : 3
बड़ा integral

2nd expression
- no integer
- no Power
- no integral

$$x = \left(\frac{h\nu}{KT}\right)$$

$$\theta_E = \left(\frac{h\nu}{k}\right)$$

$$\theta_D = \left(\frac{h\nu_m}{k}\right)$$

Remember:

$$E_{ein} = 3N\frac{h\nu}{\left(e^{\frac{h\nu}{KT}}-1\right)}$$

$$E_{debye} = \frac{9RT^4}{\theta_D^3}\int_0^{\left(\frac{\theta_D}{T}\right)}\frac{x^3}{e^x-1}dx$$

Difference between Debye and Einstein theory are:

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

① Einstein assume single frequency with with all molecules are said to be vibrating in harmonic Oscillations.

while

Debye considered a spectrum of frequency of vibration varying from 0 to ν_m.

② No. of degrees of freedom for eistein are 3

while

In Debye, no of modes of vibration are frequency dependent.

For lower temperature region, Debye's specific heat is more accurate.

For lower temperature $T \ll \theta_D$

$$\Rightarrow \frac{\theta_D}{T} \gg 1$$

$$\Rightarrow C_v \simeq 9R\left[4\left(\frac{T}{\theta_D}\right)^3 \int_0^\infty \frac{x^3}{e^x - 1} \cdot dx - \left(\frac{\theta_D}{T}\right) \frac{1}{e^{\frac{\theta_D}{T}}}\right]$$

$$\simeq 9R\left[4 \cdot \frac{T^3}{\theta_D^3} \cdot \lambda\right]$$

where $\lambda = \int_0^\infty \frac{x^3}{e^{x} - 1}$

$$\simeq \frac{36 R \lambda}{\theta_D^3} T^3$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

③ While Einstein assumed, vibrations of atoms are independent of each other, Debye postulated that the vibrations of atomic oscillators are strongly coupled with each other

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Delta Q_{required} = \int C_v dT$$

$$= \frac{36R\lambda}{\theta_D^3} \cdot \int_{T_1}^{T_2} T^3 dT$$

$$= \frac{36R\lambda}{\theta_D^3} \cdot \frac{(T_2^4 - T_1^4)}{4}$$

$$\boxed{\Delta Q = \frac{9R\lambda}{\theta_D^3}(T_2^4 - T_1^4)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q5) Debye's theory was an improved upon Einstein's theory, Debye assumed that:

i) Molecular oscillations are not independent of each ~~but~~ but rather strongly coupled with each other.

ii) There is no single frequency but rather a continuum of frequencies with which the molecules vibrate with a maxima of ν_m.

iii) Total number of modes of vibration are 3N

From phase space, we know no of modes of vibration for momentum p, is given as

$$g(p)\,dp = \frac{2\pi V}{h^3} 4\pi p^2 dp$$

Assuming $p = \frac{h\nu}{v}$, we have

$$g(\nu)\,d\nu = \frac{h^2 2\pi V 4\pi}{h^3}\left[\frac{1}{v_L^2} + \frac{2}{v_T^2}\right] d\nu = A\nu^2 d\nu$$

where v_L and v_T are longitudinal &

(*) Note that both Einstein & Debye theory use maxwell Boltzmann distribution !!!

of elastic waves

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

transverse velocities. — Factor of 2 due to 2 modes of polarization.

But total modes of vibration are 3N

$$\Rightarrow 3N = \int_0^{\nu_m} A \cdot \nu^2 \, d\nu$$

$$\Rightarrow 3N = \frac{A \cdot \nu_m^3}{3} \Rightarrow A = \frac{9N}{\nu_m^3}$$

Hence $g(\nu)\, d\nu = \frac{9N}{\nu_m^3} \nu^2 \, d\nu$

Total energy is given as $\int g(\nu)\, d\nu \, E(\nu)$

Now from Planck theory and Maxwell distribution,

$$E(\nu) = \frac{h\nu}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

$$\Rightarrow E = \int_0^{\nu_m} \frac{9N}{\nu_m^3} \nu^2 \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \, d\nu$$

$$= \frac{9Nh}{\nu_m^3} \int_0^{\nu_m} \frac{\nu^3 \, d\nu}{e^{\frac{h\nu}{kT}} - 1}$$

Put $\boxed{\frac{h\nu}{kT} = x}$ and $\boxed{\frac{h\nu_m}{k} = \theta_D}$

$$= \frac{9Nh}{\nu_m^3}\left(\frac{kT}{h}\right)^4 \int_0^{\frac{\theta_D}{T}} \frac{x^3 \, dx}{e^x - 1} = \frac{9RT^4}{\theta_D^3} \int_0^{\frac{\theta_D}{T}} \left(\frac{x^3}{e^x - 1}\right) dx$$

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Now to calculate C_v, we must know Liebnitz rule

$$\frac{d}{dx}\int_{g(x)}^{f(x)} \lambda(y)\,dy = \lambda(f(x))\,f'(x) - \lambda(g(x))\,g'(x) \quad \text{—} (1)$$

$$\Rightarrow C_v = \frac{dE}{dT}$$

$$= \frac{9R}{\theta_D^3}\cdot\left[4T^3\int_0^{\theta_D/T}\frac{x^3}{e^x-1}dx + T^4\cdot\frac{d}{dT}\int_0^{\frac{\theta_D}{T}}\frac{x^3}{e^x-1}dx\right]$$

Using (1)

$$= \frac{9R}{\theta_D^3}\left[4T^3\cdot\int_0^{\frac{\theta_D}{T}}\frac{x^3}{e^x-1}dx + T^4\cdot\frac{\left(\frac{\theta_D}{T}\right)^3}{e^{\frac{\theta_D}{T}}-1}\cdot\frac{-\theta_D}{T^2}\right]$$

$$= 9R\left[4\left(\frac{T}{\theta_D}\right)^3\int_0^{\frac{\theta_D}{T}}\frac{x^3}{e^x-1}dx - \left(\frac{\theta_D}{T}\right)\left(\frac{1}{e^{\frac{\theta_D}{T}}-1}\right)\right]$$

At $\boxed{T \gg \theta_D}$, we have

$$\frac{\theta_D}{T} \ll 1 \qquad e^{\frac{\theta_D}{T}} = 1 + \left(\frac{\theta_D}{T}\right)$$

$$\Rightarrow C_v \simeq 9R\left[4\left(\frac{T}{\theta_D}\right)^3\cdot\int_0^{\frac{\theta_D}{T}}x^2\,dx - 1\right]$$

$$= 9R\left[\frac{4}{3}\cdot\frac{T^3}{\theta_D^3}\cdot\left(\frac{\theta_D}{T}\right)^3 - 1\right]$$

$$= 9R\cdot\frac{1}{3} = \underline{\underline{3R}}$$

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Hence at higher temperature, it converges to dulong & petit law.

At $\boxed{T << \theta_D}$

$\frac{\theta_D}{T} >> 1$

$$\Rightarrow C_v \simeq 9R\left[4\left(\frac{T}{\theta_D}\right)^3 \cdot \int_0^\infty \frac{x^3}{e^x - 1}\,dx - \frac{1}{\infty}\right]$$

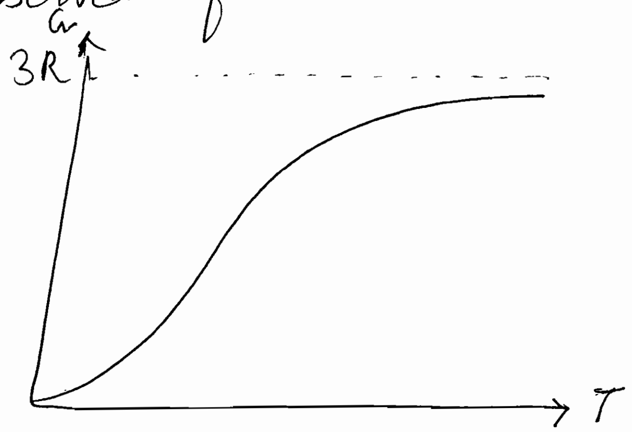
$$= 9R \cdot 4 \cdot \frac{T^3}{\theta_D^3} \cdot \lambda$$

$$= \left(\frac{36 R\lambda}{\theta_D^3}\right) \cdot T^3$$

$$\approx A T^3$$

Hence at lower temperature, the expression converges to T^3 law

$\therefore$ for both high and low temperature, debye's theory is concomitant with the observed facts.



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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

However there are some shortcomings in the theory :

1. electronic contribution is not taken

2. There is no theoretical prediction of θ_D, and moreover it is not const. with temperature, but varies widely with Temperature, except at very low Temperature.

3. The theory is valid only for isotropic solids

Q6) Given $C(T) = aT + bT^2$

Avg. specificheat $= \dfrac{\int_0^T C_v \, dt}{T}$

$$= \frac{\int_0^T (aT + bT^2) \, dT}{T}$$

$$= \frac{\left(a\frac{T^2}{2} + b\frac{T^3}{3} \right)}{T}$$

$$= \left(\frac{aT}{2} \right) + \left(\frac{bT^2}{3} \right)$$

At mid temperature $\frac{T}{2}$, $C\left(\frac{T}{2}\right) = \left(a\frac{T}{2}\right) + \left(\frac{bT^2}{4}\right)$

$\therefore$ avg. specificheat is more than mid point specific heat

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q7)

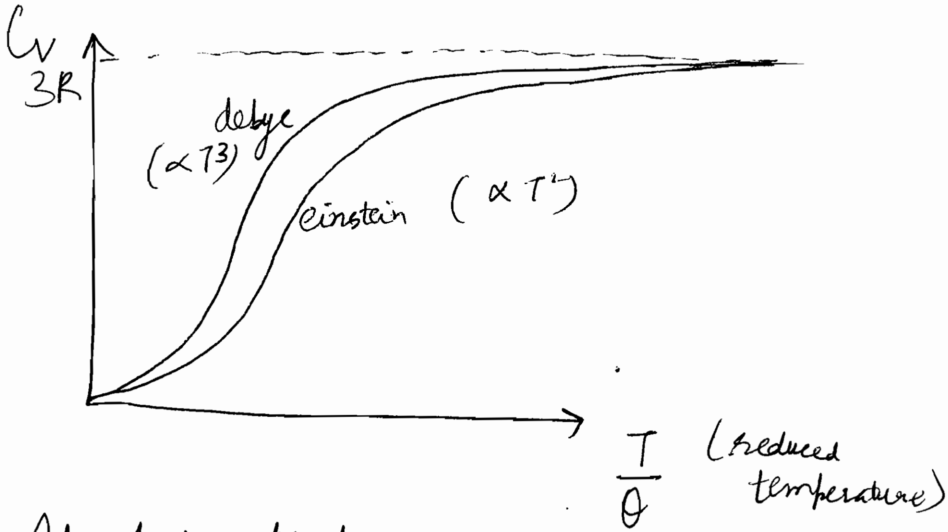
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q8)

<Q-2>

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

<Q4>



→ At higher temperature both the curves converge at 3R, which is the Dulong & Petit law for higher Temperature.

→ For lower temperature Einstein specific heat drops exponentially while the debye's specific heat varies ~~at~~ cubically T^3.

"Law of corresponding states" says that while the plots of C_V versus T may be different for different metals, however, if we ~~plot~~ C_V versus reduced Temperature $\frac{T}{\theta}$ $\left[\frac{T}{\theta_E}\right.$: einstein and $\left.\frac{T}{\theta_D} : \text{debye}\right]$, the graph is strikingly similar for almost all metals. This is due to the fact that both Debye and Einstein specific heats are of form $C_V = f\left(\frac{T}{\theta}\right)$

(*) 2 different solids at different temperature T_1, T_2 but different reduced Temperature $\frac{T_1}{\theta_1} = \frac{T_2}{\theta_2}$, will have same C_V

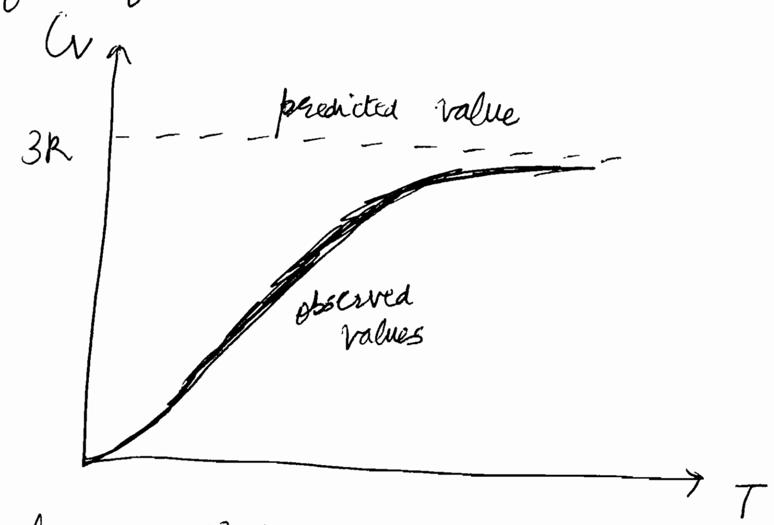
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q9) Dulong and Petit's law states that molar specific heat of all solids is given by

$$C_V = 3R$$

where R is universal gas constant

While Dulong and Petit's law agrees with experimental values at high temperatures it fails for lower temperatures.



<famous T^3 law>

Q10) <Q2>
<Q5>

$\boxed{\text{Significance of Debye's Temperature}}$

1. θ_D give upper bound to the frequency spectrum as $\theta_D = \frac{h\nu_m}{k} \Rightarrow \nu_m = \left(\frac{\theta_D k}{h}\right)$

2. Law of Correspondence states. Curves of C_V vs T for different solids are different by a single curve of C_V vs $\left(\frac{T}{\theta_D}\right)$ fits for all solids

Q1)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(60 marks)

For a maxwell distribution

Occupation index $f(\epsilon) = A e^{-\left(\frac{\epsilon}{kT}\right)}$;

density of states $g(\epsilon) = \frac{2\pi V}{h^3} \cdot (2m)^{\frac{3}{2}} \epsilon^{1/2}$

$\Rightarrow$ No. of particles with energy between ϵ and $d\epsilon$, $n(\epsilon)\, d\epsilon$ is given by

$$n(\epsilon)\, d\epsilon = f(\epsilon)\, g(\epsilon)\, d\epsilon$$

$$= A e^{-\frac{\epsilon}{kT}} \frac{2\pi V}{h^3} \cdot (2m)^{\frac{3}{2}} \epsilon^{1/2} d\epsilon$$

$$= B\, \epsilon^{1/2} e^{-\frac{\epsilon}{kT}} d\epsilon$$

(where B is a const)

But ~~total~~ no. of particles $= N$

$$\Rightarrow N = \int_0^\infty n(\epsilon)\, d\epsilon$$

$$= \int_0^\infty B\, \epsilon^{1/2} e^{-\frac{\epsilon}{kT}} d\epsilon$$

Put $\frac{\epsilon}{kT} = x$

$$= B (kT)^{\frac{3}{2}} \int x^{1/2} e^{-x} dx$$

$$= B (kT)^{\frac{3}{2}} \Gamma\left(\frac{3}{2}\right)$$

$$= B \cdot (kT)^{3/2} \cdot \frac{1}{2}\sqrt{\pi}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow B = \frac{2N}{(kT)^{3/2}\sqrt{\pi}} = \frac{2\pi N}{(\pi kT)^{3/2}}$$

$$\Rightarrow \boxed{n(\epsilon)\, d\epsilon = \frac{2\pi N}{(\pi kT)^{3/2}}\, \epsilon^{1/2}\, e^{-\frac{\epsilon}{kT}}\, d\epsilon}$$

Now for non relativistic case, the energy of molecules in motion with velocity v is given as

$$\epsilon = \frac{1}{2} m v^2$$

Hence no. of particles with velocity between v and $v+dv$ is given as

$$n(v)\, dv = \frac{2\pi N}{(\pi kT)^{3/2}} \cdot \left(\frac{1}{2} m v^2\right)^{\frac{1}{2}} \cdot e^{-\left(\frac{mv^2}{2kT}\right)} m v\, dv$$

$$\boxed{n(v)\, dv = NC\, v^2\, e^{-\left(\frac{mv^2}{2kT}\right)}\, dv}$$

where C is a const. $C = \frac{2\pi\, m^{3/2}}{(\pi kT)^{\frac{3}{2}}\sqrt{2}}$

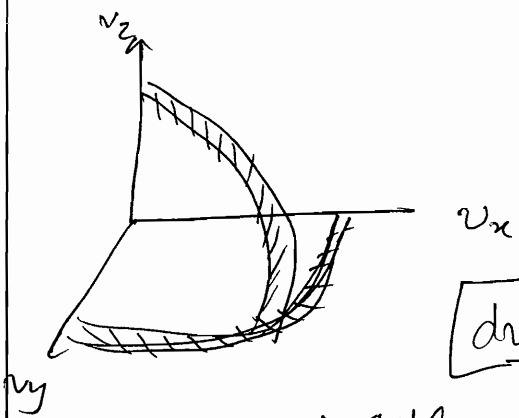
now in a phase space, we can consider velocity v to $v+dv$ can be broken into components v_x to $(v_x + dv_x)$, v_y to $(v_y + dv_y)$ and v_z to $(v_z + dv_z)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow P(v)\,dv = \frac{n(v)\,dv}{N} = C v^2 e^{-\left(\frac{mv^2}{2kT}\right)} dv$$

Also $P(v)\,dv = P(v_x)\,dv_x \; P(v_y)\,dv_y \; P(v_z)\,dv_z$

$$= P(v_x)\,P(v_y)\,P(v_z)\,dv_x\,dv_y\,dv_z$$



$$\boxed{dv_x\,dv_y\,dv_z = 4\pi v^2\,dv}$$

$\Rightarrow$ replace $4\pi v^2 dv$ by $dv_x\,dv_y\,dv_z$

Also since Probability of v_x, v_y, v_z are same $\Rightarrow P(v_x) = P(v_y) = P(v_z)$

Now we can write $P(v)\,dv$ as

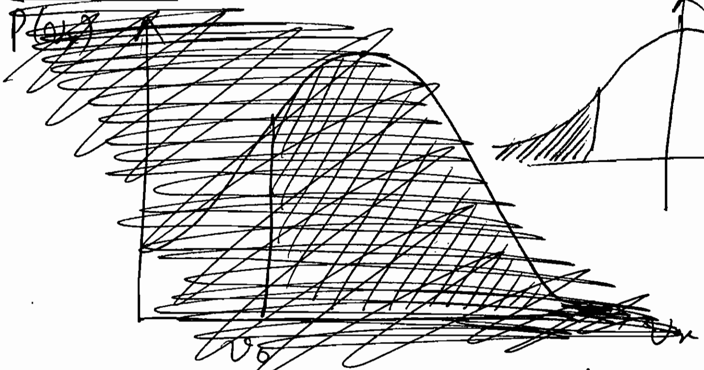
$$= \left[\sqrt{\frac{m}{2\pi kT}}\, e^{-\frac{mv_x^2}{2kT}}\right]\left[\sqrt{\frac{m}{2\pi kT}}\, e^{-\frac{mv_y^2}{2kT}}\right]\left[\sqrt{\frac{m}{2\pi kT}}\, e^{-\frac{mv_z^2}{2kT}}\right] 4\pi$$

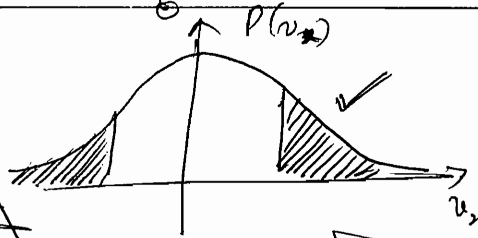
$$\Rightarrow P(v_x)\,dv_x = \sqrt{\frac{m}{2\pi kT}}\; e^{-\frac{mv_x^2}{2kT}}\,dv_x$$

$$= D\, e^{-\frac{mv_x^2}{2kT}}\,dv_x$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$$





कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

important curve

$$P(v_x > v_0) = \frac{1 - 2\int_0^{v_x} P(v_x)\, dv_x}{2}$$

in (+ve) direction

$$= \frac{1}{2}\left[1 - 2\int_0^{v_0} \sqrt{\frac{m}{2\pi kT}}\; e^{-\frac{m v_x^2}{2kT}}\, dv_x\right]$$

$$\boxed{\begin{aligned} &\text{Put } \sqrt{\frac{m v_x^2}{2kT}} = x \\ &\Rightarrow \quad dv_x = \sqrt{\frac{2kT}{m}}\, . \, dx \end{aligned}}$$

$$= \frac{1}{2}\left[1 - 2\int_0^{\sqrt{\frac{m v_0^2}{2kT}}} \sqrt{\frac{m}{2\pi kT}}\; \sqrt{\frac{2kT}{m}}\, .\, e^{-x^2}\, dx\right]$$

$$= \frac{1}{2}\left[1 - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{m v_0^2}{2kT}}} e^{-x^2}\, dx\right]$$

$$\boxed{P(v_x > v_0) = \frac{1}{2}\left[1 - \operatorname{erf}\left(\sqrt{\frac{m v_0^2}{2kT}}\right)\right]}$$

Q2)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

r.m.s. velocity for a gas molecule, v_{rms} is given as

$$v_{rms} = \sqrt{\frac{3kT}{m}} \quad — (1)$$

Escape velocity for a particle be v_{esc}, say

$$\Rightarrow \frac{1}{2}\cancel{m}v_{esc}^2 = +\frac{GM\cancel{m}}{R}$$

$$\Rightarrow v_{esc} = \sqrt{\frac{2GM}{R}}$$

We also know $\cancel{m}g = \frac{GM\cancel{m}}{R^2}$

$$\Rightarrow GM = gR^2$$

$$\Rightarrow v_{esc} = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR} \quad — (2)$$

Since $v_{esc} = v_{rms}$

$$\Rightarrow \sqrt{2gR} = \sqrt{\frac{3kT}{m}}$$

$$\Rightarrow T = \left(\frac{2gR.m}{3k}\right)$$

$$= \frac{2 \times 9.8 \times 6370 \times 10^3 \times 23.24 \times 10^{-27} \times 2}{3 \times 1.38 \times 10^{-23}}$$

$$= \boxed{1.4 \times 10^5\ k}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

do not forget to convert 'gm' to kg.

$$\eta = \frac{F/A}{\left(\frac{dv}{dz}\right)}$$

Q3)

From transport phenomenon, viscosity η is given as

$$\eta = \frac{1}{3} m n \bar{v} \lambda$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}}$$

Remember, its average speed $\bar{v} = \sqrt{\frac{8kT}{\pi m}}$

$$\frac{m}{s} \frac{kg\, \cancel{m^2}}{s^2\, \cancel{m^2}} \quad \left(\frac{kg}{ms}\right)$$

$$\Rightarrow \lambda = \frac{3\eta}{m\frac{N}{V}\sqrt{\frac{8kT}{\pi m}}} = \frac{3\eta V\sqrt{\pi}}{N\sqrt{8kTm}}$$

$$= \frac{3\eta RT\sqrt{\pi}}{N.P\sqrt{8\frac{R}{N}Tm}}$$ (assuming ideal gas)

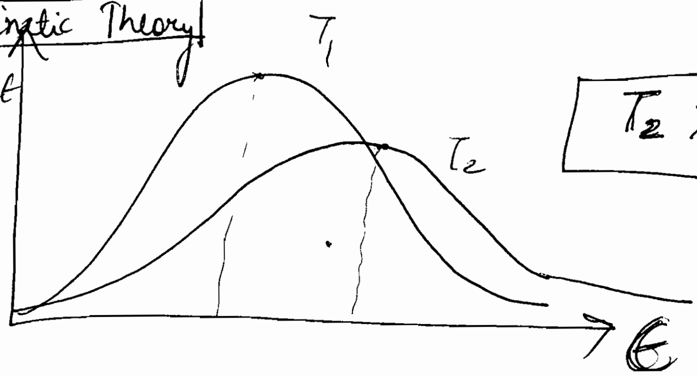
$$\boxed{\lambda = \frac{3\eta}{P}\sqrt{\frac{T\pi R}{8N^2 m}}}$$

$$= \frac{3.974 \times 10^{-7}\, m}{\sqrt{2}}$$

$$= 2.77 \times 10^{-7}$$

Q4) From kinetic Theory

$n(E)dE$



$$\boxed{T_2 > T_1}$$

From kinetic Model

$$PV = \frac{1}{3} mn v^2 = \frac{2}{3} \cdot \frac{1}{2} mnv^2$$

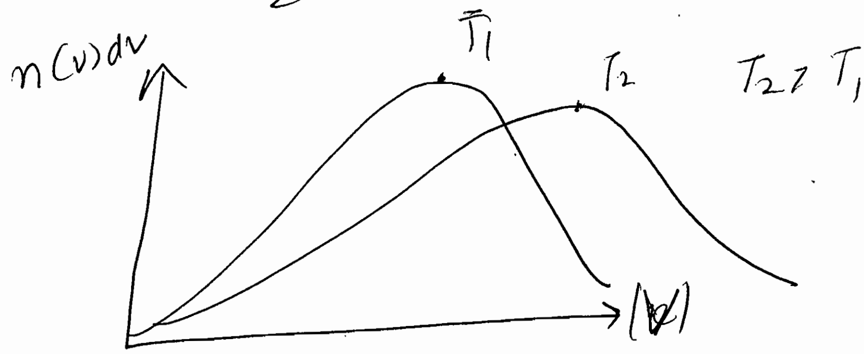
$$= \frac{2}{3} E$$

$$\Rightarrow E = \frac{3}{2} PV = \frac{3}{2} RT \quad \text{(from ideal gas)}$$

$\therefore$ more the temperature more the energy

Hence the curve moves towards right.

Also $E = \frac{1}{2} mv^2$



Also the area under the curve is equal to number of particle, hence constant, therefore peak shifts downwards also on increasing Temp.

$\boxed{\text{From distribution}}$

The peak in the Maxwell distribution curve corresponds to the most probable speed. Now,

$$n(v)\,dv = N\left(\sqrt{\frac{m}{2\pi kT}}\right)^3 4\pi v^2 e^{-\frac{mv^2}{2kT}}\,dv$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

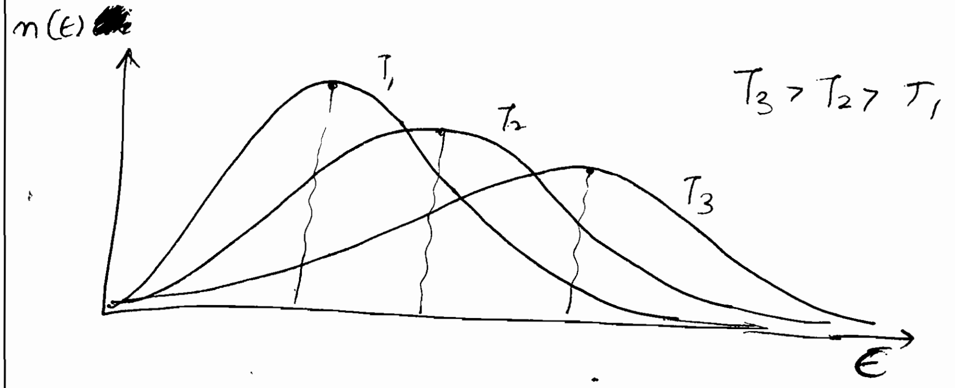
For most probable speed, $\frac{d\,n(v)}{dv} = 0$

$$\Rightarrow \left[2v\, e^{-\frac{mv^2}{2kT}} + v^2 e^{-\frac{mv^2}{2kT}}\left(-\frac{2mv}{2kT}\right)\right] = 0$$

$$\Rightarrow 1 = v^2 \cdot \frac{m}{2kT} \Rightarrow \boxed{v_{M.P.} = \sqrt{\frac{2kT}{m}}}$$

$$\Rightarrow v_{M.P.} \propto \sqrt{T}$$

When temperature is increase $v_{M.P.}$ also increases, hence curve shifts rightwards.



Q5)

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n}$$

Mean free path, λ, is defined as the average distance after which a molecules undergoes collision !!

Assuming ideas gas

$$n = \frac{N}{V} = \frac{NP}{RT}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \lambda = \frac{RT}{\sqrt{2}\,\pi\,(2\times10^{-10})^2 \times N \times P}$$

$$= \underline{\underline{2.27\times10^{-7}\ m}}$$

Q6) $\langle Q1 \rangle$ $\langle Q4 \rangle$

$$n(v)\,dv = N\left(\sqrt{\frac{m}{2\pi kT}}\right)^3 4\pi v^2 e^{-\frac{mv^2}{2kT}}\,dv$$

$$= A\,v^2 e^{-\frac{mv^2}{2kT}}\,dv$$

where $A = N\left(\sqrt{\frac{m}{2\pi kT}}\right)^3 4\pi$

$$\Rightarrow \langle v^3 \rangle = \frac{\int_0^\infty v^3\,dn}{\int_0^\infty dn}$$

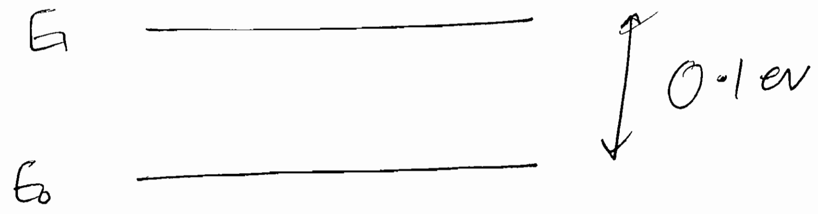
do not use denomina as N. This will avoid confusion with constants like A

$$= \frac{\int_0^\infty v^5 e^{-\frac{mv^2}{2kT}}\,dv}{\int_0^\infty v^2 e^{-\frac{mv^2}{2kT}}\,dv}$$

$$= \frac{\frac{1}{2\left(\frac{m}{2kT}\right)^{\frac{6}{2}}}\cdot\Gamma\frac{6}{2}}{\frac{1}{2\left(\frac{m}{2kT}\right)^{\frac{3}{2}}}\cdot\Gamma\frac{3}{2}} = \frac{1}{\left(\frac{m}{2kT}\right)^{\frac{3}{2}}}\;\frac{2}{\frac{1}{2}\cdot\sqrt{\pi}}$$

$$\boxed{\langle c^3 \rangle = \frac{4}{\sqrt{\pi}}\cdot\left(\frac{2kT}{m}\right)^{3/2}}$$

Q7)



कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Assuming that the assembly follows Maxwell distribution,
let no. of particle in '0' state be n_0

$\Rightarrow$ no. of particles in '1' state, n_1 is given by

$$n_1 = n_0 e^{-\frac{\Delta E}{kT}} = n_0 e^{-\left(\frac{0.1\,eV}{kT}\right)}$$

Also $n_0 + n_1 = N$

$$\Rightarrow n_0\left[1 + e^{-\frac{\Delta E}{kT}}\right] = N$$

$$\Rightarrow n_0 = \frac{N}{1 + e^{-\left(\frac{\Delta E}{kT}\right)}}$$

$$\Rightarrow n_1 = \frac{N}{1 + e^{-\frac{\Delta E}{kT}}} \cdot e^{-\frac{\Delta E}{kT}}$$

$$\Rightarrow \boxed{\left(\frac{n_1}{N}\right) = \frac{e^{-\left(\frac{\Delta E}{kT}\right)}}{1 + e^{-\left(\frac{\Delta E}{kT}\right)}}}$$

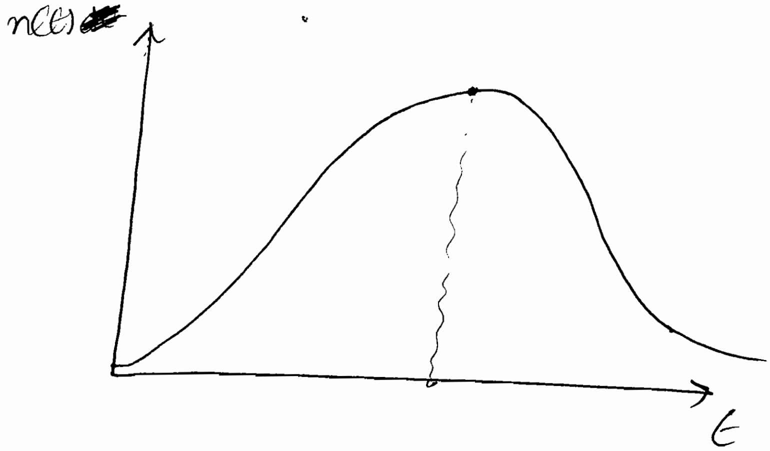
$$e^{-\frac{\Delta E}{kT}} = e^{-\frac{0.1 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}} = 0.02$$

$$\Rightarrow \frac{n_1}{N_1} = \frac{0.02}{1.02} \approx 0.02 = 2\%$$

Q8)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)



$$n(\epsilon)\, d\epsilon = \frac{2\pi N}{(\pi k T)^{3/2}} \cdot \epsilon^{1/2} \, e^{-\frac{\epsilon}{kT}} \, d\epsilon$$

This distribution peaks for $\left.\frac{dn}{d\epsilon}\right|_{\epsilon=\epsilon_0} = 0$

$$\Rightarrow \frac{1}{2}\epsilon^{-1/2} \cdot e^{-\frac{\epsilon}{kT}} + \epsilon^{1/2} e^{-\frac{\epsilon}{kT}}\left(\frac{-1}{kT}\right) = 0$$

$$\Rightarrow \frac{1}{2\sqrt{\epsilon}} = \frac{\sqrt{\epsilon}}{kT} \Rightarrow \boxed{\epsilon = \frac{kT}{2}}$$

Do not forget to mention that at $\epsilon = \frac{1}{2} kT$, $\frac{d^2 n}{d\epsilon^2} < 0$

The mean energy of molecules is given as

$$\bar{\epsilon} = \frac{\int \epsilon \, dn}{\int dn} = \frac{\int \epsilon^{3/2} e^{-\frac{\epsilon}{kT}} d\epsilon}{\int \epsilon^{1/2} e^{-\frac{\epsilon}{kT}} d\epsilon}$$

Put $\frac{\epsilon}{kT} = x$, we have

$$\bar{\epsilon} = \frac{(kT)^{\frac{5}{2}} \int x^{3/2} \cdot e^{-x} \, dx}{(kT)^{3/2} \int x^{1/2} e^{-x} \, dx} = (kT) \frac{\Gamma\frac{5}{2}}{\Gamma\frac{3}{2}} = \frac{3}{2} kT$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\bar{E} = \frac{3}{2} \times \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \text{ eV}$$

$$= \underline{\underline{0.0388 \text{ eV}}}$$

Q9) Average speed $\bar{v}$ for a molecule is given as

$$\bar{v} = \sqrt{\frac{8}{\pi} \frac{kT}{m}}$$

$$\Rightarrow \left(\frac{\bar{v}_1}{\bar{v}_2}\right) = \frac{\sqrt{\frac{T_1}{m_1}}}{\sqrt{\frac{T_2}{m_2}}}$$

if $\bar{v}_o = \bar{v}_H$

$$\Rightarrow \frac{T_o}{m_o} = \frac{T_H}{m_H}$$

$$\Rightarrow T_H = T_o \cdot \frac{m_H}{m_o} = 350 * \frac{2}{32}$$

$$= \underline{\underline{21.875 \text{ K}}}$$

Q10) Using $\frac{dN_{N.C.}}{N} = - \frac{dx}{\lambda}$

N.C. : non collided

$$\Rightarrow \boxed{N = N_0 e^{-\left(\frac{x}{\lambda}\right)}}$$

for $x = 2\lambda$

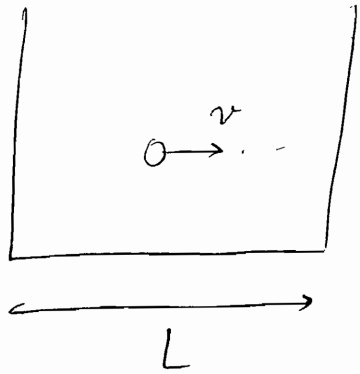
$$N = N_0 e^{-2} = \frac{6 \times 10^3}{e^2} = \underline{\underline{812}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q11)

$$\Delta p = 2 m v_x$$

$$\Delta t = \left(\frac{2L}{v}\right)$$



$$\Rightarrow \frac{\Delta p}{\Delta t} = \left(\frac{m v_x^2}{L}\right)$$

$$\Rightarrow P = \frac{F}{A} = \frac{m v_x^2}{L^3}$$

Total Pressure by all the molecules $= \frac{N m v_x^2}{L^3}$

$$= \left(\frac{m N v_x^2}{V}\right)$$

Assuming $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$

Now $v_{rms}^2 = V_x^2 + V_y^2 + V_z^2 = 3 V_x^2$

$\Rightarrow V_x^2 = \frac{V^2}{3}$ & $\frac{N}{V} = n$ $\Rightarrow \langle v_x^2 \rangle = \frac{\langle v^2 \rangle}{3} = \left(\frac{v_{rms}^2}{3}\right)$

$$\Rightarrow \boxed{P = \frac{1}{3} m n v_{rms}^2}$$

From Boltzmann distribution,

No. of molecules with velocity between v and $v + dv$

$$n(v)\, dv = N \left(\frac{m}{2\pi k T}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}} dv$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\langle v^2 \rangle = \frac{\int v^2 \, n(E)\, dE}{\int n(E)\, dE}$$

$$= \frac{\int v^4 e^{-\frac{mv^2}{2kT}} dv}{\int v^2 e^{-\frac{mv^2}{2kT}} dv}$$

$$= \frac{1}{\left(\frac{m}{2kT}\right)} \cdot \frac{\Gamma\frac{5}{2}}{\Gamma\frac{3}{2}}$$

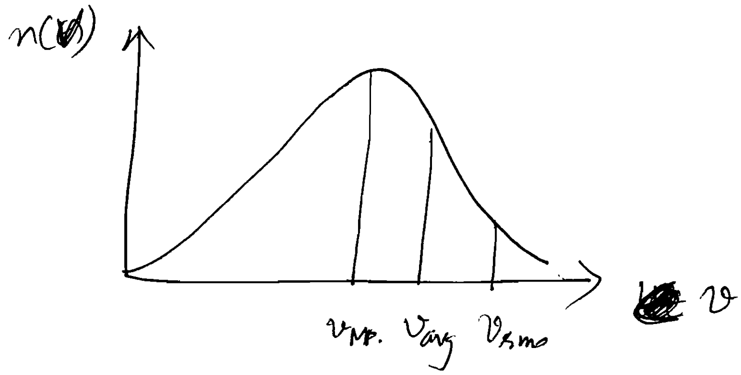
$$= \frac{3}{2} \cdot \frac{2kT}{m} = \left(\frac{3kT}{m}\right)$$

$$\Rightarrow \quad P = \frac{1}{3} \cancel{m} n \, \frac{3kT}{\cancel{m}}$$

$$\boxed{P = nkT}$$

Q12) (Q 1)

Q13) From the Maxwell distribution, we get



कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$v_{M.P.} = \sqrt{\frac{2RT}{M}} \qquad v_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow \quad v_{M.P.} = \sqrt{\frac{2 \times 8.314 \times 273}{2 \times 10^{-3}}} = 1506.5 \text{ m/s}$$

$$v_{avg} = \sqrt{\frac{8}{\pi . 2}}\, v_{MP} = 1700 \text{ m/s}$$

$$v_{rms} = \sqrt{\frac{3}{2}}\, v_{M.P} = 1845.07 \text{ m/s}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q14) X

Q15) Energy of a particle is, in general, a function of a number of parameters, Each of these parameters is called degree of freedom. If energy ϵ, associated with such a particle can be described in terms of a parameter such that

$$\epsilon = az^2$$

where a = const., then the average energy per particle associated with this parameter z, will be $\left(\frac{1}{2}kT\right)$ at equilibrium temperature T.

This is the law of equipartition of energy

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Proof :

Let the particles follow Maxwell distribution. i.e.

$$n(\epsilon)\, d\epsilon = \frac{2\pi N}{(\pi kT)^{3/2}} \epsilon^{1/2} e^{-\frac{\epsilon}{kT}} d\epsilon$$

$$= A\, \epsilon^{1/2} e^{-(\epsilon/kT)}\, d\epsilon$$

Now if $\epsilon = az^2$

$$\Rightarrow \langle \epsilon \rangle = \frac{\int \epsilon\, n(\epsilon)\, d(\epsilon)}{\int n(\epsilon)\, d(\epsilon)}$$

$$= \frac{\int az^2 . (az^2)^{\frac{1}{2}} e^{-\frac{az^2}{kT}} . 2az\, dz}{\int (az^2)^{1/2} . e^{-\frac{az^2}{kT}} . 2az\, dz}$$

Cancelling out common terms,

$$= \frac{\int z^4 . e^{-\frac{az^2}{kT}} dz}{\int z^2 e^{-\frac{az^2}{kT}} dz}$$

$$= \frac{1}{\left(\frac{a}{kT}\right)} . \frac{\Gamma\frac{5}{2}}{\Gamma\frac{3}{2}} =$$

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Proof

Let the particle follow Maxwell Dist^n

$$n(z)\,dz = A\,e^{-\frac{\epsilon(z)}{kT}}\,dz$$

Now if $\epsilon = az^2$

$$\langle \epsilon \rangle = \frac{\int \epsilon\, n(\epsilon)\, d\epsilon}{\int n(\epsilon)\, d\epsilon}$$

$$= \frac{\int az^2 .\ e^{-\frac{az^2}{kT}}\, dz}{\int e^{-\frac{az^2}{kT}}\, dz}$$

$$= a . \frac{1}{\left(\frac{a}{kT}\right)} \frac{\Gamma 3/2}{\Gamma 1/2}$$

$$= \frac{1}{2} kT$$

Short Comings :

(1) Only for energy that can be expressed as az^2 i.e. not even gravitation energy $\left(\frac{GMm}{z}\right)$

(2) For lower temperatures, it does not yield

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correct results as quantum effect take over.

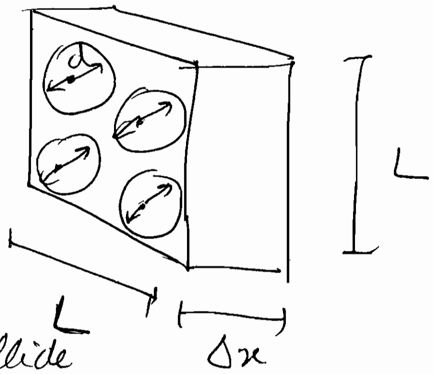
eg. Dulong & Petit law fails for low temperatures

For other higher powers integrals do not converge...

Q16) <Q4> <Q11>

Q17) Consider a thin layer of gas of dimension L, L and Δx as shown.

Let a large number of molecules N be projected towards the layer



Let ΔN molecules collide with the molecules of the layer. Let n' be the no. of molecules per unit volume, and d be their diameter. On average ratio of no. of collisions ΔN to the no. of projected molecules N is equal to the ratio of the target area of molecules in the layer to the total area of face of the layer.

$$\therefore \frac{\Delta N}{N} = \frac{(n' L^2 \Delta x)\, \pi d^2}{L^2}$$

$$\boxed{4\pi r^2 = \pi d^2}$$

$$= n' \pi d^2 \Delta x$$

In the ~~limitat~~ of very small Δx,

$$\left(\frac{dN}{N}\right) = + n' \pi d^2 \, dx$$

define $\lambda = \left(\frac{1}{n' \pi d^2}\right)$

$$\Rightarrow \left(\frac{dN_c}{N}\right) = \left(\frac{dx}{\lambda}\right)$$

c: collided molecules

$\Rightarrow \lambda = $ [redacted] $dx / \left(\frac{dN}{N}\right)$

i.e. λ is the ratio of distance travelled by the molecules to the collision probability $\left(\frac{dN}{N}\right)$; λ is called mean free path.

Other theories give correction in mean free path as,

Claussius : $\lambda = \frac{3}{4}\left(\frac{1}{n' \pi d^2}\right)$

Maxwell : $\lambda = \frac{1}{\sqrt{2}} \frac{1}{n' \pi d^2}$

Using Maxwell's result,

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$$\lambda = \frac{1}{\sqrt{2}} \frac{V}{n N_A} \cdot \frac{1}{\pi d^2}$$

$$= \frac{1}{\sqrt{2}} \frac{1}{\pi d^2} \frac{RT}{P} \cdot \frac{1}{N_A}$$

$$\Rightarrow N_A = \left(\frac{RT}{\sqrt{2} \pi d^2 P \lambda} \right)$$

$$\boxed{N_A = 6.64 \times 10^{+23}}$$

Q18)
$$10^5 * 10^{-3} = n . 8.314 \times 293$$

$$\Rightarrow n = 0.041$$ (from ideal gas equation)

$$\Rightarrow 0.041 \times 6.023 \times 10^{23} \times m_{H_2} = 0.09 \times 10^{-3}$$

$$\Rightarrow m_{H_2} = 3.64 \times 10^{-27} \text{ kg}$$

From Maxwell distribution, we know

$$v_{RMS} = \sqrt{\frac{3kT}{m}} \quad v_{MP} = \sqrt{\frac{2kT}{m}} \quad \bar{v} = \sqrt{\frac{8kT}{\pi m}}$$

$$\Rightarrow v_{RMS} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{3.64 \times 10^{-27}}} = \underline{1847 \text{ m/s}}$$

$$\bar{v} = \sqrt{\frac{8}{\pi} \times \frac{1}{3}} \, v_{RMS} = \underline{1702.27 \text{ m/s}}$$

$$v_{MP} = \sqrt{\frac{2}{3}} \times v_{RMS} = \underline{1508.06 \text{ m/s}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

NTP (1 atm, 20°C)   STP (1 atm, 0°C)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q19)

$$v_{RMS} = \sqrt{\frac{3kT}{m}}$$

$$\frac{v_{Ar}}{v_{O_2}} = \frac{\sqrt{\frac{T_{Ar}}{m_{Ar}}}}{\sqrt{\frac{T_{O_2}}{m_{O_2}}}} = \frac{\sqrt{\frac{313}{40}}}{\sqrt{\frac{273}{32}}} = 0.95$$

$$\Rightarrow v_{Ar} = 460 \times 0.95 = \underline{\underline{440.54\ m/s}}$$

$$\frac{v_{T_2}}{v_{T_1}} = \sqrt{\frac{T_2}{T_1}} \Rightarrow 2 = \sqrt{\frac{T}{293}}$$

$$\Rightarrow \boxed{T = 1172k}$$

Q20) <Q-16>

Q21) <Q-1>

Q22) Assuming Maxwell Boltzmann distⁿ and density of state g(E) to be equal for all levels.

$2kT$ ———— N_2

kT ———— N_1

———— N_0

0

$$\Rightarrow N_1 = N_0 e^{-\frac{kT}{kT}} = \left(\frac{N_0}{e}\right)$$

$$N_2 = N_0 e^{-\frac{2kT}{kT}} = \left(\frac{N_0}{e^2}\right)$$

$$\Rightarrow N_0\left[1 + 1/e + 1/e^2\right] = N \Rightarrow N_0 = \left(\frac{N}{1.5}\right) = 0.66N$$

$= \underline{66}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$N_1 = N_0 e^{-1} = 25$

$N_2 = N_0 e^{-2} = 9$

$\Rightarrow E_{Total} = E_0 N_0 + E_1 N_1 + E_2 N_2$

$= 25 kT + 9 * 2kT$

$$\boxed{E = 43 kT}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q23) <15>

Let the no. of degree of freedom of gas be f

$\Rightarrow$ Internal Energy $U = f * \frac{1}{2} kT = \frac{f}{2} kT$

$\Rightarrow dU = \frac{f}{2} k \, dT$

But we know $dU = C_v \, dT$

$\Rightarrow C_v = \frac{f}{2} k$ for 1 particle

For 1 mole,

$C_v = \frac{f}{2} k . N_A = \frac{f}{2} R$

Also for ideal gas

$C_p - C_v = R$

$\Rightarrow C_p = R + C_v = \left(\frac{f+2}{2}\right) R$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \gamma = \frac{C_p}{C_V} = \left(\frac{f+2}{f}\right)$$

For diatomic, $f = 5 \Rightarrow \boxed{\gamma = 7/5}$

For triatomic, $f = 6 \Rightarrow \boxed{\gamma = 8/6} = 4/3$

Mention : diatomic → 3 D.O.F. due to motion of COM
+
2 DOF due to rotation along axis ⊥ to internuclear axis.

triatomic → 3 DOF for motion of COM
+
3 DOF due to rotation of molecule along 3 Principal axes

(Q24) Let number of particle in $(-\epsilon)$ state by N_1 and total number of particles in (ϵ) state be N_2

Now ; $N_1 + N_2 = N$ (fixed)
$-\epsilon N_1 + \epsilon N_2 = E$ (fixed) ... given
$\Rightarrow N_1 - N_2 = -\left(\frac{E}{\epsilon}\right)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \quad N_1 = \left(\frac{N - \frac{E}{\epsilon}}{2}\right)$$

$$N_2 = \left(\frac{N + \frac{E}{\epsilon}}{2}\right)$$

Entropy of Microcanonial ~~ensemble~~ S, is defined as

$$S = k \ln \Omega$$

where Ω is thermodynamic probability which in this case is,

$$^NC_{N_1} = {^NC_{N_2}} = \frac{\lfloor N}{\lfloor N_1 \; \lfloor N_2}$$

$$= \frac{\lfloor N}{\lfloor\left(\frac{N - \frac{E}{\epsilon}}{2}\right) \; \lfloor\frac{N + \frac{E}{\epsilon}}{2}}$$

$$\Rightarrow S = k\left[\ln \lfloor N\right] - k \ln\left[\lfloor\left(\frac{N - \frac{E}{\epsilon}}{2}\right)\right] - k\left[\ln \lfloor\left(\frac{N + \frac{E}{\epsilon}}{2}\right)\right]$$

By sterling approximation

$$\ln \lfloor n = n \ln n - n$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \boxed{\begin{aligned} S = {} & k\ln N - N \\ & - \frac{k}{2}\left(N-\frac{E}{\epsilon}\right)\ln\frac{1}{2}\left(N-\frac{E}{\epsilon}\right) \\ & - \frac{k}{2}\left(N+\frac{E}{\epsilon}\right)\ln\frac{1}{2}\left(N+\frac{E}{\epsilon}\right) \\ & - N \end{aligned}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Tut 26 : FD and BE statistics

(Q1)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(30 marks)

Bose Einstein Distribution is the energy distribution of integral spin particles like photons, deuterons.

In order to find the distribution, we need to calculate the thermodynamic probability Ω, i.e. the number of possible microstates in a macrostate or a configuration

Let there be $(N_1, N_2, N_3 \ldots N_N)$ particles in Energy states $(\epsilon_1, \epsilon_2, \epsilon_3, \ldots \epsilon_N)$

Since the particles are indistinguishable, there is one way to choose which $N_1, N_2 \ldots$ particles in $\epsilon_1, \epsilon_2 \ldots$ states.

Now we need to distribute N_i particles in g_i cells such that

1. Particles are indistinguishable
2. Any of the g_i cells can hold any of the N_i particles.
3. Apriori probability of all microstates is same.

No. of such possible combinations is given by

$$\Omega_i = \frac{g_i \, \lfloor N_i + g_i - 1}{\lfloor g_i \, \lfloor N_i} = \frac{\lfloor N_i + g_i - 1}{\lfloor N_i \, \lfloor g_i - 1}$$

$$\Rightarrow \Omega = \prod_i \Omega_i = \prod_i \frac{\lfloor N_i + g_i - 1}{\lfloor N_i \, \lfloor g_i - 1}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Now at equilibrium entropy is maximum

$$\Rightarrow d(\ln \Omega) = 0 \qquad [\because dS = k(d\ln\Omega)]$$

$$\Rightarrow d\left[\sum \ln \underline{|N_i + g_i - 1} - \ln \underline{|N_i} - \ln \underline{|g_i - 1}\right] = 0 \quad \text{— (1)}$$

Note that according to sterling approximation,

$$\ln \underline{|n} = n\ln n - n$$

$$\Rightarrow d(\ln \underline{|n}) = \left(n \cdot \frac{1}{n} + \ln n - 1\right)dn = \ln n \, dn \quad \text{— (2)}$$

Using (2) in (1),

$$\sum \ln(N_i + g_i - 1)\, dN_i - \ln(N_i)\, dN_i = 0$$

$$\Rightarrow \sum \ln\left(\frac{N_i + g_i - 1}{N_i}\right) dN_i = 0 \quad \text{— (3)}$$

Also $\sum n_i = N$

$$\Rightarrow \sum dn_i = 0 \quad \text{— (4)}$$

Also $\sum \epsilon_i n_i = E$

$$\Rightarrow \sum \epsilon_i dn_i = 0 \quad \text{— (5)}$$

Multiply (4) by α, (5) by β, and perform (4) + (3) − (5), we have

$$\sum \alpha dn_i + \sum \ln\left(\frac{N_i + g_i - 1}{N_i}\right) dn_i - \sum \beta \epsilon_i dn_i = 0$$

$$\Rightarrow \alpha + \ln\left(\frac{N_i + g_i - 1}{N_i}\right) - \beta\epsilon_i = 0$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$\Rightarrow$ Approximating $N_i + g_i - 1 \approx N_i + g_i$, we have

$$\frac{N_i+g_i}{N_i} = e^{-(\alpha - \beta \epsilon_i)}$$

$$\Rightarrow \frac{g_i}{N_i} = e^{-(\alpha-\beta\epsilon_i)} - 1$$

$$\Rightarrow f(\epsilon_i) = \frac{N_i}{g_i} = \frac{1}{e^{-(\alpha-\beta\epsilon_i)} - 1}$$

Also, we know

$$S = k \ln \Omega$$

$$\Rightarrow T dS = Tk\, d(\ln \Omega)$$

$$\Rightarrow T dS = -k\cancel{T}\sum(\alpha - \beta\epsilon_i)\, dn_i$$

$$\Rightarrow \sum \epsilon_i dn_i = 0 + k\beta T \sum \epsilon_i dn_i$$

$$\Rightarrow k\beta T = 1$$

$$\Rightarrow \boxed{\beta = 1/kT}$$

(★) Note that derivation of β is same in all 3 cases as $d(\ln\Omega)$ is always $-(\alpha - \epsilon_i\beta)$.....

$$\Rightarrow f(\epsilon_i) = \frac{1}{e^{\left(\frac{\epsilon_i}{kT} - \alpha\right)} - 1}$$

Put $\alpha = \frac{\mu}{kT}$, we have

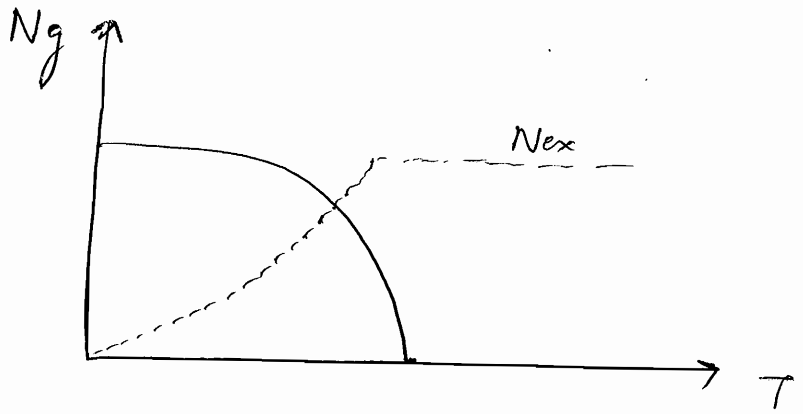
$$\boxed{f(\epsilon_i) = \frac{1}{e^{\left(\frac{\epsilon_i - \mu}{kT}\right)} - 1}}$$

(★) μ is called chemical potential of assembly of bosons

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q2)

Bose Einstein Condensate is a state of matter of dilute gas of weakly interacting bosons at temperatures close to absolute zero. Under such conditions very large fractions of bosons occupy the lowest quantum state at which point quantum effects become apparent on macroscopic scale. This transition occurs below a critical temperature T_B called Condensation Temperature.



Total number of particles for temp. before T_B is given as

$$N = N_g + N_{ex}$$

where N_{ex} have energy more than 0.

For such particles from B.E. statistics

$$N_{ex} = \int f(E)\, g(E)\, dE$$

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$= \int \frac{1}{e^{\left(\frac{\epsilon-\mu}{kT}\right)}-1} \cdot \frac{2\pi V}{h^3} \cdot (2m)^{3/2} \cdot \epsilon^{1/2}\, d\epsilon$$

A very interesting thing for $T = T_B$ is that chemical potential $\mu = 0$

$$= \int \frac{1}{e^{\frac{\epsilon}{kT}}-1} \cdot \frac{2\pi V}{h^3} \cdot (2m)^{3/2} \cdot \epsilon^{1/2}\, d\epsilon$$

Put $\frac{\epsilon}{kT} = x$, we have

$$N_{ex} = (kT)^{3/2} \frac{2\pi V}{h^3} (2m)^{3/2} \int_0^\infty \frac{1}{e^x - 1} x^{1/2} \cdot dx.$$

Also let $\int_0^\infty \frac{x^{1/2}}{e^x - 1} = \lambda$

$$\Rightarrow N_{ex} = \frac{2\pi V}{h^3} \cdot (2m)^{3/2} \lambda (kT)^{3/2}$$

At $T = T_B$, $N_{ex} = N$ as $N_g = 0$

$$\Rightarrow N = \frac{2\pi V}{h^3} \cdot (2m)^{3/2} \lambda (kT_B)^{3/2}$$

$$\Rightarrow T_B = \frac{1}{k} \left[\frac{Nh^3}{(2\pi V)(2m)^{3/2} \lambda} \right]^{2/3}$$

$$\boxed{T_B = \frac{h^2}{2mk} \left[\frac{n}{2\pi\lambda} \right]^{2/3}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Also, $N_{ex} = (kT)^{3/2} \cdot \dfrac{N}{(kT_B)^{3/2}}$

$$= N \cdot \left(\frac{T}{T_B}\right)^{3/2}$$

$$\Rightarrow N_g = N - N_{ex}$$

$$\boxed{N_g = N\left(1 - \left(\frac{T}{T_B}\right)^{3/2}\right)}$$

<draw the curve>

Helium Bose Einstein Condensate

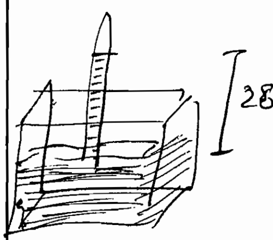
For He, T_B = [redacted] 2.17k

At atmospheric Pressure Helium condenses at 4.2k into a normal liquid. As temp is lowered further, it goes phase transition once again at 2.17k and change into another liquid of vastly different properties.

The new fluid has
(i) Zero viscosity
(ii) infinite thermal conductivity
(iii) no bubbles.

Lack of viscosity means superfluid can flow through smallest of cracks without impediment. Another effect that superfluid Helium shows are that of fountain effect & 2nd sound

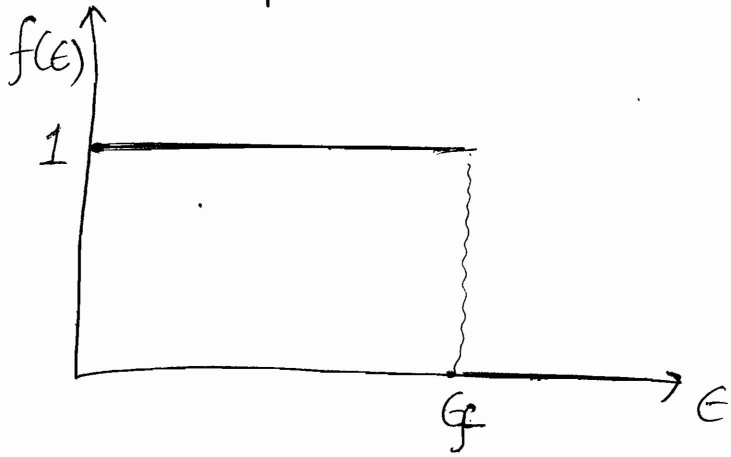
Anamolous fluid is called Helium-II



Fountain effect in capillary tubes.

Q3)

Fermi Energy is defined as the energy level for which, at $T = 0k$, all energy states below this energy level are fully occupied, and above which no state is occupied.



<u>For $T = 0k$</u>

$$f(\epsilon) = \begin{cases} 1 & \epsilon < \epsilon_{f_0} \\ 0 & \epsilon > \epsilon_{f_0} \end{cases}$$

$$g(\epsilon)\, d\epsilon = \frac{2\pi V}{h^3} \cdot (2m)^{3/2} \cdot \epsilon^{1/2}\, d\epsilon$$

Also spin degeneracy $= 2$ (fermions)

$$\Rightarrow N = \int 2 f(\epsilon)\, g(\epsilon)\, d\epsilon$$

$$= \int_0^{\epsilon_{f_0}} \frac{4\pi V}{h^3} \cdot (2m)^{3/2}\, \epsilon^{1/2}\, d\epsilon$$

$$= \left(\frac{4\pi V}{2h^3}\right) \cdot (2m)^{3/2} \cdot \frac{\epsilon_{f_0}^{3/2}}{(3/2)}$$

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \epsilon_{f_0} = \left[\frac{3}{2} N \cdot \frac{2h^3}{4\pi V (2m)^{3/2}} \right]^{2/3}$$

Also,

$$E = \int \epsilon_i \, dn_i$$

$$= \int \epsilon \, \frac{2 * 2\pi V}{h^3} \cdot (2m)^{3/2} \cdot \epsilon^{1/2} \, d\epsilon$$

$$= \int_0^{\epsilon_f} \epsilon^{3/2} \cdot \frac{3}{2} \frac{N}{\epsilon_{f_0}^{3/2}} \cdot d\epsilon$$

$$= \frac{3N}{2\epsilon_{f_0}^{3/2}} \cdot \frac{\epsilon_f^{5/2}}{\frac{5}{2}}$$

$$\boxed{E = \frac{3}{5} N \epsilon_{f_0}}$$

Hence total energy at $T = 0$, in not Zero, but proportional to fermi energy and no. of particles.

Q4) Electrons are fermion i.e. $\frac{1}{2}$ spin particles. They follow Pauli's exclusion principle.

In order to find an expression for distribution of electrons, we need to find out thermodynamic probability Ω i.e. number of microstates in a configuration.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Let $(N_1, N_2, N_3, \ldots N_N)$ particles be there in $(\epsilon_1, \epsilon_2, \epsilon_3, \ldots)$ energy states.

Since the particles are indistinguishable, there is only 1 way to choose $(N_1, N_2, \ldots)$ particle in $(\epsilon_1, \epsilon_2, \epsilon_3 \ldots)$ states

Now we need to distribute, N_i particles in g_i cells such that:

1. Particles are indinguishable
2. Pauli Exclusion Principle is followed i.e. 1 cell of g_i cells can take at maximum 1 fermion.
3. Apriori Probability of all microstates is same.

No. of such Combinations is given as

$$\Omega_i = {}^{g_i}C_{N_i} = \frac{g_i!}{N_i!\,(g_i - N_i)!}$$

use this one

$$\Rightarrow \ln\Omega = \sum\left[\ln N_i! - \ln g_i! - \ln (N_i - g_i)!\right] \quad \text{—} \quad (1)$$

~~From~~

Also we know,

$$\sum n_i = N$$

$$\Rightarrow \sum dn_i = 0 \quad \text{—} \quad (2)$$

$$\sum \epsilon_i n_i = E$$

$$\Rightarrow \sum \epsilon_i dn_i = 0 \quad \text{—} \quad (3)$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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At equilibrium, Entropy is maximum

$\Rightarrow \quad dS = 0$

$\Rightarrow \quad k\, d(\ln \Omega) = 0 \qquad (S = k \ln \Omega)$

from ①, we have

$$d \ln \Omega = \sum d\left[\ln \lfloor N_i - \ln \lfloor g_i - \ln \lfloor N_i - g_i \right] = 0 \quad — ④$$

From sterling approximation,

$$\ln \lfloor n \simeq n \ln n - n$$

$$\Rightarrow d \ln \lfloor n = \left[n \cdot \frac{1}{n} + \ln n - 1 \right] dn$$

$$\boxed{d(\ln \lfloor n) = \ln n \, dn} \quad — ⑤$$

Using ⑤ in ④, we get

$$d(\ln \Omega) = \sum \left[\ln n_i \, dn_i - \ln (N_i - g_i)\, dN_i \right]$$

$$= \sum \ln \left(\frac{n_i}{n_i - g_i} \right) dn_i \quad — ⑥$$

Multiply ② by α, ③ by β and performing ② + ⑥ − ③, we get

$$\sum \left(\alpha + \ln \left(\frac{n_i}{n_i - g_i} \right) - \beta \epsilon_i \right) dn_i = 0$$

$$\Rightarrow \alpha + \ln \left(\frac{n_i}{n_i - g_i} \right) - \beta \epsilon_i = 0$$

$$\Rightarrow \left(\frac{n_i}{n_i - g_i} \right) = e^{-(\alpha - \beta \epsilon_i)}$$

$$\Rightarrow 1 - \frac{g_i}{n_i} = e^{(\alpha - \beta \epsilon_i)} \quad \Rightarrow \frac{n_i}{g_i} = \frac{1}{1 - e^{(\alpha - \beta \epsilon_i)}}$$

Corrected $\Rightarrow$ $\boxed{\dfrac{n_i}{g_i} = \dfrac{1}{1 + e^{-(\alpha - \beta \epsilon_i)}}} = \boxed{\dfrac{1}{1 + e^{\left(\frac{\epsilon - \epsilon_F}{kT}\right)}}}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$N_i = f(\epsilon_i)\, g_i$$

$$= \frac{g_i}{e^{\frac{\epsilon - \epsilon_f}{kT}} + 1} = \frac{g_i}{A e^{\frac{\epsilon}{kT}} + 1}$$

where $A = e^{-\frac{\epsilon_f}{kT}}$

$$\frac{g_i}{N_i} = A e^{\left(\frac{\epsilon}{kT}\right)} + 1$$

For Maxwell distribution

$$\frac{g_i}{N_i} = A e^{+\left(\frac{\epsilon}{kT}\right)}$$

$\Rightarrow$ they both converge, when

$$\boxed{\frac{g_i}{N_i} \gg 1}$$

We can also find a quantum mechanical condition. For particles to be distinguishable (Maxwell), we should have

$$\lambda_{debroglie} < \left(\frac{V}{N}\right)^{1/3}$$

$$\frac{h}{\sqrt{2mE}} \leq \left(\frac{V}{N}\right)^{1/3}$$

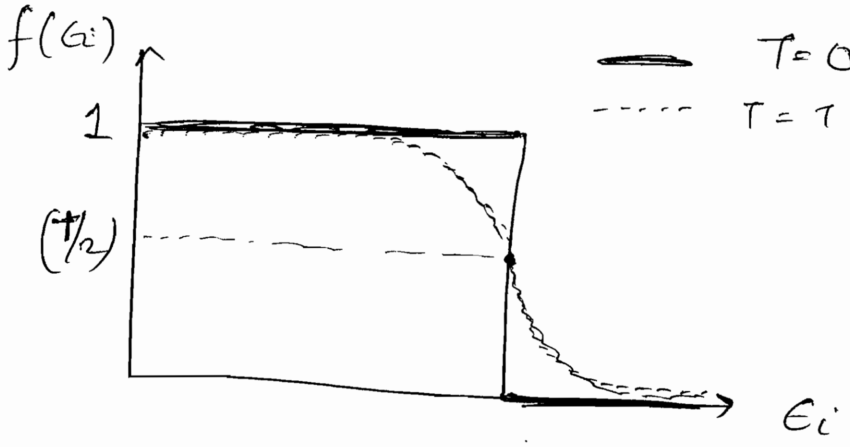
Approximating E as kT, we have

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\left(\frac{N}{V}\right)^{\frac{1}{3}} \frac{h}{\sqrt{2mkT}} < 1$$

$$\Rightarrow \boxed{T > \left(\frac{N}{V}\right)^{2/3} \frac{h^2}{2mk}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

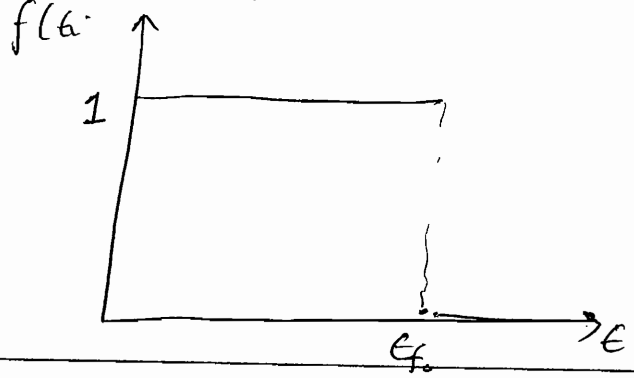


Q5)(6) , Q1

Q7) , Q4

Q8) Let the fermi momentum be given as p_{f_0}. We have fermi energy ϵ_{f_0}, given as $\epsilon_{f_0} = \left(\frac{p_{f_0}^2}{2m}\right)$

At absolute zero, occupation index f_i for fermions is given as shown below



कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Also density of state

$$g(\epsilon) = \frac{2\pi V}{h^3}\cdot (2m)^{3/2}\, \epsilon^{1/2}$$

Also spin degeneracy for fermions = 2

⇒ No. of fermions between energy ϵ and $\epsilon+d\epsilon$ are given by

$$dn = 2f(\epsilon_i)\, g(\epsilon)d\epsilon$$

$$\Rightarrow N = \int dn = \frac{4\pi V}{h^3}\cdot(2m)^{3/2}\int_0^{\epsilon_{f_0}} \epsilon^{1/2}\, d\epsilon$$

$$\Rightarrow \boxed{N = \frac{4\pi V\,(2m)^{3/2}}{h^3}\cdot\frac{(\epsilon_{f_0})^{3/2}}{(3/2)}}$$

Now $\epsilon_{f_0} = \frac{p_{f_0}^2}{2m}$

$$\Rightarrow \boxed{N = \frac{8\pi V}{3h^3}\cdot p_{f_0}^3} \quad \text{—— (1)}$$

Now energy of electrons, E is given as

$$E = \int \epsilon\, dn$$

$$= \int \frac{4\pi V\,(2m)^{3/2}}{h^3}\cdot \epsilon^{3/2}\, d\epsilon$$

$$= \frac{3}{2}N\cdot(\epsilon_{f_0})^{-3/2}\cdot\int_0^{\epsilon_{f_0}} \epsilon^{3/2}\, d\epsilon$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$= \frac{3}{2} . N . \frac{\epsilon_{f_0}^{-3/2}}{\frac{5}{2}} \epsilon_{f_0}^{5/2} = \frac{3}{5} N \epsilon_{f_0}$$

$$= \frac{3}{5} . N . \frac{p_{f_0}^2}{2m}$$

$$\Rightarrow \boxed{E = \frac{3}{10} \frac{N p_{f_0}^2}{m}}$$

Also $PV = \frac{2}{5} E$

$$\Rightarrow P = \frac{2}{5} \cdot \frac{3}{10} \cdot \frac{N}{V} \frac{p_{f_0}^2}{m}$$

$$= \frac{2}{5} \cdot \frac{3}{10} \cdot n \frac{p_{f_0}^2}{m}$$

From (1)

$$P = \frac{3}{25} \frac{n}{m} \cdot \left(\frac{3 h^3 N}{8\pi V}\right)^{2/3}$$

$$= A . n^{5/3}$$

Its good habit to define const.

$$A = \frac{3}{25m} \left(\frac{3h^3}{8\pi}\right)^{2/3}$$

Hence $\boxed{P \propto n^{5/3}}$

(Q9) Q3

(Q10) Given $n_i = g_i e^{-\alpha - \beta \epsilon_i}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

From equipartition theorem, the average energy per degree of freedom per molecule is $(1/2)kT$. For 3 degrees of freedom of a free molecule, we have

$$\langle \epsilon \rangle = \frac{3}{2} kT \quad \text{—— (1)}$$

Also from classical statistical mechanics,

$$g_i = \frac{2\pi V}{h^3} \cdot (2m)^{3/2} \, \epsilon_i^{1/2}$$

$$= A \, \epsilon_i^{1/2}$$

where $A = \frac{2\pi V}{h^3} (2m)^{3/2}$ = const. for the given system.

$$\Rightarrow \langle \epsilon \rangle = \frac{E}{N} = \frac{\int \epsilon_i n_i}{\int n_i} = \frac{\int \epsilon_i A \epsilon_i^{1/2} e^{-\beta \epsilon_i} d\epsilon_i}{\int A \epsilon_i^{1/2} d\epsilon_i}$$

$$= \frac{\int e^{-\beta \epsilon_i} \epsilon_i^{3/2} \, d\epsilon}{\int e^{-\beta \epsilon_i} \epsilon_i^{1/2} \, d\epsilon}$$

$$= \cancel{\left(\frac{1}{\beta}\right)^{3/2} \int e^{-\beta \epsilon_i}}$$

$$= \frac{\dfrac{3\sqrt{\pi}}{4 \beta^{5/2}}}{\left(\dfrac{1}{\beta}\right)^{3/2} \cdot \dfrac{\sqrt{\pi}}{2}} = \frac{3}{4} \cdot \frac{1}{\beta} \cdot 2 = \frac{3}{2} \frac{1}{\beta} \quad \text{—— (2)}$$

Comparing (1) and (2), we have

$$\boxed{\beta = \frac{1}{kT}}$$

Q11) This is similar to distribution of fermions. Its given by

$$^{15}C_{10} = \underline{\underline{3003}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q12) For fermions,

$$f(\epsilon) = \frac{1}{e^{\left(\frac{\epsilon - \epsilon_f}{kT}\right)} + 1}$$

Now for $\epsilon = \epsilon_f$, we have

$$f(\epsilon) = \frac{1}{e^{0} + 1} = \left(\frac{1}{2}\right)$$

$$\Rightarrow \frac{n_i}{g_i} = \frac{1}{2}$$

i.e. half of the energy states are filled for ~~every~~ energy level equal to fermi energy.

$\therefore$ Probability of ~~energy~~ occupation of quantum state is $(1/2)$

Q13) According to F.D,

$$f(\epsilon) = \frac{1}{e^{\left(\frac{\epsilon - \epsilon_f}{kT}\right)} + 1} < 1$$

$\Rightarrow$ for every cell, occupancy is less than 1 $\Rightarrow$ a cell can't have 2 fermions $\Rightarrow$ every fermion in unique energy state

कृपया इस स्थान में कुछ न लिखें।
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Mechanics

Tut Sheet 1

Q1)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For a γ ray, Energy = 1 MeV

Momentum $p = \frac{E}{c} = \frac{1\times10^6}{3\times10^8}$ eV

$= 3.3\times10^{-3}$ eV/ms⁻¹

In order to conserve momentum,

Momentum of Particle $= 3.3\times10^{-3}$ eV s/m

$= 5.28\times10^{-22}$ kg m/s

$\Rightarrow$ Energy of the particle $= \sqrt{p^2c^2 + m_0^2c^4}$

$= \sqrt{2.5\times10^{-26} + 8.1\times10^{-19}}$ J

$= 9\times10^{-10}$ J

$= 5625$ MeV

$m_0c^2 >> p^2c^2$

$\Rightarrow$ ~~$T = E - m_0c^2$~~

~~$= \sqrt{(m_0c^2)^2 + (pc)^2} - m_0c^2$~~

~~$= m_0c^2\sqrt{1+\left(\frac{pc}{m_0c^2}\right)^2} - m_0c^2$~~

~~$= m_0c^2\left[\frac{1}{2}\left(\frac{pc}{m_0c^2}\right)^2\right]$~~

~~$= \frac{1}{2}\frac{p^2c^2}{m_0c^2} = \frac{1}{2}\frac{p^2}{m_0}$~~

$\Rightarrow T = \left(\frac{p^2}{2m}\right)$

$= 88.88$ eV

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q2)

Differential Cross section refers to the number of particles scattered per unit time per solid angle as a fraction of no. of particles incident per unit time per unit area.

mention the symbols in the definition

$$\sigma(\theta) = \frac{dN}{I\, d\Omega}$$

where dN: No. of particles scattered per unit time

$d\Omega$: solid angle

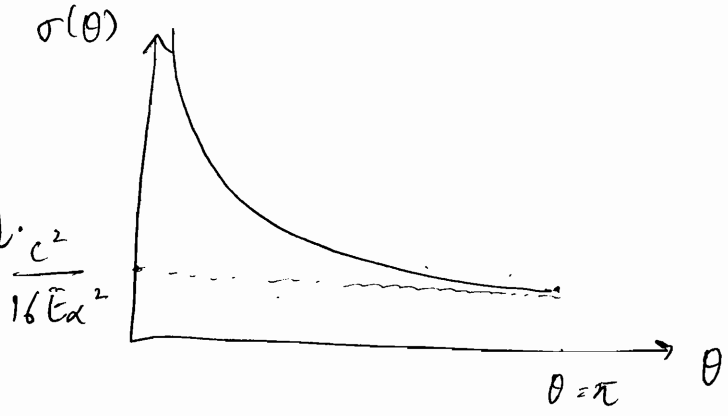
I: No. of particles incident per unit area per unit time

For Rutherford scattering,

$$\sigma(\theta) = \frac{c^2}{16 E_\alpha^2} \operatorname{cosec}^4\left(\frac{\theta}{2}\right) \quad \text{where } c = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}$$

θ: scattering angle

E_α: energy of α particle



Also note that this curve signifies that scattering around $\theta = 0$ is maximum $\Rightarrow$ nucleus is extremely small as most of the particles remain unscattered!!

$$\sigma = \frac{c^2}{16 E_\alpha^2} \int_0^\pi \operatorname{cosec}^4\left(\frac{\theta}{2}\right) . 2\pi \sin\theta \, d\theta$$

$$= \frac{4\pi c^2}{16 E_\alpha^2} \int_0^\pi \frac{1}{\sin^4\left(\frac{\theta}{2}\right)} \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) d\theta$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

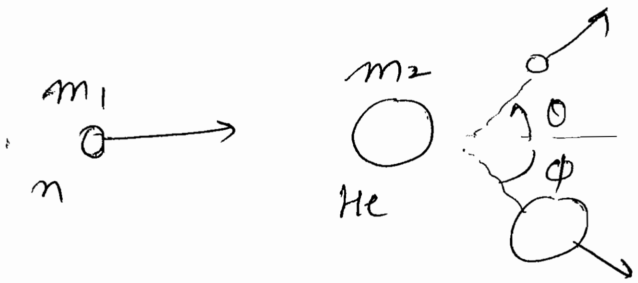
Put $\sin\frac{\theta}{2} = x$, we have

$$\sigma = \frac{4\pi C^2}{16 E_\alpha^2} \int_0 \frac{2\,dx}{x^3} \longrightarrow \infty$$

The range of the Coulumb force is infinite. Hence infinite area of cross section where force is felt.

However in reality, electrons screen the nucleus and effectively force is zero beyond a certain distance, and therefore a finite scattering cross section is there.

(Q3)



velocity of neutron $u_1 = \sqrt{\frac{2E}{m}}$

In the centre of mass system

$$u_1' = u_{1,CM} = u_1 - u_{CM} = u_1 - \frac{m_1 u_1}{m_1+m_2} = \left(\frac{m_2 u_1}{m_1+m_2}\right)$$

$$u_2' = u_{2,CM} = u_2 - u_{CM} = 0 - \frac{m_1 u}{m_1+m_2} = -\left(\frac{m_1 u_1}{m_1+m_2}\right)$$

$$\boxed{u_1' = -\frac{m_2}{m_1} u_2'}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Now from momentum conservation,

$m_1 v_1' \cos\theta' + m_2 v_2' \cos\phi' = m_1 u_1' + m_2 u_2' = 0.$

$\Rightarrow m_1 v_1' \cos\theta' = -m_2 v_2' \cos\phi'$ — (1)

Also

$m_1 v_1' \sin\theta' - m_2 v_2' \sin\phi' = 0$

$\Rightarrow m_1 v_1' \sin\theta' = m_2 v_2' \sin\phi'$ — (2)

From (1) and (2)

$$\boxed{\theta' = \pi - \phi'}$$

Also from energy conservation,

$m_1 v_1'^2 + m_2 v_2'^2 = m_1 u_1'^2 + m_2 u_2'^2$ — (3)

Replacing v_1' by $\frac{m_2}{m_1} v_2'$ and u_1' by $-\frac{m_2}{m_1} u_2'$, we have

$$\boxed{\begin{aligned} |v_1'| &= |u_1'| \\ |v_2'| &= |u_2'| \end{aligned}}$$

Hence the magnitude of velocities in COM frame before & after collision remain same.

Now in COM frame

$u_1' = \frac{m_2 u_1}{m_1 + m_2} = \frac{4}{1+4} u_1 = \left(\frac{4}{5}\right) u_1$

$u_2' = \frac{-m_1 u_1}{m_1 + m_2} = -\left(\frac{1}{5}\right) u_1$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow |\text{Momentum}_{\text{Neutron}}| = \frac{4}{5} m_N u_1 = \frac{4}{5}\cdot\sqrt{2mE}$$

$$= 1.84\times 10^{-20}\ \text{kg m s}^{-1}$$

$$|\text{Momentum}_{\text{Helium}}| = \frac{1}{5}\cdot m_{He} u_1 = \frac{4}{5}\cdot m u_1$$

$$= 1.84\times 10^{-20}\ \text{kg m s}^{-1}$$

Also the momentum of individual particles before & after the collision remain same.

(Q4) <Q2>

$$\sigma(\theta) = A e^{-B(\vec{k_i} - \vec{k_f})^2}$$

$$|\vec{k_i} - \vec{k_f}|^2 = k_i^2 + k_f^2 - 2k_i k_f \cos\theta$$

$$= 2k^2(1-\cos\theta)$$

$$= 4k^2 \sin^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \sigma(\theta) = A e^{-4Bk^2 \sin^2\left(\frac{\theta}{2}\right)}$$

$$\sigma = \int_0^{\pi} \sigma(\theta)\, 2\pi \sin\theta\, d\theta$$

$$= 2\pi A \int_0^{\pi} e^{-4Bk^2 \sin^2\left(\frac{\theta}{2}\right)} \sin\theta\, d\theta$$

Put $\sin^2\frac{\theta}{2} = x \Rightarrow 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cdot\frac{1}{2}d\theta = dx$

$$\sigma = 2\pi A \int_0^{1} e^{-4Bk^2(x)}\, 2dx$$

$$\Rightarrow \sigma = 4\pi A \int_0^1 e^{-4Bk^2 x}\, dx = 4\pi A \frac{(1-e^{-4Bk^2})}{4Bk^2}$$

$$= \frac{\pi A}{Bk^2}(1 - e^{-4Bk^2})$$

(Q5) From (Q-3)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$u_1' = -\frac{m_2}{m_1} u_2'$$

$$v_1' = \frac{m_2}{m_1} v_2'$$

$$|u_1'| = |v_1'|$$

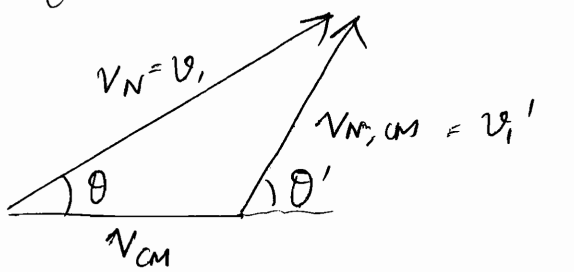
$$|u_2'| = |v_2'|$$

$$\theta' = \pi - \phi'$$

Simply apply Cosine theorem

$$-\cos\theta' = \frac{v_{CM}^2 - v_1'^2 - v_1^2}{2 v_{CM} v_1'}$$

Now for neutron, we can see



$$v_1' \cos\theta' + v_{CM} = v_1 \cos\theta$$

$$v_1' \sin\theta' = v_1 \sin\theta$$

Also, $\frac{E'}{E} = \frac{v_1^2}{u_1^2} = \frac{(v_1'\sin\theta')^2 + (v_1'\cos\theta' + v_{CM})^2}{u_1^2}$

$$= \frac{v_1'^2 + v_{CM}^2 + 2 v_{CM} v_1' \cos\theta'}{u_1^2}$$

$$= \frac{u_1'^2 + \left(\frac{m_1 u_1}{m_1+m_2}\right)^2 + 2\frac{m_1}{m_1+m_2} u_1'^2 \cos\theta'}{u_1^2}$$

$$u_1' = \left(\frac{m_2 u_1}{m_1+m_2}\right)$$

$$= \left(\frac{m_2}{m_1+m_2}\right)^2 + \left(\frac{m_1}{m_1+m_2}\right)^2 + \frac{2 m_1 m_2 \cos\theta'}{(m_1+m_2)^2}$$

Now $m_2 = A m_1$

$$\therefore \frac{E'}{E} = \left(\frac{A}{1+A}\right)^2 + \left(\frac{1}{1+A}\right)^2 + \frac{2A}{(1+A)^2}\cos\theta'$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \frac{E'}{E} = \frac{1 + A^2 + 2A\cos\theta}{(1+A)^2}$$

$$\Rightarrow \boxed{\frac{E'}{E} = \frac{1 + A^2 + 2AMc}{(1+A)^2}}$$

(Q6) Centre of the mass of a system of particles refers to the point where
(i) whole mass of the system is considered to be concentrated
(ii) net Force ~~acting~~ acting on the system is supposed to act.

Let a Consider a system of n particles such that each experience a force $\vec{F_i}$ and has position vector $\vec{r_i}$

i.e. $\vec{F_{total}} = \vec{F_1} + \vec{F_2} + \dots \vec{F_n}$

$$= m_1 \frac{d^2}{dt^2}\vec{r_1} + m_2 \frac{d^2}{dt^2} d\,\vec{r_2} + \dots$$

Now at a point called C.O.M, whole Mass is concentrated, and net Force is supposed to act.

$$\Rightarrow \vec{F_{total}} = (M_1 + M_2 + \dots M_n)\frac{d^2\vec{R}_{CM}}{dt^2}$$

$$\Rightarrow \frac{d^2 R_{CM}}{dt^2} = \frac{d^2}{dt^2}\frac{[m_1\vec{r_1} + m_2\vec{r_2} + \dots m_n\vec{r_n}]}{[M_1 + M_2 + M_3 \dots]}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \boxed{\vec{R}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}}$$

velocity of C.O.M $= \frac{d\vec{r}_{CM}}{dt}$

$$= \frac{d}{dt}\vec{R}_{CM} = \frac{d}{dt}\frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{\sum m_i \frac{d\vec{r}_i}{dt}}{\sum m_i}$$

$$\boxed{v_{CM} = \frac{\sum m_i v_i}{\sum m_i}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q7) $\sigma(\theta) = \alpha \operatorname{cosec}^4\left(\frac{\phi}{2}\right) = \frac{d\sigma}{d\Omega}$

$\therefore$ Now $= d\sigma = \sigma(\phi) \cdot 2\pi \sin\phi \, d\phi$

$$= \alpha \cdot \operatorname{cosec}^4\left(\frac{\phi}{2}\right) 2\pi \sin\phi \, d\phi$$

$$d\sigma = 4\pi\alpha \frac{\cos(\phi/2)}{\sin^3(\phi/2)} d\phi$$

$$\boxed{\frac{d\sigma}{d\phi} = \frac{4\pi\alpha \cos(\phi/2)}{\sin^3(\phi/2)}}$$

Q8) Thrust $= v_{ex}\frac{dm}{dt} = mg$

$$\Rightarrow 10000 = 4.5 \times 10^3 \left(\frac{dm}{dt}\right)$$

$$\Rightarrow \left(\frac{dm}{dt}\right) = 2.22 \text{ kg/s}$$

$$\text{Thrust} = v_e\left(\frac{dm}{dt}\right) = 4.5\times10^3 \times 2.50$$

$$= 11250\ N$$

$$\vec{a} = \frac{\text{Thrust} - mg}{m} = 1.25\ m/s^2$$

$$\Rightarrow \vec{v}_f = \vec{v}_i + \vec{a}t = 0 + 1.25 * 8$$

$$\Rightarrow \boxed{v_f = 10\ m/s}$$

Q9)

Ignoring relativistic effects,

$$E = \frac{p^2}{2m} \Rightarrow p_\alpha = \sqrt{2m_\alpha \cdot E}$$

$$p_{Nucleus} = \sqrt{2m_\alpha E}$$

$$\Rightarrow E_{nucleus} = \frac{2m_\alpha E}{2m_{Nucleus}} = \frac{2.4.E}{2(m_o-4)}$$

$$= \frac{4E}{(m_o-4)}$$

Energy evolved during disintegration $= Q$

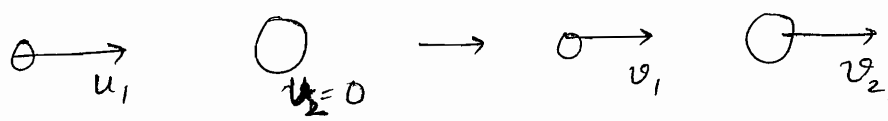
$$= (K.E)_{Product} - (K.E.)_{reactants}$$

$$= (K.E)_{products}$$

$$= E + \frac{4E}{m_o-4} = \frac{(m_o \cancel{-4})E}{(m_o-4)}$$

* Note that there is nothing wrong to ignore relativistic effects in nuclear reactions.

Q10)



From conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 = m_1v_1 + m_2v_2 \quad \text{—(1)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Also for an elastic collision coefficient of restitution, $e = 1$

$\Rightarrow$ Velocity of approach of particles = velocity of their separation

$\Rightarrow \quad u_1 = v_2 - v_1 \quad$ — (2)

From (1) and (2),

$$2m_1u_1 = m_2v_2 + m_1v_2$$

$$\Rightarrow \boxed{v_2 = \frac{2m_1u_1}{m_1+m_2}}$$

Also $v_1 = v_2 - u_1 = \dfrac{2m_1u_1}{m_1+m_2} - u_1$

$$\Rightarrow \boxed{v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1}$$

$$\frac{\text{decrease in k.E}}{\text{k.E initial}} = -\frac{\left[\left(\frac{m_1-m_2}{m_1+m_2}\right)u_1^2 - u_1^2\right]}{u_1^2}$$

$\Rightarrow$ fractional decrease $= \dfrac{4m_1m_2}{(m_1+m_2)^2}$

Now $\text{neutron}_{\text{Mass}} = H_{\text{Mass}}$

$$\Rightarrow \boxed{v_1 = 0}$$

if Hydrogen nucleus is used to slow down neutrons in a nuclear reactor.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

But D_2O is used as :

(1) Deuterium has better Cross section of absorption.

(2) Unlike H, Deuterium does not absorb neutrons.

Q 11) <Q6>

Proof of Unique Centre of Mass

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Usefulness of centre of mass lies in the fact that when no external force acts, the Centre of Mass remains in const. motion. When we shift the frame of reference to Centre of Mass itself, the net momentum becomes zero, and no. of unknown variables reduce and analysis is easy in case of collisions or scattering or annihilation when net force external = 0

Also, in order to study motion of a system all we need to know is motion of Centre of Mass, b'coz however complicated the system is internal forces cancel each other, Centre of mass behaves as if the whole mass is concentrated on it, and the external forces directly apply to it.

Q12) Take Carbon Atom as Origin,

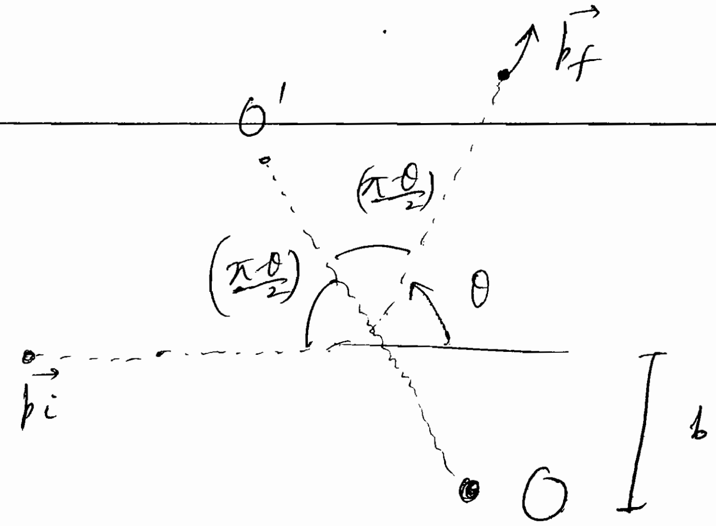
$$\vec{R}_{CM} = \frac{m_c \vec{r}_c + m_o \vec{r}_o}{m_c + m_o}$$

$$= \frac{12 * 0 + 16 * 1.2 \text{Å}}{28}$$

$= \underline{\underline{0.685 \text{ Å}}}$ away from Carbon atom

---

Q13) Let the Origin of Central force be O.
Since the force is central, torque = 0
hence angular momentum remains conserva



The net force acts along OO'.

Also impulse = $\Delta\vec{p}$

i.e. $\int F_{OO'}\, dt = \vec{p_f} - \vec{p_i}$

Since the collision is elastic, $|\vec{p_f}| = |\vec{p_i}|$

we have

$$\vec{p_f} - \vec{p_i} = \sqrt{p_f^2 + p_i^2 - 2p_f p_i \cos\theta}$$

$$= 2p \sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \boxed{\int F_{OO'}\, dt = 2p \sin\left(\frac{\theta}{2}\right)} \quad \text{--- } (1)$$

Also the angular momentum J remains conserved

$J_{initial} = p_i b$

$$J(r) = mr^2\left(\frac{d\phi}{dt}\right)$$

$$\Rightarrow p_i b = mr^2\left(\frac{d\phi}{dt}\right) \quad \text{--- } (2)$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Using in (2), we have

$$\int F_{00'} \cdot \frac{mr^2}{p_i b} d\phi = 2p \sin\left(\frac{\theta}{2}\right)$$

Now $F_{00'} = F\cos\theta = \frac{k}{r^2}\cos\theta$

$$\Rightarrow \int \frac{k}{\cancel{r^2}} \cos\theta \frac{m\cancel{r^2}}{p_i b} \cdot d\phi = 2p \sin\left(\frac{\theta}{2}\right)$$

Also ϕ varies from $-\left(\frac{\pi-\theta}{2}\right)$ to $\left(\frac{\pi-\theta}{2}\right)$

$$\Rightarrow \int_{-\left(\frac{\pi-\theta}{2}\right)}^{\left(\frac{\pi-\theta}{2}\right)} \frac{km}{p_i b} \cdot \cos\phi \, d\phi = 2p_i \sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \frac{km}{p_i b} \cdot 2\cos\left(\frac{\theta}{2}\right) = 2p_i \sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \cot\left(\frac{\theta}{2}\right) = \frac{p_i^2 b}{km} = \frac{mv_0^2 b}{k}$$

Hence Proved !!

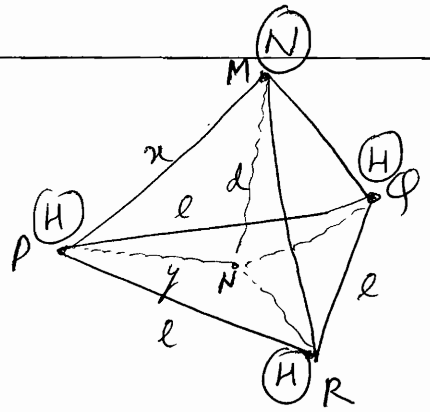
(Q14) Q57

$$v_1 \cos\theta = v_1' \cos\theta' + v_{cm}$$

$$v_1 \sin\theta = v_1' \sin\theta'$$

$$\Rightarrow \tan\theta = \frac{\sin\theta'}{\cos\theta' + \frac{v_{cm}}{v_1'}} = \left[\frac{\sin\theta'}{\cos\theta' + \left(\frac{m_1}{m_2}\right)}\right]$$

Q15)



$y = 0.939$ Å

$x = 1.014$ Å

$d = \sqrt{x^2 - y^2}$

$\boxed{d = 0.38 \text{ Å}}$

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

wrt. N atom, we can see from symmetry that COM will lie on MN

$$\Rightarrow y_{CM} = \frac{y_N + y_{H_1} + y_{H_2} + y_{H_3}}{m_N + 3m_H}$$

$$= \frac{0 + 0.38 * 3}{14 + 3} = \frac{0.38 * 3}{17}$$

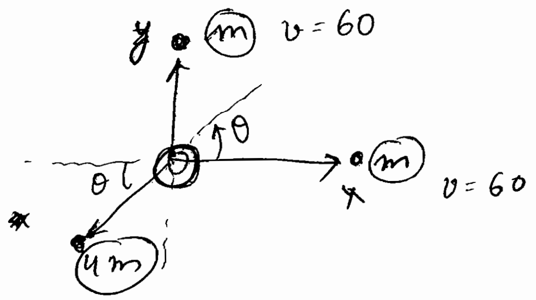
$= \underline{\underline{0.067 \text{ Å}}}$ from N atom

Q16) From Newton's 2^{nd} law,

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

if $\vec{F}_{ext} = 0 \Rightarrow \vec{p} = \text{Const.}$

Hence if net external force $= 0$, the total momentum of a system of particles remains conserved.



Since initially bomb is stationary, the

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

momentum $= 0$

$\Rightarrow$ Final momentum $= 0$ (no external force)

Let the direction of velocity be θ s.t.

$$v_x = -v\cos\theta\,\hat{i} - v\sin\theta\,\hat{j}$$

(from figure 1)

To ~~conserve~~ conserve momentum

$$p_x = 0$$

$$\Rightarrow \cancel{m}\,60 - 4\cancel{m}\,.\,v\cos\theta = 0$$

$$\Rightarrow v\cos\theta = \frac{60}{4} = 15 \text{ m/s}$$

$$p_y = 0$$

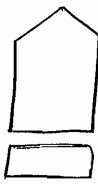
$$-4m\,v\sin\theta + m\,.\,60 = 0$$

$$\Rightarrow v\sin\theta = 15 \text{ m/s}$$

$$\Rightarrow \boxed{\theta = 45^\circ}$$

$$\boxed{v = 15\sqrt{2}} \text{ m/s}$$

(Q 17) For the rocket motion



$M_0 - m'$: mass after ejection

m' : mass of gases

$$M = M_0 - m'$$

$$\Rightarrow \boxed{dM = -\,dm'} \quad \text{— (1)}$$

Now, we know

$$\vec{F}_{ext}\,dt = \frac{d\vec{p}}{dt}$$

$$\Rightarrow d\vec{F}_{ext}\,.dt = \vec{p}_f - \vec{p}_i$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Let dm mass be removed and the velocity increases by dv. Let $u_{exhaust}$ be the velocity of removed mass w.r.t. rocket.

$$\Rightarrow d\vec{F}dt = (\cancel{M} - dm)(\vec{v} + d\vec{v}) + dm(\vec{u}_{ex} + \vec{v} + d\vec{v}) - M\vec{v}$$

$$\Rightarrow d\vec{F}dt = M d\vec{v} - dm\,\vec{v} + dm\,\vec{u}_{ex} + dm\,\vec{v}$$

$$d\vec{F}dt = M d\vec{v} + dm\,\vec{u}_{ex}$$

Taking components along motion of rocket,

$$-\cancel{M}g\cdot dt = M dv - u_{ex}\,dm$$

Also $dm = -dM$, we have,

$$-\cancel{M}g = M dv + u_{ex}\,dM$$

$$\Rightarrow dv = -g\,dt - u_{ex}\frac{dM}{M}$$

$$\Rightarrow \int dv = -gt - u_{ex}\int_{M_0}^{M}\frac{dM}{M}$$

$$\Rightarrow \boxed{v_2 = v_1 - gt + u_{ex}\ln\left(\frac{M_0}{M}\right)}$$

Given $-\frac{dM}{dt} = k$

M_f: Mass of fuel

M_0: initial Mass of rocket w/. fuel

$$\Rightarrow \cancel{+}kt = (M_f + M_0) - (M_0)$$

$$\Rightarrow t = \left(\frac{M_f}{k}\right)$$

$$\Rightarrow v_2 = v_1 - \frac{gM_f}{k} + u_{ex}\ln\left(\frac{M_0 + M_f}{M_0}\right)$$

$$\Rightarrow \boxed{v_2 = v_1 - \frac{gM_f}{k} + u_{ex}\ln\left(1 + \frac{M_f}{M_0}\right)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q18) Q12)

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

Q19)

$\vec{P_i} = \vec{P_f}$

$\Rightarrow 40 * 4 - 60 * 2 = 100 . v$

$\Rightarrow \boxed{v = 0.4\ m/s}$ in direction of Ram's travel.

loss in k.E. $= \frac{1}{2} . 40 . 16 + \frac{1}{2} . 60 . 4 - \frac{1}{2} . 100 . \frac{16}{100}$

$= 320 + 120 - 8$

$= \underline{\underline{432\ J}}$

Q20) Scattering Cross section is the number of particles scattered per unit time as a fraction of no. of particles per unit time per unit area. It's a measure of area of cross section responsible for scattering of particles.

<Q 13>

$b = \frac{c}{2E_\alpha} \cot\left(\frac{\theta}{2}\right) \Rightarrow \sigma(\theta) = \frac{-b}{\sin\theta}\left(\frac{db}{d\theta}\right)$

$\cancel{\int \sigma(\theta)\, 2\pi \sin\theta\, d\theta}$

$\sigma(\theta) = \frac{c^2}{16E_\alpha^2} \operatorname{cosec}^4(\theta/2)$

$\Rightarrow \sigma = \int \sigma(\theta) . 2\pi \sin\theta\, d\theta$

$= \infty$

Q 21)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Rocket equation,

$$v_f = v_i - gt + u_{ex} \ln\left(\frac{M_0}{M}\right)$$

$$\frac{dm}{dt} = -\frac{m}{200}$$

* do not neglect the "$-gt$" factor

$$\Rightarrow \int dm = -\frac{m}{200} \int dt$$

$$\Rightarrow (1 - 0.27) m = \frac{m}{200} t$$

$$\Rightarrow t = 200 * 0.73 = \underline{\underline{146 s}}$$

$$\Rightarrow v_f = 0 - 146 * 9.8 + 2500 \ln\left(\frac{m}{0.27m}\right)$$

$$\boxed{v_f = \underline{\underline{1842.5\ m/s}}}$$

Q 22)

< Q 13 >

$$b = \frac{c}{2E\alpha} \cdot \cot\left(\frac{\theta}{2}\right)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{Z_1 Z_2 e^2}{2E\alpha} \cot\left(\frac{\theta}{2}\right)$$

$$= \frac{9\times10^9 \times 2Z \times (1.6\times10^{-19})^2}{2\times 6\times10^{-13}} \times \cot(15)$$

$$= \underline{\underline{1.43 Z \times 10^{-15}}}$$

Q 23)

$$8\times10^3 = -9.8 * 30 + 3000 \ln\left(\frac{M_0}{M}\right)$$

$$\Rightarrow \left(\frac{M_0}{M}\right) = 15.87$$

Q 26)

< Q 13 >

Q27)
Q28)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

<Q17>

Let velocity of cm be v_{CM}.

$$\vec{v}_{i,CM} = \vec{v}_i - \vec{v}_{CM}$$

$$\vec{p}_{i,CM} = m_i[\vec{v}_i - \vec{v}_{CM}]$$

$$\vec{p}_{system,CM} = \sum_i m_i[v_i - \vec{v}_{CM}]$$

$$= \sum_i m_i v_i - \sum m_i v_{CM}$$

$$= \sum m_i v_{CM} - \sum m_i v_{CM}$$

$$= 0$$

Q29) For a conservative force field F, $\vec{\nabla}\times\vec{F} = 0$

For the given Force

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy+z^3 & x^2 & 3xz^2 \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(3xz^2) - \frac{\partial}{\partial z}(x^2)\right]$$
$$+\hat{j}\left[\frac{\partial}{\partial z}(2xy+z^3) - \frac{\partial}{\partial x}(3xz^2)\right]$$
$$+\hat{k}\left[\frac{\partial}{\partial x}x^2 - \frac{\partial}{\partial y}(2xy+z^3)\right]$$

$$= i[0] + \hat{j}[3z^2 - 3z^2] + \hat{k}[2x - 2x]$$

$$= 0$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Hence a conservative force.

For this force, let Potential be V

$$\Rightarrow \vec{F} = -\vec{\nabla} V$$

$$\Rightarrow V = -\int F_x \, dx + f_1(y,z) \quad — (1)$$

$$V = -\int F_y \, dy + f_2(x,z) \quad — (2)$$

$$V = -\int F_z \, dz + f_3(x,y) \quad — (3)$$

From (1),

$$V = -x^2 y - z^3 x + C_1(y,z) \quad — (4)$$

From (2)

$$V = -x^2 y + C_2(x,z) \quad — (5)$$

from (3)

$$V = -xz^3 + C_3(x,y) \quad — (6)$$

From (4), (5), (6), we have

$$\boxed{V = -x^2 y - z^3 x + C}$$

where C is const.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

Tut Sheet 2 Rotating Frame of Reference

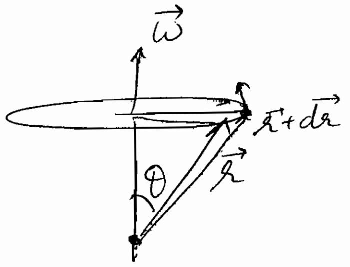
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q1) A rotating frame of reference is a non inertial frame of refrence. Therefore in order to validate the use of Newtons laws in such a frame, Pseudo Forces needed to be applied. Coriolis force is one such pseudo force, that is applied on particles that are moving on a rotating body.

$$C.F. = -2m(\vec{\omega} \times \vec{v})$$

where $\vec{\omega}$: angular velocity vector of rotating body

$\vec{v}$: velocity of particle as seen on rotating body.



Now Consider a ~~moving~~ rotating both in the above figure

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{r\sin\theta \, d\phi}{dt} = r\sin\theta\left(\frac{d\phi}{dt}\right)$$

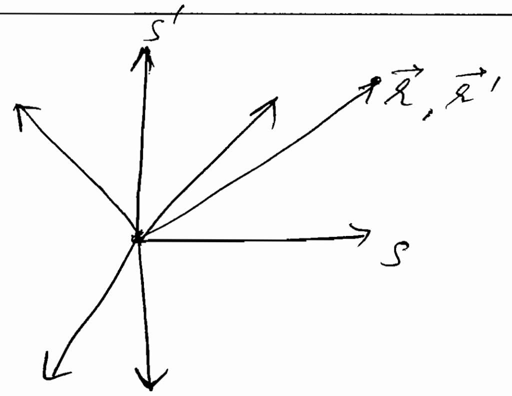
$$= \omega r \sin\theta$$

$$= \vec{\omega} \times \vec{r}$$

$\therefore$ for any vector $\vec{\lambda}$ fixed in rotating body's frame of refrence

$$\left(\frac{d\vec{\lambda}}{dt}\right) = \vec{\omega} \times \vec{\lambda}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)



Consider two frames S and S' where S' is rotating at angular velocity ω.

Now since origin is common

$$\vec{r} = \vec{r}'$$

i.e. $x\hat{i} + y\hat{j} + z\hat{k} = x'\hat{i}' + y'\hat{j}' + z'\hat{k}'$

The velocity vector $\frac{d\vec{r}}{dt}$ is given as

$$\frac{d\vec{r}}{dt} = \left(\frac{dx'}{dt}\hat{i} + \frac{dy'}{dt}\hat{j} + \frac{dz'}{dt}\hat{k}\right) + x'\frac{di'}{dt} + y'\frac{dj'}{dt} + z'\frac{dk'}{dt}$$

$$= \left(\frac{d\vec{r}'}{dt}\right)_r + x'(\vec{\omega} \times \hat{i}') + y'(\vec{\omega} \times \hat{j}') + z'(\vec{\omega} \times \hat{k}')$$

$$\boxed{\frac{d\vec{r}}{dt} = \left(\frac{d\vec{r}}{dt}\right)_r + \vec{\omega} \times \vec{r}}$$

velocity measured from inertial frame

velocity measured from rotating frame

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Similarly for velocity vector $\vec{v}$,

$$\frac{d\vec{v}}{dt} = \left(\frac{d\vec{v}}{dt}\right)_r + \vec{\omega}\times\vec{v}$$

Now $\vec{v} = v' + \vec{\omega}\times\vec{r}$

$$\Rightarrow \frac{d\vec{v}}{dt} = \left(\frac{d(v'+\vec{\omega}\times\vec{r})}{dt}\right)_r + \vec{\omega}\times(v'+\vec{\omega}\times\vec{r})$$

$$\Rightarrow \vec{a} = \vec{a}' + \left(\frac{d\vec{\omega}}{dt}\times\vec{r}\right)_r + \left(\vec{\omega}\times\frac{d\vec{r}}{dt}\right)_r + \vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

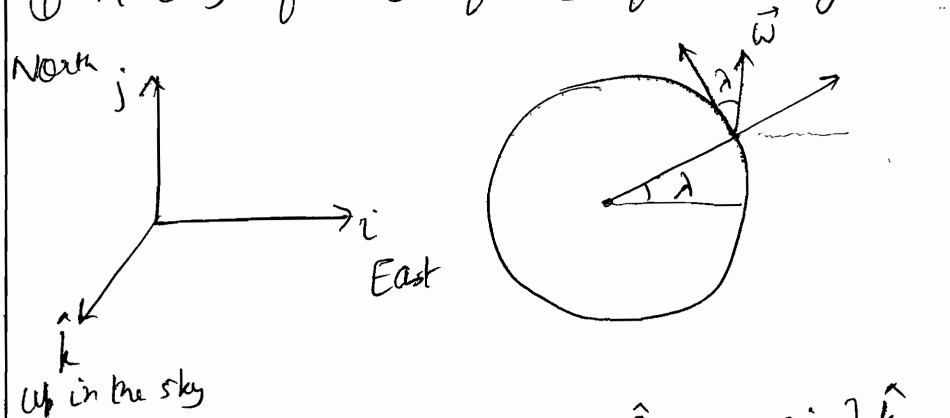
$$\Rightarrow \boxed{\vec{a} = \vec{a}' + 2(\vec{\omega}\times v') + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \left(\frac{d\vec{\omega}}{dt}\right)\times\vec{r}}$$

$$\Rightarrow \vec{a}' = \vec{a} - 2(\vec{\omega}\times\vec{v}) - \vec{\omega}\times(\vec{\omega}\times\vec{r}) - \left(\frac{d\vec{\omega}}{dt}\right)\times\vec{r}$$

Q2) <Q1)

<u>2 examples</u>

① A body falling freely from heigh H.



$$\vec{\omega} = \omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Coriolis Force $= -2m(\vec{\omega}\times\vec{v})$

Now

$v = -gt\,\hat{k}$

$\Rightarrow C.F. = -2m[\omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}] \times -gt\,\hat{k}$

$= 2mgt\,\omega\cos\lambda\,\hat{i}$

$\Rightarrow \vec{a}_{East} = 2gt\omega\cos\lambda$

~~$s = u_{East}t + \frac{1}{2}at^2$~~

~~$= \frac{1}{2}2gt\omega\cos\lambda\,t^2$~~

~~$s = gt\omega\cos\lambda\,t^2$~~

$\Rightarrow \vec{S}_{east} = \frac{2gt^3\omega\cos\lambda}{6}$

$= \frac{gt^3\omega\cos\lambda}{3}$

Also $H = \frac{1}{2}gt^2$, $t = \left(\frac{2H}{g}\right)^{1/2}$

$$\Rightarrow \boxed{S_{east} = \frac{g\omega\cos\lambda}{3}\cdot\left(\frac{2H}{g}\right)^{3/2}}$$

② Ferrel's law

let $\vec{v}$ on the surface be $[v_E\,\hat{i} + v_N\,\hat{j}]$

$\Rightarrow C.F. = -2m[\omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}] \times [v_E\,\hat{i} + v_N\,\hat{j}]$

$(C.F.)_{i\text{-}j\,plane} = -2m\omega[\sin\lambda\,v_E\,\hat{j} - \sin\lambda\,v_N\,\hat{i}]$

$\vec{F}_{on\,earth\,surface} = 2m\omega\sin\lambda[v_N\,\hat{i} - v_E\,\hat{j}]$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

∴ we have 2 observations

① more λ, more coriolis force.
∴ no tropical cyclones on Equator

② in N-H. $\sin\lambda > 0 \Rightarrow$ force towards right of motion.
in S-H $\sin\lambda < 0 \Rightarrow$ force towards left of motion

This is called Ferrel's law and explains direction of winds and ocean currents.

(Q3) (Q4)

Q 2

For a rotating frame, we have Centrifugal pseudo force acting on every body.

$$\text{Cf. force} = -m\,\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$= -m\,[\omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}] \times [\,[\omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}] \times [r\,\hat{k}]\,]$$

$$= -m\,[\omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}] \times [\omega r\cos\lambda\,\hat{i}]$$

$$= +m\,\omega^2 r\cos^2\lambda.\,[\hat{k}] - m\omega^2 r\cos\lambda\sin\lambda.\,\hat{j}$$

Taking the component along earth's earth centre

$$\text{Cf. force} = m\omega^2 r\cos^2\lambda$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$g_{eff} = \frac{mg - m\omega^2 r \cos^2\lambda}{m} = g - \omega^2 r \cos^2\lambda$$

For $g_{eff} = 0$, we have

$$g = \omega_1^2 r \cos^2\lambda$$

$$\omega_1 = n\omega$$

$$\Rightarrow g = n^2\omega^2 \cos^2\lambda \; r$$

At equation $\lambda = 0$

$$\Rightarrow g = n^2\omega^2 r$$

Currently $9.8 = g - \omega^2 r$

$$\Rightarrow 9.8 + \omega^2 r = n^2\omega^2 r$$

$$\Rightarrow n^2 - 1 = \frac{9.8}{\omega^2 r}$$

$$= \frac{9.8}{\left[\frac{2\pi}{24\times60\times60}\right]^2 \times 6400\times10^3}$$

$$= 289.83$$

$$\boxed{n = 17}$$

(Q5) ⟨Q2⟩

(Q6) ⟨Q1⟩

(Q7) ⟨Q2⟩

(Q8a) ⟨Q1⟩ ⟨Q1⟩ Right banks are more eroded in NH.

Tut Sheet 3 Gravitation & Central Force Motion

Q1)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Kepler's laws of planetary motion :

(1) Planets revolve around the sun in elliptical paths with sun as one of the foci.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Radial vector from the sun to a planet sweeps equal areas in equal time i.e. area velocity is constant

(2) Areal velocity of the planetary motion is const. i.e. Area of the orbit swept by the planet per unit time remains same.

(3) Time Period of rotation (T) is proportional to radius of motion (r) as

$$T^2 \propto r^3$$

The square of the planets time Period of complete revolution around the sun is proportional to cube of its semi-major axis

Now, $F = \frac{GMm}{r^n}$

For circular orbital motion,

$$F = \frac{GMm}{r^n} = \frac{mv^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{GM}{r^{n-1}}}$$

$$\Rightarrow \text{Time Period} = \frac{2\pi r}{v} = \frac{2\pi r\sqrt{r^{n-1}}}{\sqrt{GM}}$$

Now this should be of form $\propto r^{3/2}$

$$\Rightarrow 1 + \frac{n-1}{2} = 3/2$$

$$\Rightarrow \boxed{n = 2}$$

(Q2)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For a satellite motion,

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow \text{Time Period} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi}{\sqrt{GM}} r^{3/2}$$

Also $\frac{GM\cancel{m}}{R_e^2} = \cancel{m}g$

$$\Rightarrow GM = gR_e^2$$

$$\Rightarrow \text{Time Period} = \frac{2\pi}{\sqrt{gR_e^2}} \cdot r^{3/2}$$

$$\Rightarrow T = \frac{2 \times 3.14 \times (3.884 \times 10^8)^{3/2}}{\sqrt{9.8 \times (6400 \times 1000)^2}}$$

$$= \underline{\underline{27.76 \text{ days}}}$$

For geostationary satellite, $T = 1$ day

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow r_2 = r_1 \cdot \left(\frac{T_2}{T_1}\right)^{2/3} = 3.884 \times 10^5 \left(\frac{1}{27.76}\right)^{2/3}$$

$$= \underline{\underline{4.23 \times 10^4 \text{ km}}}$$

$$\approx \underline{\underline{42,300 \text{ km}}}$$

Height = $\underline{35,900 \text{ km}}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q3) We know on moon gravitation is 1/6 th that of Earth.

$$\Rightarrow \frac{GMm}{R^2} = mg \Rightarrow g = \frac{GM_{Earth}}{R^2_{Earth}}$$

$$\frac{g}{6} = \frac{GM_{moon}}{R_M^2}$$

$$\boxed{\frac{M_{moon}}{R_{Moon}^2} = \frac{1}{6} \cdot \frac{M_{Earth}}{R_{Earth}^2}}$$

From Newton's law,

$$\frac{GM_{moon}\,m}{(R_{moon}+H)^2} - \frac{GM_{Earth}\,m}{(d-R_{moon}-H)^2} = m\vec{a}$$

Ignoring earth's effect as $d >> R_{Moon}$ we have,

$$\frac{GM_{oon}}{(R_{moon}+H)^2} = a = \frac{dv}{dt}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Let $R_{Moon} + H = r$

$$\Rightarrow \boxed{-\frac{GM}{r^2} = \ddot{r}}$$

Multiply both sides by $2\dot{r}$

$$\Rightarrow -\frac{GM}{r^2} \cdot 2\dot{r} = 2\dot{r}\,\ddot{r}$$

$$\Rightarrow 2GM \cdot d\left(\frac{1}{r}\right) = d(\dot{r}^2)$$

$$\Rightarrow 2GM \int d\left(\frac{1}{r}\right) = \int d\,(\dot{r}^2)$$

$$\Rightarrow \frac{2GM}{r} + C = \dot{r}^2$$

But at $r \to \infty$, $\dot{r} = 0$

$$\Rightarrow C = 0$$

$$\Rightarrow \frac{2GM}{r} = \dot{r}^2$$

$$\Rightarrow \dot{r} = -\sqrt{\frac{2GM}{r}}$$

[$\dot{r} < 0$ as r reduces with time]

$$\Rightarrow v = \dot{r} = \sqrt{\frac{2GM}{R}}$$: Speed when falls

Radius of moon = 1720 km

$$\Rightarrow v = \sqrt{2\frac{g}{6}\frac{R_M^2}{R_M}} = \sqrt{\frac{g R_M}{3}} = 2370 \text{ m/s}$$

Write accelaration as $\left(\frac{dv}{dt}\right)$

$$a = \frac{dv}{dt} = v\left(\frac{dv}{dr}\right)$$

$$\Rightarrow \frac{GM}{r^2} = v\frac{dv}{dr}$$

$$\Rightarrow \int \frac{GM}{r^2} dr = \int v\,dv$$

$$\Rightarrow GM\left(\frac{1}{r} - \frac{1}{R}\right) = \frac{v^2}{2}$$

$$\Rightarrow \boxed{v \approx \sqrt{\frac{2GM}{R}}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q4)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

At new moon

$$\frac{GM_S \cancel{m_m}}{r_{SM}^2} - \frac{GM_E \cancel{m_m}}{r_{EM}^2} = \cancel{m_m} a_{moon}$$

$$\Rightarrow \boxed{\vec{a}_{moon} = G\left[\frac{M_S}{r_{SM}^2} - \frac{M_E}{r_{EM}^2}\right]}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Solar eclipse को new moon होता है !!

Q5)

For $v < v_0$: fall back to Earth

$v_0 < v < v_E$: Orbit around Earth

$v > v_E$: satellite can go out of Earth's attractive force.

Using the conservation of energy,

$$K.E_i + P.E_i = K.E_f + P.E_f$$

$$K_i + P_i = K.E_f + 0$$

For v_i minimum, $K.E_f$ should just be 0

$$\Rightarrow \frac{1}{2} m v_i^2 - \frac{GMm}{R_E} = 0$$

$$\Rightarrow \boxed{v_i = v_{esc} = \sqrt{\frac{2GM}{R_E}}}$$

$G = 6.67 \times 10^{-11}$

Q7)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

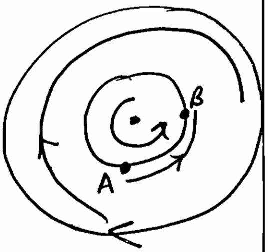
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Time Period of rotation $= \dfrac{2\pi}{\sqrt{g R_e^2}} \times r^{3/2}$

$= \dfrac{2 \times 3.14 \times (20 \times 10^6)^{3/2}}{\sqrt{9.8 \times (6.4 \times 10^6)^2}}$

$= \underline{\underline{0.32 \text{ days}}} = \underline{\underline{7.8 \text{ hours}}}$

→ Point on Earth is also revolving



$\omega_{satellite} = \dfrac{2\pi}{T} = \dfrac{2\pi}{7.8}$ (hrs^{-1})

$\omega_{Point} = \dfrac{2\pi}{24}$ (hrs^{-1})

east to west होना चाहिए!!

$\omega_{satellite, P} = 2\pi \left[\dfrac{1}{7.8} + \dfrac{1}{24} \right] hrs^{-1}$

$\Rightarrow T_{satellite, P} = 5.9 \text{ hrs} \approx 6 \text{ hours}$

Q6)

$\dfrac{\cancel{m} v^2}{\cancel{r}} = \dfrac{GM\cancel{m}}{r^{\cancel{2}}} \Rightarrow v = \sqrt{\dfrac{GM}{r}}$

$= \sqrt{\dfrac{6.67 \times 10^{-11} \times 1.9 \times 10^{27}}{70000}}$

$= 42.54 \text{ km/sec}$

Q8)

«Q1»

Areal velocity $= \dfrac{d}{dt}\left(\dfrac{1}{2} r \times \Delta r\right)$

$= \dfrac{1}{2} r \times \dfrac{\Delta r}{dt} = \dfrac{1}{2} r v$

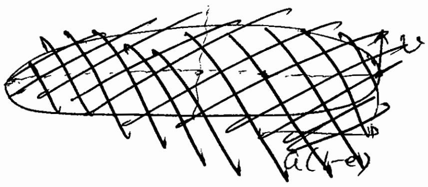
$= \left(\dfrac{mvr}{2m}\right) = \text{const.} = \left(\dfrac{J}{2m}\right)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Also

$$T = \frac{2\pi ab}{vr} = \frac{2\pi a m b}{\sqrt{\frac{k\mu b^2}{a}}} = \frac{2\pi\sqrt{m}}{\sqrt{k}}\, a^{3/2}$$

$$= \propto a^{3/2}$$



$$\boxed{\frac{J^2}{k\mu} = \frac{b^2}{a}} \Rightarrow J = mvr = \sqrt{\frac{k\mu b^2}{a}}$$

Remember this !!

Q9) Additional velocity: $v_{esc} - v_{orb}$

Now for v_{esc}, we have total $E = 0$

$$\Rightarrow \frac{1}{2} m v_{esc}^2 - \frac{GMm}{R_e} = 0$$

$$\Rightarrow v_{esc} = \sqrt{\frac{2GM}{R_e}}$$

For v_{orb} near the surface of earth,

$$\frac{GMm}{r^2} = \frac{m v_{orb}^2}{r}$$

$$\Rightarrow v_{orb} = \sqrt{\frac{GM}{R_e}}$$

$\Rightarrow$ Additional velocity required $= (\sqrt{2} - 1)\sqrt{\frac{GM}{R_e}}$

Also $mg = \frac{GMm}{R^2} \Rightarrow GM = gR_e^2$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

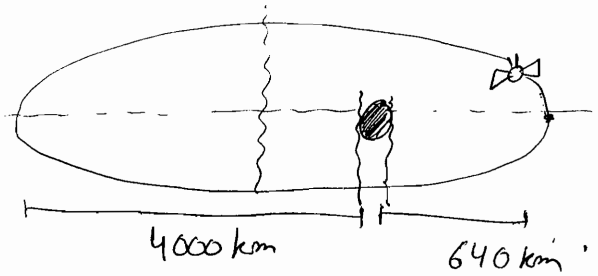
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$\Rightarrow$ Additional velocity $= (\sqrt{2}-1)\sqrt{gR_e}$

$= \underline{\underline{3.27 \text{ km/s}}}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

10



e and a ?

Do not do the same mistake again & again..... Its given from surface of earth... not centre of earth!!!!

Now

$a(1-e) = 640\times10^3 + 6400 = 7040 \text{ km}$

$a(1+e) = 4000\times10^3 + 6400 = 10{,}400 \text{ km}$

$\Rightarrow \frac{1+e}{1-e} = \frac{10400}{7040}$

$\Rightarrow \frac{2e}{2} = \frac{336}{1744} = \underline{\underline{0.192}}$

$\Rightarrow a = \frac{640\times10^3 + 6400\times10^3}{0.807} = \underline{\underline{8720 \text{ km}}}$

11

Let a particle be moving under a central force F.

Since force is central $\vec{F} = F(r)\,\hat{r}$

not $F\hat{\theta}$ is 0

$\Rightarrow \vec{a_\theta} = 0$

$\Rightarrow \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = 0$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$\Rightarrow \quad r^2\dot{\theta} = \text{Const.} \quad — (1)$

Also since torque is 0, we have angular momentum as constant

$\Rightarrow \quad m\dot{\theta}r^2 = \text{Const.} \quad - (2)$

From (1) and (2), let

$m\dot{\theta}r^2 = J$ (Const

~~In order to find trajectory~~

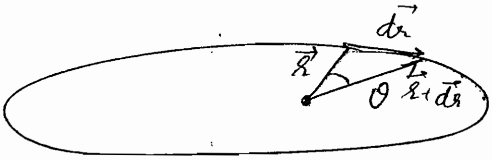
Equation of motion is

$\vec{F}(r) = m a_r \hat{r}$

$\Rightarrow \quad F(r) = m[\ddot{r} - \dot{\theta}^2 r]$

$$\Rightarrow \quad \boxed{\frac{d^2r}{dt^2} - \left(\frac{d\theta}{dt}\right)^2 r - \frac{F(r)}{m} = 0}$$

The aerial velocity is the area swept by the radial vector per unit time,



$d\vec{A} = \frac{1}{2}\vec{r} \times d\vec{r}$

$\frac{d\vec{A}}{dt} = \frac{1}{2}\vec{r} \times \frac{d\vec{r}}{dt} = \frac{1}{2}\frac{\vec{r} \times m\vec{v}}{m} = \left(\frac{\vec{J}}{2m}\right)$

$\Rightarrow \left|\frac{d\vec{A}}{dt}\right| = \left(\frac{J}{2m}\right) = \text{const.}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(12)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For an elliptical orbit,
Time Period T is given by

$$T = \frac{\pi ab}{J} 2\mu$$

Also semilactus rectum $l = \frac{b^2}{a} = \frac{J^2}{k\mu}$

$$\Rightarrow T = \frac{\pi ab\, 2\mu}{\sqrt{\frac{b^2}{a} k\mu}}$$

Now $k = GMm$

$$\Rightarrow T = \frac{\pi ab \cdot 2\mu}{\sqrt{\frac{b^2}{a} \cdot GM\mu^2}} = \frac{2\pi a\cancel{b}\sqrt{a}}{\cancel{b}\sqrt{GM}}$$

$$= \frac{2\pi}{\sqrt{GM}} a^{3/2}$$

$$75 \times 365 \times 24 \times 60 \times 60 = \frac{2\pi}{\sqrt{6.67 \times 10^{-11} \times 2 \times 10^{30}}} \times a^{3/2}$$

$$\boxed{a = 2.664 \times 10^{12}\ m}$$

$$\boxed{= 2.664 \times 10^{9}\ km}$$

Now, ~~Ignoring~~

$$a(1-e) = 8.9 \times 10^{7}$$

$$a(1+e) = 5.239 \times 10^{9}\ km$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q 13)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Let the mass of the particle be m.
Since Force is Central

$$\vec{a}_\theta = 0$$

$$\vec{a}_r = (\ddot{r} - \dot{\theta}^2 r)\hat{r}$$

$$F(r) = m(\ddot{r} - \dot{\theta} r) \quad \text{— (1)}$$

Also since torque $= 0$, $\Rightarrow$ angular momentum remains conserved

$$\Rightarrow m\dot{\theta} r^2 = J \quad (\text{const})$$

$$\Rightarrow \dot{\theta} = \frac{J}{mr^2}$$

Now, Put $r = \frac{1}{u} \Rightarrow \frac{dr}{du} = \frac{-1}{u^2}$

$$\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{J}{mr^2} \cdot \left(\frac{dr}{du}\right)\left(\frac{du}{d\theta}\right)$$

$$= -\frac{J}{m} u^2 \cdot \frac{1}{u^2}\left(\frac{du}{d\theta}\right)$$

$$\boxed{\frac{dr}{dt} = -\frac{J}{m}\left(\frac{du}{d\theta}\right)}$$

$$\frac{d^2 r}{dt^2} = -\frac{J}{m} \cdot \frac{d^2u}{d\theta^2} \cdot \frac{d\theta}{dt} = -\frac{J^2}{m^2} u^2 \left(\frac{d^2u}{d\theta^2}\right)$$

From (1),

$$\frac{F}{m} = -\frac{J^2}{m^2} u^2 \left(\frac{d^2u}{d\theta^2}\right) - \frac{J^2}{m^2} u^3$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\boxed{\frac{d^2u}{d\theta^2} + u = -\frac{m F}{J^2 u^2}} \quad — (2)$$

Now, for gravitation like central force, let $F = \frac{-k}{r^2} = -ku^2$

$$\Rightarrow \frac{d^2u}{d\theta^2} + u = +\frac{m.k}{J^2}$$

$$\Rightarrow u = \frac{mk}{J^2} + A\cos(\theta-\theta_0) \quad \text{where } A : \text{const.}$$

$$\Rightarrow \frac{(J^2/mk)}{r} = 1 + \left(\frac{AJ^2}{mk}\right)\cos(\theta-\theta_0) \longrightarrow (3)$$

comparing with conic section,

$\frac{l}{r} = 1 + e\cos(\theta-\theta_0)$, we have

(3) as a plane motion equation, we

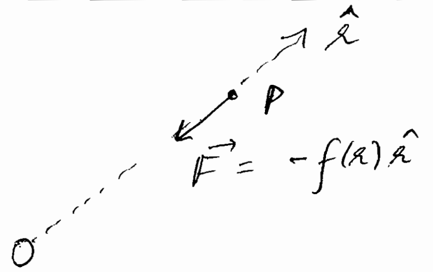
semi latus rectum $= \left(\frac{J^2}{mk}\right)$ and

$$e = \left(\frac{AJ^2}{mk}\right)$$

Constant A can be found out using total Energy E.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

2nd Solution



Let the force experienced by the particle be $-f(r)\,\hat{r}$

Now,

$$\vec{L} = m\vec{r} \times \vec{v}$$
$$= m\vec{r} \times (\dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta})$$
$$= m\vec{r} \times r\dot{\theta}\,\hat{\theta}$$
$$= mr^2\dot{\theta}\,(\hat{r} \times \hat{\theta})$$

Now $\vec{\tau} = \dfrac{d\vec{L}}{dt}$

Also $\vec{\tau} = \vec{F} \times \vec{r} = -f(r)\,\hat{r} \times r\,\hat{r} = 0$

$\Rightarrow \tau = 0$

$\Rightarrow \vec{L} = \text{const.}$

Also as $\vec{L} = mr^2\dot{\theta}\,(\hat{r} \times \hat{\theta})$

$\therefore$ $\vec{L}_0$ is perpendicular to both $\hat{r}$ and $\hat{\theta}$. As $\vec{L}_0$ is constant, $\therefore$ $\hat{r}$ and $\hat{\theta}$ always lie in the plane perpendicular to $\vec{L}_0$, which implies that the path of the particle is a planar curve.

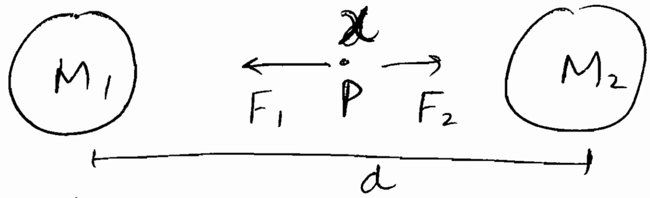
कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

Q14) From kepler 3rd law, $T^2 \propto R^3$

$$\Rightarrow \frac{R_N}{R_E} = \left(\frac{T_N}{T_E}\right)^{2/3} = \left(\frac{165}{1}\right)^{2/3}$$

$$\Rightarrow \boxed{R_N = 30 R_E}$$

Q15)



At P, $F_1 = F_2$

$$\Rightarrow \sqrt{\frac{M_1}{(x)^2}} = \sqrt{\frac{M_2}{(d-x)^2}}$$

$$\Rightarrow \frac{d-x}{x} = \sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow \frac{d}{x} = \frac{\sqrt{M_2} + \sqrt{M_1}}{\sqrt{M_1}}$$

$$\Rightarrow \boxed{x = \frac{\sqrt{M_1}\, d}{\sqrt{M_2} + \sqrt{M_1}}} \qquad d-x = \frac{\sqrt{M_2}\, d}{\sqrt{M_1} + \sqrt{M_2}}$$

$$V = \left(-\frac{GM_1}{x}\right) + \left(-\frac{GM_2}{d-x}\right)$$

$$= -\frac{GM_1(\sqrt{M_1}+\sqrt{M_2})}{\sqrt{M_1}\, d} - \frac{GM_2(\sqrt{M_1}+\sqrt{M_2})}{\sqrt{M_2}\, d}$$

$$= -\frac{G\sqrt{M_1}+\sqrt{M_2}}{d}\left[\sqrt{M_1}+\sqrt{M_2}\right]$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$V = -\frac{G}{d}\left[M_1 + M_2 + 2\sqrt{M_1 M_2}\right]$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Tut Sheet 4 : Rigid Body Dynamics

Q1) For a top, lets us a consider it as symmetrical body.

i.e. $I_{xx} = I_{yy}$ and Cross terms are 0

$$\Rightarrow \boxed{\vec{J} = I\vec{\omega}}$$

○ Phenomenon of rotation of axis of rotation when a torque acts perpendicular to the axis of rotation is called Precession....

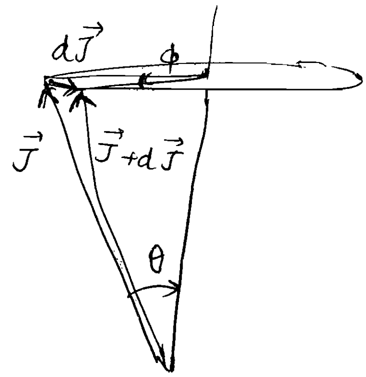


Fig 1

For such a spinning top, torque τ be applied perpendicular to Angular Momentum. Such a torque will not change the magnitude of Angular Momentum but only its direction.

Let such a torque change the angular momentum $\vec{J}$ by $d\vec{J}$, as shown in Fig 1.

$$\Rightarrow \vec{\tau} = \frac{d\vec{J}}{dt}$$

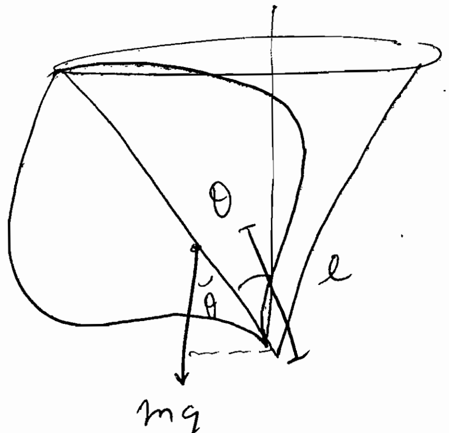
Also $|d\vec{J}| = J\sin\theta \cdot d\phi$
(from Fig 1)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow d\tau = \frac{J \sin\theta \, d\phi}{dt} = J \sin\theta \left(\frac{d\phi}{dt}\right)$$

$\left(\frac{d\phi}{dt}\right)$ is defined as precessional velocity. ω_p.

$$\Rightarrow \boxed{\omega_p = \frac{\tau}{J \sin\theta}}$$



For the shown top,

$\tau = mgl \sin\theta$

$$\Rightarrow \omega_p = \frac{mgl}{J} = \left(\frac{mgl}{I\omega}\right)$$

Now for solid sphere $\boxed{I = \frac{2}{5} mr^2}$

$$\Rightarrow \omega_p = \frac{\cancel{m} g l}{\frac{2}{5} \cancel{m} r^2 \omega} = \frac{g (5+20)\times 10^{-3}}{\frac{2}{5} \times 4\times 10^{-4} \times 20 \times 2 \times \pi}$$

$$= \underline{12.19 \text{ rad/s}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q2)

(Q1)

$$\omega_p = \frac{mgl}{I\omega} = \frac{\cancel{m}gl}{\cancel{m}k^2\omega} = \left(\frac{gl}{k^2\omega}\right)$$

$$= \frac{9.8 \times \cancel{\pi} \times 100}{25 \times 20 \times 2\cancel{\pi}} = \underline{\underline{0.98 \text{ rad/s}}}$$

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

(Q3) For a single particle, angular momentum $\vec{L}$ is defined as moment of the linear momentum $\vec{p}$

i.e. $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

For a rotating rigid body, let it be composed of N particles.

$$\Rightarrow \vec{L} = \sum_i \vec{r_i} \times m_i \vec{v_i}$$

Also $\vec{v_i} = \vec{\omega} \times \vec{r_i}$

where $\vec{\omega}$ is the angular velocity of the rotating body

$$\Rightarrow \vec{L} = \sum_i \vec{r} \times m_i (\vec{\omega} \times \vec{r_i})$$

$$= \sum_i m_i \left[\vec{\omega}(r_i^2) - \vec{r_i}(\vec{r} \cdot \vec{\omega})\right]$$

$$= \sum_i m_i \left[(\omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k})(x_i^2 + y_i^2 + z_i^2) - (x_i \hat{i} + y_i \hat{j} + z_i \hat{k})(\omega_x x_i + \omega_y y_i + \omega_z z_i)\right]$$

$$\Rightarrow \vec{L}_x = \sum_i m_i \left[\omega_x (x_i^2 + y_i^2 + z_i^2) - x_i [\omega_x x_i + \omega_y y_i + \omega_z z_i]\right]$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow L_x = \sum_i m_i (y_i^2 + z_i^2)\, \omega_x - \sum_i x_i m_i (\omega_y y_i + \omega_z z_i)$$

$$= I_{xx}\, \omega_x + I_{xy}\, \omega_y + I_{xz}\, \omega_z$$

where

$$I_{xx} = \sum m_i (y_i^2 + z_i^2)$$

$$I_{xy} = -\sum m_i (x_i y_i)$$

$$I_{xz} = -\sum m_i (x_i z_i)$$

Similarly

$$\vec{L} = \overleftrightarrow{I}\, \vec{\omega}$$

where

$$\overleftrightarrow{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

Time derivative of Angular Momentum is ~~[illegible]~~ given by

i.e. $\left(\frac{d\vec{L}}{dt}\right)$

$$= \frac{d}{dt} \sum (\vec{r}_i \times m_i \vec{v}_i)$$

$$= \sum m_i \left[\frac{d\vec{r}_i}{dt} \times \vec{v}_i\right] + \vec{r}_i \times m_i \left(\frac{d\vec{v}_i}{dt}\right)$$

$$\boxed{= \sum (\vec{r}_i \times \vec{F}_i)}$$ = nothing but Torque acting on the body

Hence $\vec{\tau} = \left(\frac{d\vec{L}}{dt}\right)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

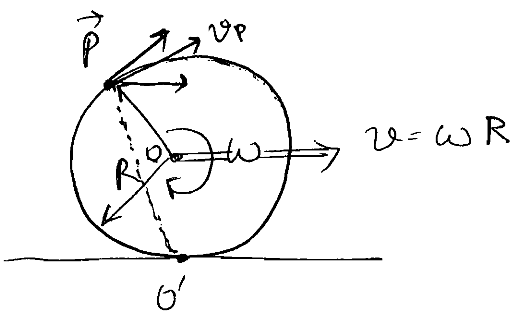
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

If $\tau = 0$

$$\Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{Const.}$$

i.e. Law of Conservation of Angular Momentum says that in absence of external torque, Angular Momentum of a body remains conserved. It finds application in motion of all bodies under central force where torque $= 0$

Q4)



For a rolling body, $v = \omega R$

$$\vec{v}_P = \vec{v}_{P,CM} + \vec{v}_{CM}$$

$$= \vec{\omega} \times \vec{r}_{OP} + \vec{\omega} \times \vec{R}_{CM}$$

$$= \vec{\omega} \times (R_{CM} + \vec{r})$$

$$= \underline{\omega \times \vec{r}_{O'P}}$$

Hence $\vec{v}_P$ is $\perp$ to $r_{O'P}$ i.e. equivalent to rotation about point of contact.

Kinetic Energy =

$$= \sum \frac{1}{2} m_i v_i^2$$

$$= \sum \frac{1}{2} m_i (\vec{\omega} \times \vec{r}_{O'P})^2$$

$$= \sum \frac{1}{2} m_i \omega^2 d_i^2 \qquad [d_i = \overline{O'P'}]$$

Prove for

1 scalar

K.E.

and

1 vector

velocity

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$= \left(\sum \frac{1}{2} m_i d_i^2\right) \omega^2$$

$$\boxed{T = \frac{1}{2} I_P \omega^2}$$

$I_P = \sum' m_i d_i^2$

= moment of inertia about point of contact.

For a circular plate or type (not sphere)

$$I_P = I_C + m r^2$$

$$\Rightarrow T = \frac{1}{2} (I_C + m r^2) \omega^2$$

$$= \frac{1}{2} I_C \omega^2 + \frac{1}{2} m (\omega r)^2$$

$$\boxed{T = \frac{1}{2} I_C \omega^2 + \frac{1}{2} m v_c^2}$$

First term represents k.E. due to rotation about axis through COM, and other term represents translation of COM (1)

(Q5) For a body with principal axes x, y, z, we have

$$\vec{J} = \omega_x I_x \hat{i} + \omega_y I_y \hat{j} + \omega_z I_z \hat{k}$$

(3) Now $$\tau = \left(\frac{dJ}{dt}\right) = \left(\frac{dJ}{dt}\right)_r + (\vec{\omega} \times \vec{J})$$

$$\vec{\omega} \times \vec{J} = \begin{vmatrix} i & j & k \\ \omega_x & \omega_y & \omega_z \\ J_x & J_y & J_z \end{vmatrix} = \sum i [\omega_y J_z - \omega_z J_y]$$

$$\Rightarrow N_x = I_x \dot{\omega}_x - (I_y \omega_y \omega_z - I_z \omega_y \omega_z)$$

(2)

(*) Now this equation holds for all inertial frames. Considering an intertial frame S. Let S' be the frame attached to the body $\Rightarrow \left(\frac{d\vec{L}_0}{dt}\right)_S = \left(\frac{dL_0}{dt}\right)_{S'} + \vec{\omega} \times \vec{L}_0$

(*) 1st line will be

$$\vec{N}_{ext,0} = \frac{d\vec{L}_0}{dt}$$

where O is the fixed point on the body, $\vec{N}_{ext,0}$ and $\vec{L}_0$ are torques & angular momentum of the body about the fixed point

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow N_x = I_x \dot{\omega}_x - (I_y - I_z)\,\omega_y \omega_z$$

$$N_y = I_y \dot{\omega}_y - (I_z - I_x)\,\omega_z \omega_x$$

$$N_z = I_z \dot{\omega}_z - (I_x - I_y)\,\omega_x \omega_y$$

These are called Euler's Equations of a rigid body.

For a $\boxed{\text{spherical top}}$, $I_x = I_y = I_z = I$

$$\Rightarrow \vec{N} = I \vec{\omega}$$

Since the reaction at the point of contact exerts no Torque, we have

$$\vec{J} = \text{Const} = I\vec{\omega}.$$

For a $\boxed{\text{symmetrical top}}$, $I_x = I_y \neq I_z$

$$\Rightarrow \cancel{N_x = 0}$$

$$I_x \dot{\omega}_x = (I_y - I_z)\,\omega_y \omega_z$$

$$\dot{\omega}_x = \left[\left(1 - \frac{I_z}{I_x}\right)\omega_z\right]\omega_y$$

define $\left(1 - \dfrac{I_z}{I_x}\right)\omega_z = \Omega$

$$\Rightarrow \dot{\omega}_x = \left(1 - \frac{I_z}{I_x}\right)\omega_z \omega_y = \Omega\,\omega_y \quad \text{--- (1)}$$

Also $I_y \dot{\omega}_y = (I_z - I_x)\,\omega_z \omega_x$

$$\Rightarrow \dot{\omega}_y = \left(\frac{I_z}{I_y} - 1\right)\omega_z \omega_x$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \dot{\omega}_y = -\Omega\,\omega_x \quad \text{— (2)}$$

From (1), $\ddot{\omega}_x = \Omega\,\dot{\omega}_y$

$$= \underline{\underline{-\Omega^2\omega_x}} \quad (\text{from (2)}) \quad \text{— (3)}$$

Similarly $\ddot{\omega}_y = -\Omega\,\dot{\omega}_x$

$$= \underline{\underline{-\Omega^2\omega_y}} \quad \text{— (4)}$$

Also $I_z\dot{\omega}_z = (I_x - I_y)\,\omega_x\omega_y$

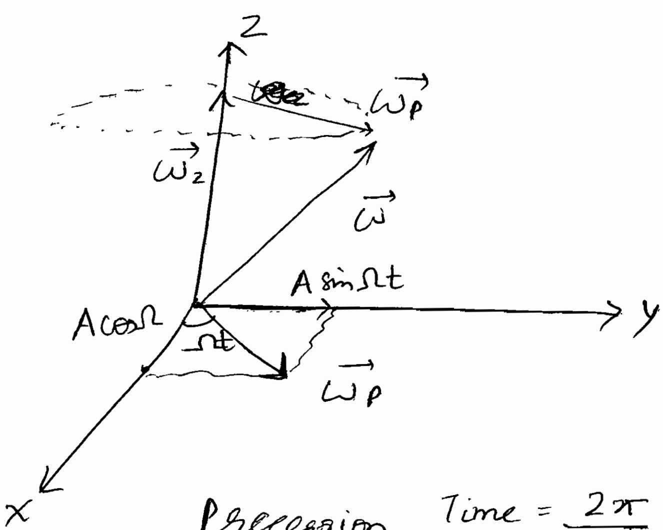
$$= 0$$

$$\Rightarrow \underline{\underline{\omega_z = \text{const}}} \quad \text{— (5)}$$

$$\boxed{\vec{\omega} = A\cos\Omega t\,\hat{i} + A\sin\Omega t\,\hat{j} + \omega_z\,\hat{k}}$$

$$= \vec{\omega}_p + \omega_z\,\hat{k}$$

$$|\vec{\omega}_p| = A = \text{const}$$



Precession Time $= \dfrac{2\pi}{\Omega} = \dfrac{2\pi}{\omega_3\left(1 - \dfrac{I_z}{I_x}\right)}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q6) <Q3>

Elements $I_{xx} = \sum (y_i^2 + z_i) m_i$

$I_{yy} = \sum (z_i^2 + x_i^2) m_i$

$I_{zz} = \sum (x_i^2 + y_i^2) m_i$

are called Moments of Inertia

while the elements

$I_{xy} = -\sum m_i x_i y_i$

$I_{yz} = -\sum m_i y_i z_i$

$I_{zx} = -\sum m_i z_i x_i$

are called Products of Intertia.

Q7) <Q5>

Q8) <Q3>

For a spherical top, $I_{xx} = I_{yy} = I_{zz} = I$

& $I_{xy} = I_{yz} = I_{zx} = 0$

$\therefore L_x = I\omega_x$

$L_y = I\omega_y$

$L_z = I\omega_z$

$\Rightarrow \vec{L} = I\vec{\omega}$

But for free rotation $\Rightarrow$ NO Torque

$\frac{d\vec{L}}{dt} = 0$

$\Rightarrow \vec{L} = $ Constant

$\Rightarrow \vec{\omega} = $ const

$\therefore$ The most general free rotation of a spherical top is a uniform rotation about an axis fixed in space whose direction is given by direction of $\vec{\omega}$.

Q9)

For principle axes,

$$\vec{J} = I_x \omega_x \hat{i} + I_y \omega_y \hat{j} + I_z \omega_z \hat{k}$$

$$K.E. = \frac{1}{2} \vec{J} \cdot \vec{\omega}$$

$$K = \frac{1}{2} \left[I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 \right]$$

For no external Torque, kinetic energy will remain same,

$$\Rightarrow \frac{\omega_x^2}{\left(\frac{2K}{I_x}\right)} + \frac{\omega_y^2}{\left(\frac{2K}{I_y}\right)} + \frac{\omega_z^2}{\left(\frac{2K}{I_z}\right)} = 1$$

This is called <u>Ellipsoid of Inertia</u> or <u>Poinsot Equation of Motion</u>.

Principle axes are those axis of the body about which the cross terms of moment of inertia vanish.

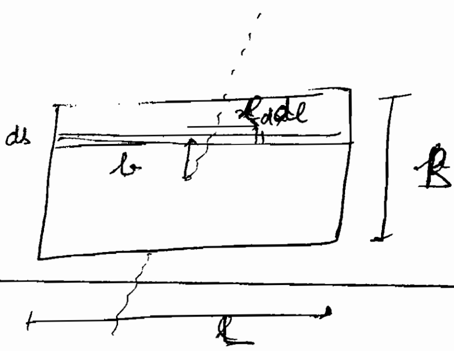
i.e. $I_{xy} = I_{yz} = I_{zx} = 0$

The moments about the principle axes are called Principal moments viz.

I_{xx}, I_{yy}, I_{zz}

For the strip shown

$$dI = dm\,(b^2 + l^2)$$
$$= \sigma\, ds\, dl \,.(b^2 + l^2)$$



कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

$$I_{ds} = \sigma\, ds \int_{-L/2}^{L/2} \left[b^2 + l^2\right] dl$$

$$= \sigma\, ds \,.\left[b^2 L + \frac{L^3}{12}\right]$$

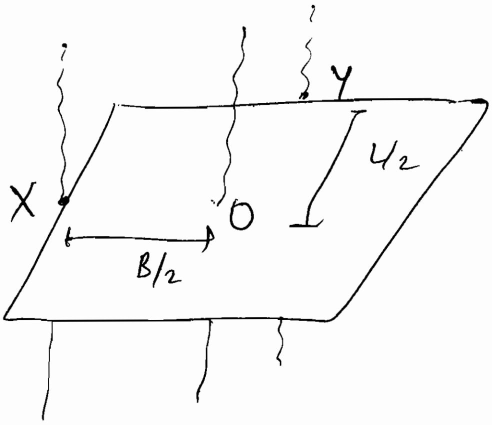
$$I = \int I_{ds} = \int_{-B/2}^{B/2} \sigma b^2 L\, ds + \frac{\sigma L^3}{12} \int_{-B/2}^{B} ds$$

$$= \sigma \,.\, L \,.\, \frac{B^3}{12} + \frac{\sigma L^3}{12} \,.\, B$$

$$= \sigma \,.\, \frac{BL}{12}\left[B^2 + L^2\right]$$

$$= \frac{M}{\cancel{BL}} \,.\, \frac{\cancel{BL}}{12}\left[B^2 + L^3\right]$$

$$\boxed{I = \frac{M}{12}\left[B^2 + L^2\right]}$$



$$I_X = I_0 + m\frac{B^2}{4}$$

$$= \frac{M}{12}\left[B^2 + L^2\right] + \frac{MB^2}{4}$$

$$= \frac{M}{12}\left[4B^2 + L^2\right]$$

$$I_y = I_0 + m \,.\, \frac{L^2}{4}$$

$$= \frac{M}{12}\left[4L^2 + B^2\right]$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q10)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

No. of independent variables = 3×3
$= 9$

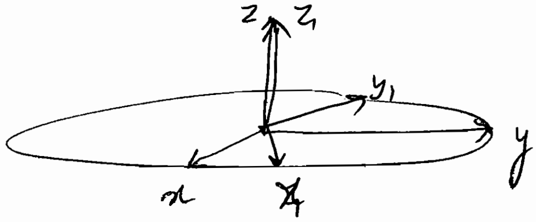
(No. of particles * No. of coordinates to describe them)

No. of Contraint equation = 3
($\because$ d_{12}, d_{23}, d_{13} are fixed)

$\Rightarrow$ No. of independent variable = Degree of freedom = $9 - 3 = 6$

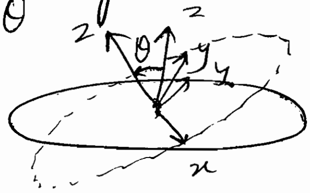
The Euler Angles ϕ, θ, ψ are defined as:
Let x, y, z be the three space axes & x', y', z' be the three body axes. The set (x, y, z) is transformed to (x', y', z') by three successive rotations. These angles of the rotations, namely, Euler Angles define the orientation of (x', y', z') w.r.t. (x, y, z).

First rotation is about z-axis by angle ϕ, to give x_1, y_1, z_1

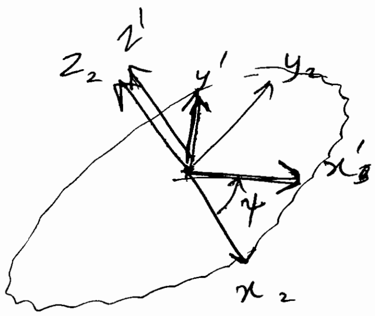


Second rotation is about either of the x or

y_1 axis say, x_1 axis. to give (x, y_2, z_2) by angle θ



The third rotation is about z_2 axis to give (x', y', z') by angle ψ



<Q1 Tut 27

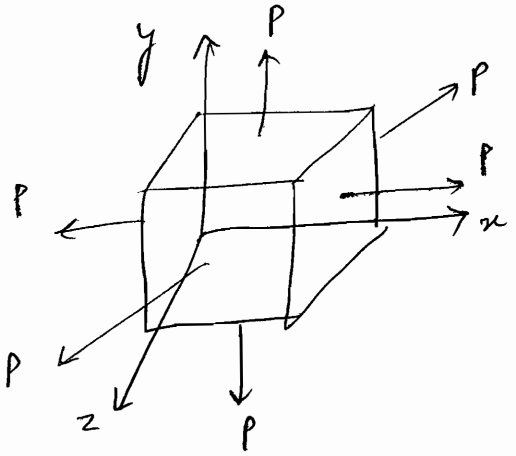
कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

Tut 5 : Mechanics of Continuous Media

Q1) (30 marks)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Consider a cube of length l_o, on which a Force is applied equally on all faces such that a stress P is created.



Now Young's modulus $Y = \left(P/\frac{dl}{l}\right)$

and Poisson Ratio $\sigma = \dfrac{(dl_\perp/l_\perp)}{(dl_\parallel/l_\parallel)}$

$\Rightarrow$ due to stress along x direction

$$l_x = l_o + \frac{l_o P}{Y}$$

$$l_y = l_o - \frac{l_o P}{Y}\sigma$$

$$l_z = l_o - \frac{l_o P\sigma}{Y}$$

Similarly considering stress along all directions

$$l_x = l_o + \frac{l_o P}{Y} - \frac{2\sigma l_o P}{Y}$$

$$l_y = l_o + \frac{l_o P}{Y} - \frac{2\sigma l_o P}{Y}$$

$$l_z = l_o + \frac{l_o P}{Y} - \frac{2\sigma l_o P}{Y}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$\Rightarrow V' = \cancel{l_x} l_y l_z$

$= \left[l_0 + \frac{l_0 P}{Y}(1-2\sigma) \right]^3$

$= l_0^3 \left[1 + \frac{3P}{Y}(1-2\sigma) \right]$ (from Binomial Approximation)

$\Rightarrow \frac{\Delta V}{V} = \frac{3P}{Y}(1-2\sigma)$

Also, $K = \frac{P}{\left(\frac{dV}{V}\right)}$

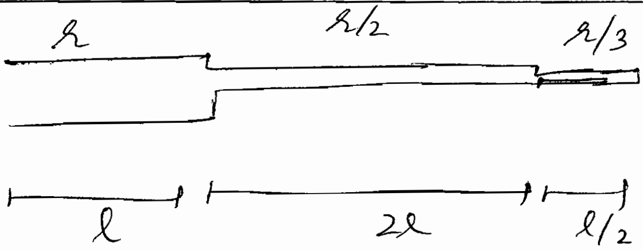
$\Rightarrow \frac{1}{k} = \frac{3}{Y}(1-2\sigma)$

$\Rightarrow \boxed{Y = 3k(1-2\sigma)}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q2) The flow of the fluid is said to be streamlined or laminar flow, if the velocity at every point in the fluid remains constant, in magnitude as well as direction.

Fluid flow remains streamlined till velocity does not exceed a limiting value called the critical ~~velocity~~ velocity, beyond which flow becomes turbulent.



Now the flow of liquid through a capillary tube is defined by Poissliew's formula as

$$Q = \left(\frac{P\pi a^4}{8\eta l}\right)$$

where a: radius of capillary
P: Pressure across capillary
η: Viscosity of fluid
l: length of capillary.

$$\Rightarrow P = \left(\frac{8Q\eta l}{\pi a^4}\right)$$

Now since the tubes are in series, the flow in each will be same.

$$\Rightarrow P \propto \left(\frac{l}{a^4}\right)$$

$$\Rightarrow P_1 : P_2 : P_3 = \frac{l_1}{a_1^4} : \frac{l_2}{a_2^4} : \frac{l_3}{a_3^4}$$

$$= \frac{l}{r^4} : \frac{2l}{\frac{r^4}{16}} : \frac{l}{2 \cdot \frac{r^4}{81}}$$

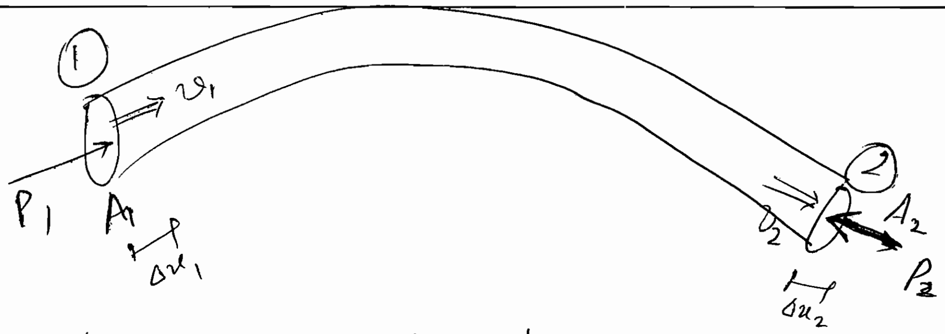
$$= 1 : 32 : \left(\frac{81}{2}\right)$$

$$\Rightarrow \underline{\underline{P_2 = 32P}}, \quad \underline{\underline{P_3 = \frac{81}{2}P}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

3

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)



Let us consider two cross sections across the flow of a ~~capillary~~ fluid. Let be area, velocity of flow, Pressure and their heights be (A_1, v_1, P_1, h_1) and (A_2, v_2, P_2, h_2)

Let in Δt time, fluid flow be Δx_1 across 1st cross section and Δx_2 across 2nd crossection,

Work Done in movement of liquid $= F_1 \Delta x_1 - F_2 \Delta x_2$

$= P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$

$= P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t$

Since fluid is incompressible

$$\Delta m = A_1 \Delta x_1 \rho = A_2 \Delta x_2 \rho$$

$$\Rightarrow \boxed{A_1 v_1 = A_2 v_2}$$

This is called Continuity equation.

$$\Rightarrow \Delta W = P_1 \frac{\Delta m}{\rho} - P_2 \frac{\Delta m}{\rho}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Now work done ΔW results in change in energy

$$\Rightarrow \Delta W = \frac{1}{2}\Delta m(v_2^2 - v_1^2) + \Delta m g(h_2 - h_1)$$

$$\Rightarrow \frac{P_1 \Delta m}{\rho} - \frac{P_2 \Delta m}{\rho} = \frac{1}{2}\Delta m(v_2^2 - v_1^2) + \Delta m g(h_2 - h_1)$$

$$\Rightarrow P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$= \text{Const.}$$

Hence Total energy i.e. sum of Pressure energy, kinetic Energy and Potential Energy remains constant througout the flow.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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Tut Sheet 6 STR

Q1) For a frame S' moving in +x direction w.r.t. a frame S, the Lorentz transformation give out,

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

Since the two event are simultaneous in S $\Rightarrow t_1 = t_2$

$$\Rightarrow t_1' = \frac{t_1 - \frac{v}{c^2}x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_2' = \frac{t_2 - \frac{v}{c^2}x_2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow t_1' - t_2' = \frac{v}{c^2}\frac{(x_2 - x_1)}{\sqrt{1-\frac{v^2}{c^2}}}$$

Since events do not take place at a constant point $\Rightarrow x_1 \neq x_2$

$\Rightarrow t_1' - t_2' \neq 0 \Rightarrow t_1' \neq t_2' \Rightarrow$ Not simultaneous.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

1 000,000,000

Q 2)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

1 BeV = 1 billion eV
= 1 GeV

Now $E = m_0c^2 + T = \sqrt{p^2c^2 + (m_0c^2)^2}$

For proton $m_0c^2 = 938$ MeV
$= 0.938$ BeV

$\Rightarrow 0.938 + 1 = \sqrt{p^2c^2 + (0.938)^2}$

$\sqrt{2.876} = 1.695$ BeV $= p^2c^2$

$\Rightarrow p = 1.695$ BeV/c

$= \underline{\underline{9.04 \times 10^{-19}}}$ kg m/s

Q3)

From lorentz transform, for a frame S' travelling in +x direction with velocity v w.r.t. frame S, we have

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow x'^2 + y'^2 + z'^2 - c^2 t'^2$$

$$= \frac{(x - vt)^2 - c^2\left(t - \frac{v}{c^2}x\right)^2}{\left[1 - \frac{v^2}{c^2}\right]} + y^2 + z^2$$

$$= \frac{x^2 + v^2t^2 - \cancel{2xvt} - c^2t^2 - \frac{v^2}{c^2}x^2 + \cancel{2tvx}}{1 - \frac{v^2}{c^2}} + y^2 + z^2$$

$$= \frac{x^2\left[1 - \frac{v^2}{c^2}\right] - c^2t^2\left[1 - \frac{v^2}{c^2}\right]}{1 - \frac{v^2}{c^2}} + y^2 + z^2$$

$$= x^2 + y^2 + z^2 - c^2t^2$$

$\therefore$ $x^2 + y^2 + z^2 - c^2t^2$ is lorentz invariant

Q4) From inverse Lorentz transform

$$x = \frac{x' + vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$y = y'$

$z = z'$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow x = \frac{60 + \frac{3c}{5} \cdot 8 \times 10^{-8}}{\sqrt{1 - \left(\frac{3}{5}\right)^2}}$$

$$= \frac{60 + \frac{3}{5} \times 3 \times 8}{\left(\frac{4}{5}\right)} = \underline{\underline{93\ m}}$$

$$t = \frac{8 \times 10^{-8} + \frac{3\cancel{c}}{5c^{\cancel{2}}} \times 60}{\sqrt{1 - \left(\frac{3}{5}\right)^2}}\ s$$

$$= \frac{8 \times 10^{-8} + 12 \times 10^{-8}}{\left(\frac{4}{5}\right)}\ s$$

$$= \frac{\overset{5}{\cancel{20}}}{\cancel{4}} \times 5 \times 10^{-8}\ s$$

$$= \underline{\underline{25 \times 10^{-8}\ s}}$$

Q5) From lorentz transforms, for a frame . . . .

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Taking differentials

$$\Rightarrow dx' = \frac{dx - v\,dt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$dy' = dy$$

$$dz' = dz$$

$$dt' = \frac{dt - \frac{v}{c^2}\,dx}{\sqrt{1-\frac{v^2}{c^2}}}$$

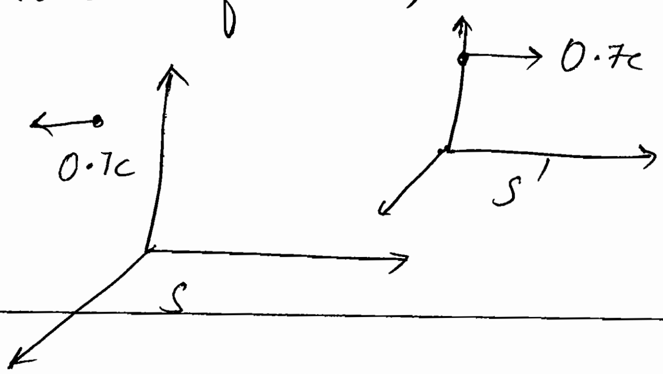
From these equations

$$u_x' = \left(\frac{dx'}{dt'}\right) = \frac{dx - v\,dt}{dt - \frac{v}{c^2}\,dx} = \left(\frac{u_x - v}{1 - \frac{v u_x}{c^2}}\right)$$

$$u_y' = \frac{dy'}{dt'} = \frac{dy\sqrt{1-\frac{v^2}{c^2}}}{dt - \frac{v}{c^2}\,dx} = \left(\frac{u_y\sqrt{1-\frac{v^2}{c^2}}}{1 - \frac{v u_x}{c^2}}\right)$$

$$u_z' = \frac{dz'}{dt'} = \left(\frac{u_z\sqrt{1-\frac{v^2}{c^2}}}{1 - \frac{v u_x}{c^2}}\right)$$

These are referred to as addition theorems of velocity.



कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Consider two frames S and S' where S' moves at $v = 0.7c$ wrt from S.
Let us consider a particle moving in frame S with $u_x = -0.7c$,
u_x' of the particle wrt. frame S' will be,

$$u_x' = \frac{-0.7c - 0.7c}{1 + \frac{0.7c \; 0.7c}{c^2}}$$

$$= \frac{-1.4}{1+0.49} c$$

$$= \underline{\underline{-0.939c}}$$

$\Rightarrow$ By an analogy, $v_{A,B} = -0.939c$
i.e. they will see each other as moving away with $v = 0.939c$

(Q6) The half life of μ meson in its own frame of reference (proper time) is 2×10^{-8} s

When observed by the person on a stationary [redacted] frame, like on earth, time dilation will be observed.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow T_{earth} = \frac{2\times10^{-8}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2\times10^{-8}}{\sqrt{1-\frac{1}{4}}}$$

$$= \underline{\underline{2.31\times10^{-8}}}\ s$$

Q7)

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

If $m = 2m_0$

$$\Rightarrow 2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 1-\frac{v^2}{c^2} = \frac{1}{4}$$

$$\Rightarrow \frac{v}{c} = \frac{\sqrt{3}}{2} \Rightarrow \boxed{v = \frac{\sqrt{3}}{2}c}$$

Q8) For a frame S' moving at velocity v wrt. frame S at a speed of v in +x direction, we have by Lorentz equations

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} ; y' = y ; z' = z ;$$

$$t' = \frac{\left(t - \frac{v}{c^2}x\right)}{\sqrt{1-\frac{v^2}{c^2}}}$$

For $\frac{v}{c} << 1$ $\quad 1-\frac{v^2}{c^2} \approx 1$

$$\Rightarrow \left\{ \begin{array}{l} x' = x - vt \ ; y' = y \ ; z' = z \\ t' = t \end{array} \right\}$$ These are Galilean transformations

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

The physical significance of Lorentz Transformations are :

(1) They preserve speed of light in addition of velocities.

(2) Using Lorentz transformation, length contraction, time dilation & relativity of simultaneity can be proved.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q9) For a body (cube) moving at velocity v wrt frame S in body frame of reference S', we have

$$\text{density} = \left(\frac{m_0}{l_0^3}\right)$$

In frame of S, we have

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$l_x = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$l_y = l_0$$

$$l_z = l_0$$

$$\Rightarrow \text{density} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}\; l_0^3 \sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{d_0}{\left(1-\frac{v^2}{c^2}\right)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Hence density increases for a moving body.

Q10)

$$\frac{19.3 \times 10^3}{\left(\sqrt{1-(0.9)^2}\right)^2} \ kg/m^3 = 101.57 \times 10^3 \ kgm^{-3}$$

Q11) Let the ~~relative~~ increase in velocity be dv. The ~~relative~~ increase in Energy

$$dE = d\left[\frac{m_0 C^2}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$= \frac{1}{2}\frac{m_0 c^2}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \cdot \frac{-1}{c^2} \cdot 2v \, dv$$

$$= \frac{E}{\left(1-\frac{v^2}{c^2}\right)} \cdot \frac{v}{c^2} \cdot dv$$

$$\boxed{dE = \frac{E v \, dv}{(c^2-v^2)}}$$

$$\cancel{\Rightarrow \frac{dE}{dv} = \left(\frac{E v}{c^2-v^2}\right)}$$

$$\boxed{\left(\frac{dE}{E}\right) = \left(\frac{v^2}{c^2-v^2}\right)\left(\frac{dv}{v}\right)}$$

$\therefore$ relative increase in Energy $\left(\frac{dE}{E}\right)$ is $\left(\frac{v^2}{c^2-v^2}\right)$ times the relative increase in velocity.

★ change $= \Delta x$

relative change $= \left(\frac{\Delta x}{x}\right)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q 12)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$0.51 + 1 = \sqrt{p^2c^2 + (0.5)^2}$$

(*) Always prefer $E = mc^2$ to $E = \sqrt{p^2c^2 + (m_0c^2)^2}$

$$\Rightarrow pc = 1.42 \text{ MeV}$$

$$\Rightarrow p = (1.42 \text{ MeV}/c)$$

Also $1.51 \text{ MeV} = mc^2$

$$\Rightarrow m = 1.51 \frac{\text{MeV}}{c^2}$$

$$\Rightarrow \text{speed} = \frac{p}{m} = \frac{1.42 \text{ MeV}/c}{1.51 \text{ MeV}/c^2}$$

$$= \underline{\underline{0.94c}}$$

$$= 2.82 \times 10^8$$

$1.51 \text{ MeV} = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

$\Rightarrow \boxed{v = 0.94c}$

Q 13)

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{25 - 0.6c \cdot 5\times10^{-8}}{\sqrt{1-(0.6)^2}}$$

$$= \frac{16}{0.8} m = 20m$$

$$t' = \frac{t - \frac{v}{c^2} \cdot x}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{5\times10^{-8} - \frac{0.6}{3\times10^8} \times 25}{0.8}$$

$$= 0$$

$\boxed{S'(20m, 0, 0, 0)}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q14) Q-97 931 MeV

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q15)

From frame 1 bound to earth

$$t = \frac{6 \times 10^3}{0.99 \times 3 \times 10^8} = \underline{2.02 \times 10^{-5} s}$$

From fram bound to meson (proper time)

$$t_0 = 2.02 \times 10^{-5} \sqrt{1 - (0.99)^2}$$

$$= \underline{2.84 \times 10^{-6} s}$$

(Q16)

$1 eV = 1.6 \times 10^{-19} J$ ⊛ 1eV is the energy acquired by an e- when it is accelarated through 1V Potential.

$1.3 MeV = mc^2$

$$\Rightarrow m = 1.3 \left(\frac{MeV}{c^2}\right)$$

Also $1.3 MeV = \sqrt{p^2c^2 + (0.51)^2}$ MeV

$$\Rightarrow 1.195 MeV = pc$$

$$\Rightarrow p = 1.195 \left(\frac{MeV}{c}\right)$$

$$\frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = E$$

$$\Rightarrow v = \frac{p}{m} = \frac{1.195 \frac{MeV}{c}}{1.3 \frac{MeV}{c^2}}$$

$$\frac{1.3}{0.51} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$v = 2.75 \times 10^8$

$$= \underline{2.75 \times 10^8 \ m/s}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

⊙ Hz
⊙ Michelson Morley

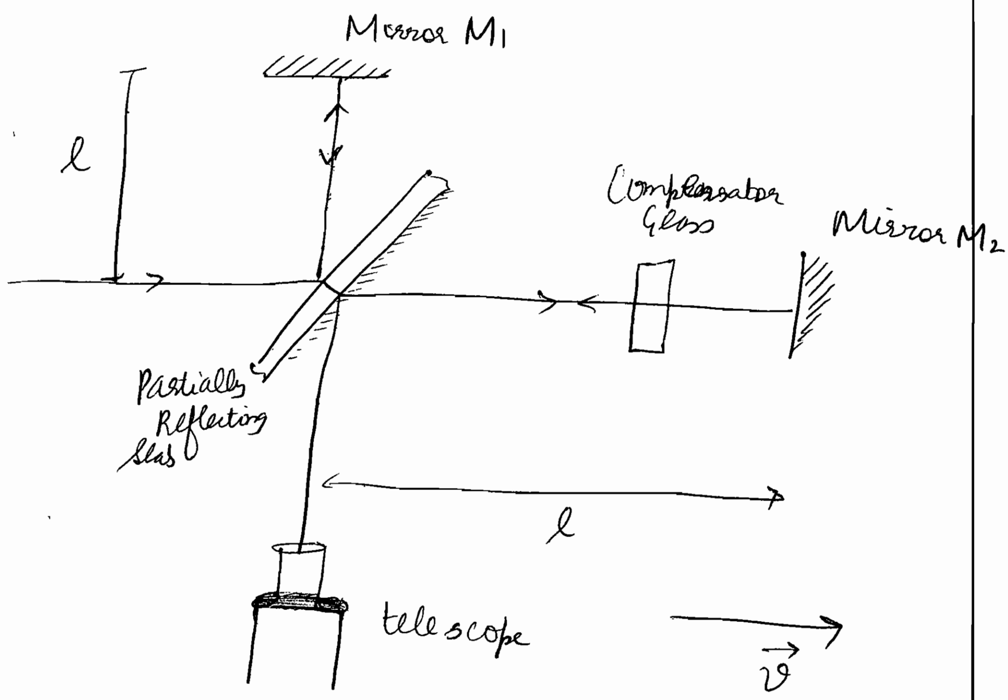
⊙ Davisson Germer (wave Particle duality)
⊙ Stern Gerlach ($s = 1/2$)
⊙ Yang & Hu's Ex. (Parity Violation)

Q17) Q-3

Q18)

Null Result of Michelson Morley Experiment

Michelson Morley Experiment negated the existence of light and is explicit proof of constancy of speed of light. The set up of the experiment uses Michelson Interferometer. (as shown below)



Michelson interferometer is based upon the principle that when path difference b/w waves travelling in perpendicular directions exist, a fring pattern is formed. When the instrument is rotated through 90°, the path difference reverses and fringe shift in the pattern should be observed. But no fringe shift was observed. The result of 0 fring shift is called Null Result. Einstein's STR successfully explained the results.

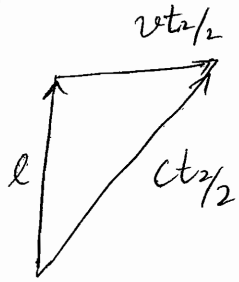
The Path difference can be calculated as. For Horizontal wave,

$$t_1 = \frac{l}{c-v} + \frac{l}{c+v} = \left(\frac{2cl}{c^2-v^2}\right)$$

$$= \frac{2l}{c}\left[1-\frac{v^2}{c^2}\right]^{-1}$$

$$= \frac{2l}{c}\left[1+\frac{v^2}{c^2}\right]$$

For vertical ray,



$$\left(\frac{ct_2}{2}\right)^2 = \left(\frac{vt_2}{2}\right)^2 + l^2$$

$$\Rightarrow t_2^2\left[c^2-v^2\right] = 4l^2$$

$$\Rightarrow t_2 = \frac{2l}{c\sqrt{1-\frac{v^2}{c^2}}} = \frac{2l}{c}\left[1+\frac{v^2}{2c^2}\right]$$

$$\Delta x = c(t_1 - t_2)$$

$$= 2l\left[\frac{v^2}{2c^2}\right] = \left(\frac{lv^2}{c^2}\right)$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

when the experiment is rotated,

$$\Delta x = \left(\frac{l v^2}{c^2}\right)$$

Hence we should get fringe shift corresponding to $\left(\frac{2lv^2}{c^2}\right)$

But Here, no shift was observed and the explanation was constancy of velocity of light, as given by Einstein.

$$t_1 = t_2 = \left(\frac{2l}{c}\right)$$

(Q19) If a length as seen in the frame at rest wrt. the body is l_0. If the same length is viewed from a frame moving at velocity v wrt. body, the length will appear to be smaller by a factor of $\sqrt{1-\frac{v^2}{c^2}}$

i.e. $$\boxed{l = l_0\sqrt{1-\frac{v^2}{c^2}}}$$

This is called Lorentz Contraction!!

For an observer, its length will be

$$\frac{l_0}{\cancel{\sqrt{1-\frac{v^2}{c^2}}}}\sqrt{1-\frac{v^2}{c^2}}$$

$\therefore$ time to pass $= \frac{l_0}{v}\sqrt{1-\frac{v^2}{c^2}} = 4\,\mu s$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \frac{100}{v}\sqrt{1-\frac{v^2}{c^2}} = 4\times10^{-6}$$

$$\Rightarrow 100\sqrt{\frac{1}{v^2}-\frac{1}{c^2}} = 4\times10^{-6}$$

$$\Rightarrow \sqrt{\frac{1}{v^2}-\frac{1}{c^2}} = 4\times10^{-8}$$

$$\frac{1}{v^2}-\frac{1}{c^2} = 1.6\times10^{-15}$$

$$\frac{1}{v^2} = 1.11\times10^{-17} + 1.6\times10^{-15}$$

$$\frac{1}{v^2} = 1.611\times10^{-15}$$

$$v = 0.249\times10^{8}\ m/s$$

(Q20) For an object moving with velocity v wrt. frame S, the mass m is given as

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

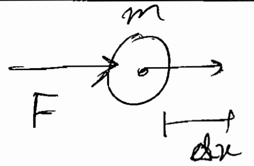
where m_0 is the mass of particle at rest

$$\Rightarrow dm = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{+3/2}}\cdot\frac{-1}{2}\cdot\frac{-2v}{c^2}\cdot dv$$

$$= \frac{m}{1-\frac{v^2}{c^2}}\frac{v}{c^2}\,dv = \frac{m v\, dv}{c^2-v^2}$$

Now consider the object being applied upon by a Force F s.t. Work Done

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)



$$dW = F \cdot dx$$

$$= \frac{d(p)}{dt} \, dx$$

$$= \frac{dm}{dt} \cdot v \, dx + m \cdot \frac{dv}{dt} \cdot dx$$

$$= v^2 dm + m \cdot v \, dv$$

$$= v^2 dm + dm(c^2 - v^2)$$

$$= c^2 dm$$

Now $dW = \Delta k \cdot E$

$$\Rightarrow \int dT = \int_{m_0}^{m} c^2 dm$$

$$\Rightarrow T = c^2(m - m_0)$$

$\Rightarrow E = T +$ rest mass energy

$$= c^2(m - m_0) + (m_0 c^2)$$

$$\Rightarrow \boxed{E = mc^2}$$

Q21) $M_{\mu_0} = 207 \, M_e$

Proper Mean life time $= 2 \cdot 2 \times 10^{-6} s$

Time measured in lab $= \dfrac{\tau_0}{\sqrt{1 - \frac{v^2}{c^2}}} = 6 \cdot 9 \times 10^{-6}$

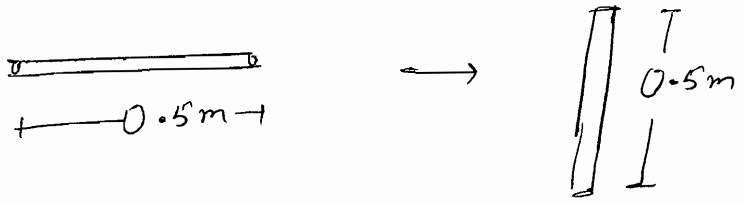
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\frac{2.2\times10^{-6}}{6.9\times10^{-6}} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\boxed{v = 0.948\,c}$$

Speed of muons in lab = 2.84×10^{8} m/s

(Q22)



The observation on the ship will observe simple rotation of rod.
The observed on the earth will also experience an increase in the length of the rod by $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ factor [जाए]

$$l_{initial} = 0.5\sqrt{1-\frac{v^2}{c^2}}\ \text{m}$$

$$l_{final} = 0.5\ \text{m}$$

(Q23) <Q-57

$$\frac{u_x+v}{1+\frac{u_x v}{c^2}} = \frac{1.8c}{1+0.81} = \underline{\underline{0.99c}}$$

(Q24) Let B be moving at speed $+v$ wrt. A

$$\Rightarrow x_B = \left[\frac{x_A - v_B t_A}{\sqrt{1-\left(\frac{v_B}{c}\right)^2}}\right]; \quad y_B = y_A; \quad z_B = z_A;$$

$$t_B = \left[\frac{t_A - \frac{v_B}{c^2} x_A}{\sqrt{1-\frac{v_B^2}{c^2}}}\right]$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$t_1 - t_2 = 10^{-6} s$$

$$t_1' - t_2' = \frac{(t_1 - t_2) + 0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow 10^{-6} \times 3 = \frac{10^{-6}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \left[1 - \frac{v^2}{c^2}\right] = \frac{1}{9} \Rightarrow \boxed{v = \sqrt{\frac{8}{9}}\, c}$$

$$x_1 - x_2 = 0$$

$$x_1' - x_2' = \frac{(x_1 - x_2) - (v)(t_1 - t_2)}{\sqrt{1-\frac{v^2}{c^2}}}$$ ✓

$$= \frac{-\sqrt{\frac{8}{9}}\, c .\ 10^{-6}}{\sqrt{(1/9)}}$$

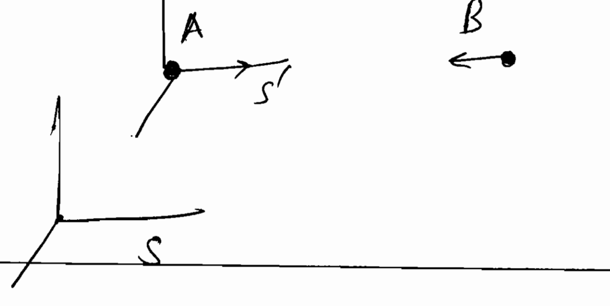
$$= \sqrt{8}\, c \times 10^{-6}$$

$$= \frac{2545}{3} m$$

$$= \underline{848.52 m}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q 25)



कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Let us attach frame S' to Body A. This implies velocity of frame $S' \Rightarrow v = \left(\frac{4c}{5}\right)$

Writing lorentz equation

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow u_x' = \frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - \frac{v}{c^2}dx}$$

$$= \left(\frac{u_x - v}{1 - \frac{v u_x}{c^2}}\right)$$

Now let us consider motion of particle B.

wrt S, $u_B = -\left(\frac{4c}{5}\right)$

$$\Rightarrow u_{B,A} = u_{B,S'} = u_x' = \frac{-\frac{4c}{5} - \frac{4c}{5}}{1 + \frac{4c}{5}\cdot\frac{4c}{5}\,\frac{1}{c^2}}$$

$$\Rightarrow u_x' = -\frac{8}{5}\,\frac{c}{1+\frac{16}{25}} = -\frac{40c}{41} = -0.97c$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

In Frame S

$$k.E._B = mc^2 - m_0c^2$$

$$= \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2 = m_0c^2\left[\frac{1}{\sqrt{1-\left(\frac{4}{5}\right)^2}} - 1\right]$$

$$= m_0c^2\left[\frac{5}{3} - 1\right] = \underline{\frac{2}{3} m_0c^2}$$

$$k.E._B{}' = m_0c^2\left[\frac{1}{\sqrt{1-\left(\frac{40}{41}\right)^2}} - 1\right]$$

$$= m_0c^2\left[\frac{41}{9} - 1\right]$$

$$= m_0c^2 \cdot \left(\frac{32}{9}\right)$$

$$= \underline{\frac{32}{9} m_0c^2}$$

Q26) In non relative physics, velocity is not const. but relative

⇒ Source and observer will encounter different velocities of wave hence doppler effect is given as

⊛ Here it matters if Source moves w.r.t. observer or observer moves w.r.t. source.

$$\nu = \nu_0 \sqrt{\frac{v - v_0}{v - v_s}}$$

⊛ Asymmetric

कृपया इस स्थान में कुछ न लिखें।
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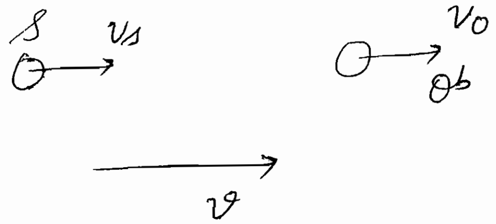


In relativistic phenomenon, velocity of light is same for both source and observer

here $\nu = \nu_0 \frac{\sqrt{1 - \frac{v}{c}}}{\sqrt{1 + \frac{v}{c}}}$

⊛ Here it does not matter whether source moves relative to observer or vice versa as there is no way out.

⊛ Symmetric

⊛ Even transverse doppler effect is observe which has no classical analogue.

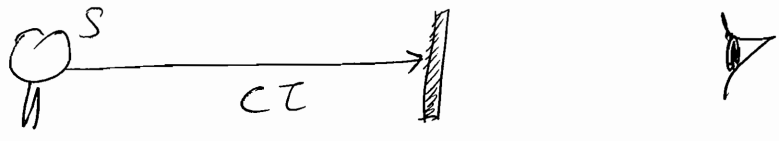
Longitudinal Doppler Effect

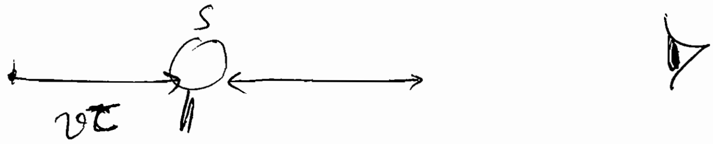
$$\alpha T' = T + \frac{\Delta x}{c}$$

$$\alpha T' = T + \frac{\Delta x' + \alpha v \Delta t'}{c}$$

$$\alpha T' \left[1 - \frac{v}{c} \right] = T$$

$$\nu' = \nu \frac{\sqrt{1 - \frac{v}{c}}}{\sqrt{1 + \frac{v}{c}}}$$



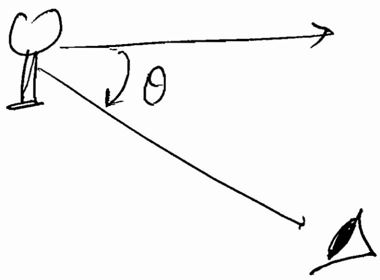


$$\lambda = (c-v)\tau = (c-v)\alpha\,\tau_0$$

$$\Rightarrow \frac{c}{\nu} = c\left(1-\frac{v}{c}\right)\alpha\,\frac{1}{\nu_0}$$

$$\Rightarrow \nu = \nu_0 \Big/ \left(1-\frac{v}{c}\right)\alpha = \nu_0\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

Transverse Doppler



$$\lambda = (c - v\cos\theta)\,\tau$$

$$\frac{c}{\nu} = c\left[1-\frac{v}{c}\cos\theta\right]\alpha\cdot\frac{1}{\nu_0}$$

$$\Rightarrow \nu = \nu_0 \Big/ \alpha\left[1-\frac{v}{c}\cos\theta\right]$$

$$= \frac{\nu_0\sqrt{1-\frac{v^2}{c^2}}}{\left(1-\frac{v}{c}\cos\theta\right)}$$

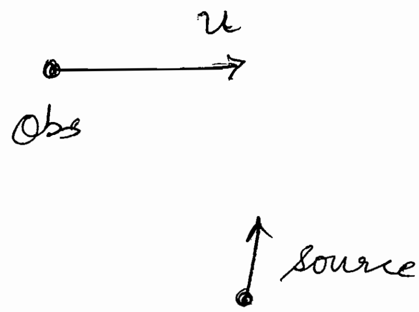
कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

⇛ For $\theta = 90^\circ$

$$\boxed{\gamma = \gamma_0 \sqrt{1 - \frac{v^2}{c^2}}}$$

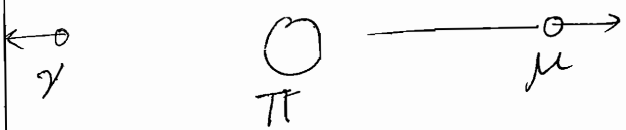
In the given problem



In observer frame,

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q27)



From conservation of momentum,

$$p_\mu = p_\gamma \quad — (1)$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Note this standard way to solve

$E_a + E_b$ ✓

$E_a^2 - E_b^2$ ✓

$E_a - E_b$ ✓

$\Rightarrow E_a$ ✓, E_b ✓

$$E_\mu + E_\gamma = \pi c^2 \quad — (2)$$

$$E_\mu^2 = p_\mu^2 c^2 + \mu^2 c^4$$

$$E_\gamma^2 = p_\gamma^2 c^2$$

$$\Rightarrow E_\mu^2 - E_\gamma^2 = \mu^2 c^4$$

$$\Rightarrow E_\mu - E_\gamma = \frac{\mu^2 c^4}{\pi c^2} = \frac{\mu^2}{\pi} c^2$$

$$\Rightarrow E_\mu = \frac{1}{2}\left[\pi c^2 + \frac{\mu^2 c^2}{\pi}\right]$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow k \cdot E_{\mu} = E_{\mu} - \mu c^2$$

$$= \frac{\pi c^2}{2} + \frac{\mu^2 c^2}{2\pi} - \mu c^2$$

$$= \frac{\pi^2 c^2 + \mu^2 c^2 - 2\pi \mu c^2}{2\pi}$$

$$\boxed{k \cdot E_{\mu} = \frac{c^2}{2\pi} (\pi - \mu)^2}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q28)
(Q30)

Q20

Let a consider a rod of length L_0 travelling at velocity v relative to a frame S. Let us attach frame S' to the moving body.

From Lorentz transformation,

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

In order to measure the length correctly, the measurements of two end points must be simultaneously from from S

i.e. $t_1 = t_2$

$$\Rightarrow x_1' = \frac{x_1 - vt_1}{\sqrt{1 - v^2/c^2}}$$

$$x_2' = \frac{x_2 - vt_2}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow (x_1' - x_2') = \frac{(x_1 - x_2)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Since measurement made from frame S' that is at rest w.r.t. rod,

$x_1' - x_2'$ is the proper length L_0.

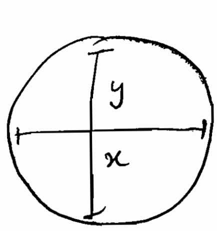
$$\Rightarrow L_0 = \frac{L_{obs}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \boxed{L = L_0 \left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

Also note that there is no affect on length $\perp$ to motion of particle as,

$$dy' = dy$$
$$dz' = dz$$

Hence for sperical ball



$x = y = d$ in frame of S'

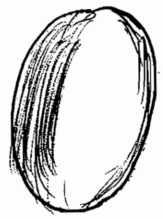
$$x \longrightarrow x\sqrt{1 - \frac{v^2}{c^2}} = d\sqrt{1 - \frac{v^2}{c^2}}$$

$$y \longrightarrow y = d$$
$$z \longrightarrow z = d$$

Hence ball will appear as spheroid ~~oblate or~~ prolate or ellipsoid.

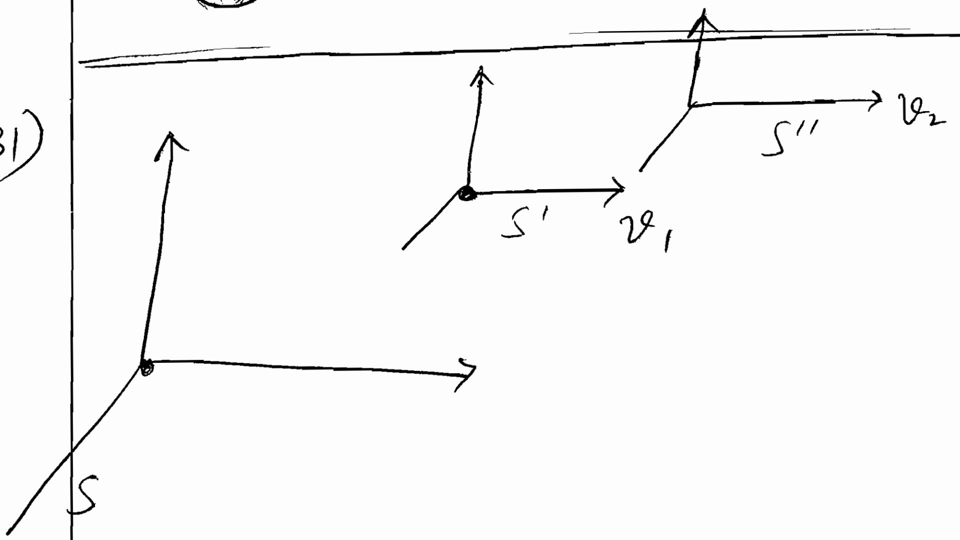
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)



Prolate Spheroid

Q31)



Let frame S' moves at velocity v_1 w.r.t. S in ⊕ve x direction.

$$\Rightarrow \quad x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z'$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Now let us suppose S'' frame moves w.r.t. S' at velocity v_2 in ⊕ve x direction

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow x'' = \frac{x' - v_2 t'}{\sqrt{1 - v_2^2/c^2}}$$

$$y'' = y'$$

$$z'' = z'$$

$$t'' = \frac{t' - \frac{v_2}{c^2} . x'}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

$$\text{Now, } x'' = \frac{\dfrac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} - \dfrac{v_2 . \left(t - \frac{v}{c^2} x\right)}{\sqrt{1 - \frac{v^2}{c^2}}}}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

$$= \frac{x \dfrac{\left[1 + \frac{v_2 v}{c^2}\right]}{\sqrt{1 - \frac{v^2}{c^2}}} - t\left[\dfrac{v + v_2}{\sqrt{1 - \frac{v^2}{c^2}}}\right]}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

$$= \frac{x - t\left(\dfrac{v + v_2}{1 + \frac{v v_2}{c^2}}\right)}{\dfrac{\sqrt{1 - \frac{v^2}{c^2}}\sqrt{1 - \frac{v_2^2}{c^2}}}{\left(1 + \frac{v v_2}{c^2}\right)}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$u := \left(\frac{v_1+v_2}{1+\frac{v_1v_2}{c^2}}\right) \Leftarrow \text{We have to reach here...}$$

$$U_x = \left(\frac{U_x' + v}{1+\frac{v\,U_x'}{c^2}}\right)$$

Such double transformations should be done by matrix method and fortunately we have MINKOWSKY MATRIX which we can initial write as equivalent of lorentz transform

Aim is to find u_3 s.t.

$\alpha_3 = \dfrac{1}{\sqrt{1-\frac{v_3^2}{c^2}}}$

&

$\beta_3 = \left(\dfrac{v_3}{c}\right)$

$$\begin{bmatrix} \alpha & 0 & 0 & i\alpha\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\alpha\beta & 0 & 0 & \alpha \end{bmatrix} = \begin{bmatrix} \alpha_2 & 0 & 0 & i\alpha_2\beta_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\alpha_2\beta_2 & 0 & 0 & \alpha_2 \end{bmatrix}\begin{bmatrix} \alpha_1 & 0 & 0 & i\alpha_1\beta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\alpha_1\beta_1 & 0 & 0 & \alpha_1 \end{bmatrix}$$

Now $\alpha_1 = \dfrac{1}{\sqrt{1-\frac{v_1^2}{c^2}}}$; $\alpha_2 = \dfrac{1}{\sqrt{1-\frac{v_2^2}{c^2}}}$; $\beta_1 = \dfrac{v_1}{c}$; $\beta_2 = \dfrac{v_2}{c}$

$$= \begin{bmatrix} \alpha_1\alpha_2 + \alpha_1\alpha_2\beta_1\beta_2 & 0 & 0 & i\alpha_1\alpha_2(\beta_1+\beta_2) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\alpha_1\alpha_2(\beta_1+\beta_2) & 0 & 0 & \alpha_1\alpha_2 + \alpha_1\alpha_2\beta_1\beta_2 \end{bmatrix}$$

Now $\alpha_1\alpha_2 \left[1+\beta_1\beta_2\right]$

$$= \frac{1}{\sqrt{1-\frac{v_1^2}{c^2}}\sqrt{1-\frac{v_2^2}{c^2}}}\left[1+\frac{v_1v_2}{c^2}\right]$$

$$= \frac{c^2+v_1v_2}{\sqrt{(c^2-v_1^2)(c^2-v_2^2)}} = \frac{1}{\sqrt{\frac{c^4+v_1^2v_2^2-(v_1^2+v_2^2)c^2}{(c^2+v_1v_2)^2}}}$$

$$= \frac{1}{\sqrt{\frac{(c^2+v_1v_2)^2 - 2c^2v_1v_2 - v_1^2c^2 - v_2^2c^2}{(c^2+v_1v_2)^2}}}$$

$$= \frac{1}{\sqrt{1-\frac{(v_1+v_2)^2c^2}{c^4\left(1+\frac{v_1v_2}{c^2}\right)^2}}} = \sqrt{\frac{1}{1-\frac{\left(\frac{v_1+v_2}{1+\frac{v_1v_2}{c^2}}\right)^2}{c^2}}}$$

$\equiv \alpha$ Say

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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(Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

Also,

$$\alpha_1 \alpha_2 [\beta_1 + \beta_2]$$

$$= \frac{\alpha}{1+\beta_1\beta_2} [\beta_1 + \beta_2]$$

$$= \alpha \cdot \frac{\left(\frac{v_1}{c} + \frac{v_2}{c}\right)}{\left(1 + \frac{v_1 v_2}{c^2}\right)} = \alpha \frac{\left(\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right)}{c}$$

let $\dfrac{\left(\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right)}{c} = \beta$

$\Rightarrow$ Minkowsky Matrix, is

$$\begin{bmatrix} \alpha & 0 & 0 & i\alpha\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\alpha\beta & 0 & 0 & \alpha \end{bmatrix}$$

where $\alpha = \dfrac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ where $v = \left(\dfrac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right)$

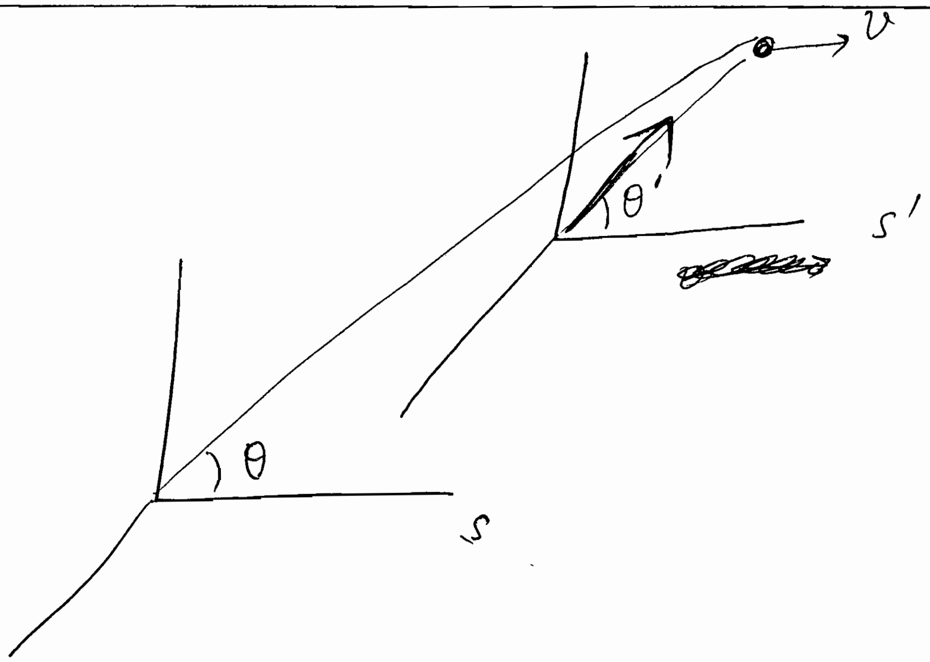
$$\beta = \left(\frac{v}{c}\right)$$

Hence 2 transformation equal to 1 transformation with relativistic composition of velocities.

Q32)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)



To prove : $\tan\theta = \dfrac{\sin\theta' \sqrt{1-\frac{v^2}{c^2}}}{\cos\theta' + \left(\frac{v}{c}\right)}$

For the set up as shown above,

$$u_x' = + c\cos\theta' \quad — (1)$$
$$u_y' = + c\sin\theta' \quad — (2)$$

Now from inverse lorentz transform,

$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y = y' \qquad z = z'$$

$$\Rightarrow u_x = \frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v}{c^2}dx'} = \left(\frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}\right) \quad — (3)$$

$$u_y = \frac{dy}{dt} = \frac{dy'\sqrt{1-\frac{v^2}{c^2}}}{dt' + \frac{v}{c^2}dx'} = \frac{u_y'}{1 + \frac{v u_x'}{c^2}}\sqrt{1-\frac{v^2}{c^2}} \quad — (4)$$

Using (1) and (2) in (3) and (4)

$$u_x = \left(\frac{+c\cos\theta' + v}{1 + \frac{v}{c^2}\cos\theta} \right) = +c\cos\theta$$

$$u_y = \frac{+c\sin\theta'}{1 + \frac{v}{c}\cos\theta} \sqrt{1 - \frac{v^2}{c^2}} = +c\sin\theta$$

$$\Rightarrow \tan\theta = \frac{\sin\theta'}{\cos\theta' + \left(\frac{v}{c}\right)} \sqrt{1 - \frac{v^2}{c^2}}$$

Q 33)

$v = 0.6c$

$+y$

m_0

For body moving with $v = 0.6c$, mass m is

$$m = \frac{m_0}{\sqrt{1-(0.6)^2}} = \frac{m_0}{0.8} = \left(\frac{5m_0}{4}\right)$$

Now $v_x = 0$

$v_y = 0.6c$

$v_z = 0$

$$E = mc^2 = \frac{5m_0}{4}c^2$$

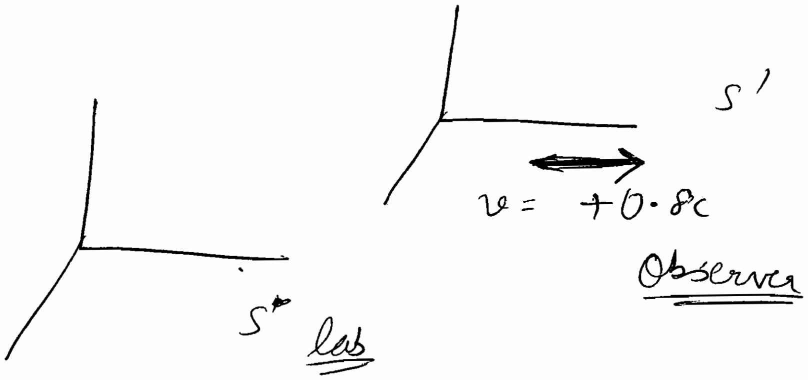
$$p_\mu = \left[0, \frac{3}{4}m_0 c, 0; \; i\frac{5m_0 c}{4} \right]$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Now the Momentum 4 vector Obeys Minkowsky Transformation,

For frame ~~of~~ of Observer ~~by~~ S'



$$p'_\mu = \alpha_{\mu\nu} p_\mu$$

$$\alpha_{\mu\nu} = \begin{bmatrix} \alpha & 0 & 0 & i\alpha\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\alpha\beta & 0 & 0 & \alpha \end{bmatrix}$$

$\Rightarrow$ Now $\alpha = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-(0.8)^2}} = \dfrac{1}{0.6} = \left(\dfrac{5}{3}\right)$

$$\beta = \frac{v}{c} = 0.8 = \left(\frac{4}{5}\right)$$

$$\Rightarrow \begin{bmatrix} p_x' \\ p_y' \\ p_z' \\ \frac{iE'}{c} \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 & i\alpha\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\alpha\beta & 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} 0 \\ \frac{3}{4} m_0 c \\ 0 \\ i\frac{5}{4} m_0 c \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{5}{4}\alpha\beta m_0 c \\ \frac{3}{4} m_0 c \\ 0 \\ i\frac{5}{4}\alpha m_0 c \end{bmatrix} = \begin{bmatrix} -\frac{5}{3} m_0 c \\ \frac{3}{4} m_0 c \\ 0 \\ i\frac{25}{12} m_0 c \end{bmatrix}$$

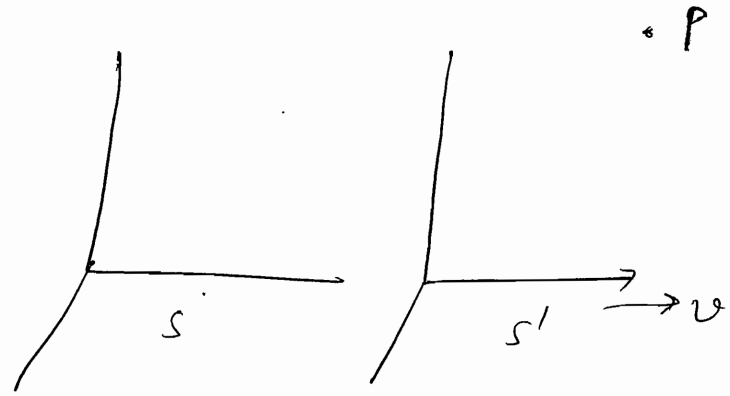
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q 34) ① All Basic laws of Physics take the same physical form in all inertial frames.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

② velocity of light is constant in all inertial frames.

Let us consider two frames S and S', latter moving at velocity v wrt. former in positive x-direction. Let the two frame coincide at $t=0$.



An event occurs at Point P which is noticed by S and S' at t and t'

Since velocity of light is constant for both frames,

$$x^2+y^2+z^2-c^2t^2 = x'^2+y'^2+z'^2-c^2t'^2 \quad —①$$

Let $x' = \alpha(x-vt)$

and $t' = (\beta t + \gamma x)$

Putting in ①, we get

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\alpha^2 x^2 + \alpha^2 v^2 t^2 - 2\alpha^2 v x t + y'^2 + z'^2$$
$$- c^2\beta^2 t^2 - c^2\gamma^2 x^2 - 2c^2\beta\gamma\, xt$$
$$= x^2 + y^2 + z^2 - c^2 t^2$$

Comparing coefficients,

$$\alpha^2 - c^2\gamma^2 = 1 \quad \text{— (2)}$$

$$\alpha^2 v^2 - c^2\beta^2 = -c^2 \quad \text{— (3)}$$

$$-2\alpha^2 v - 2c^2\beta\gamma = 0 \quad \text{— (4)}$$

$$\boxed{\alpha^2 = 1 + c^2\gamma^2}$$

$$v^2 + v^2 c^2\gamma^2 + c^2 = c^2\beta^2$$

$$\Rightarrow \boxed{\beta^2 = \left(1 + v^2\gamma^2 + \frac{v^2}{c^2}\right)}$$

$$4\alpha^4 v^2 = c^4\beta^2\gamma^2$$

$$v^2 + \cancel{c^4\gamma^4 v^2} + 2c^2\gamma^2 v^2 = c^4\gamma^2 + \cancel{c^4\gamma^4 v^2} + \gamma^2 v^2 c^2$$

$$v^2 = c^4\gamma^2 - \gamma^2 v^2 c^2$$

$$\Rightarrow \gamma = -\sqrt{v^2/(c^4 - v^2c^2)} = -\frac{v}{c^2}\sqrt{\frac{1}{1 - v^2/c^2}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\alpha = \sqrt{1 + \frac{v^2}{c^2}\frac{1}{\left(1-\frac{v^2}{c^2}\right)}}$$

$$= \sqrt{1 + \frac{v^2}{c^2-v^2}} = \sqrt{\frac{c^2}{c^2-v^2}}$$

$$\alpha = \sqrt{\frac{1}{1-\left(\frac{v^2}{c^2}\right)}}$$

$$\beta = \sqrt{1 + \frac{v^4}{c^4}\frac{1}{\left(1-\frac{v^2}{c^2}\right)} + \frac{v^2}{c^2}}$$

$$= \sqrt{1 + \frac{v^4}{c^4 - v^2c^2} + \frac{v^2}{c^2}}$$

$$= \sqrt{\frac{c^4 - \cancel{v^2c^2} + \cancel{v^4} + \cancel{c^2v^2} - \cancel{v^4}}{c^4 - v^2c^2}}$$

$$\beta = \sqrt{\frac{1}{1-\frac{v^2}{c^2}}}$$

Hence, $$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

As there is no preferential frame, we can have S travelling with velocity $(-v)$ w.r.t. S'. Writing Lorentz inverse transform

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Now let a clock be kept at x'

let $t_1' - t_2' = T_0$ (Proper time as the clock is stationary)

From inverse transforms,

$$t_1 - t_2 = \frac{\frac{v}{c^2}(0) + T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \boxed{\Delta T_{obs} = \frac{\Delta T_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q 35) Time $= \dfrac{5\mu s}{\sqrt{1-(0.8)^2}} = \dfrac{5}{0.6}\mu s = 8.33\mu s$

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

Q36) Let a rod be moving with a velocity v w.r.t. frame S along the $+x$ direction. Also let S' be frame attached to the rod. Let P be fixed mark in S frame & A, B be two end points of rod.

Consider the two events

(I) Front End A coincides with P

(II) Back End B coincides with P

<u>In frame S'</u>

length of rod $= l_0$ (measured when it is at rest)

velocity of P is $-v$ in S' frame

$\Rightarrow \Delta t_0' = \left(\dfrac{l_0}{v}\right)$

<u>As observed in frame S</u>

$\Delta t = \cancel{\dfrac{l_0}{v}} = \dfrac{L}{v}$

~~$\Rightarrow$~~ l_0

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Now Δt is proper time as for both events occur at same position P $(\Delta x = 0)$

$$\Rightarrow \Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \frac{l_0}{v} = \frac{L_{obs}}{v\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \boxed{L_{obs} = l_0\sqrt{1-\frac{v^2}{c^2}}}$$

Hence length contraction......

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q37)
Q38)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q - 27

Let S and S' be two frames.
dx be length measured in ~~[illegible]~~ stationary frame and dx' be length measured in the moving frame i.e. when the body is at rest w.r.t. frame $\Rightarrow dx'$: Proper length

$$dx = \sqrt{1-\frac{v^2}{c^2}} \cdot dx' \quad \text{(length contraction)}$$

Let time dt' be measured in moving frame with x' const. where clock is kept at rest w.r.t. frame $\Rightarrow dt'$: proper time

$$dt = \frac{d\tau}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{(time dilation)}$$

$$\Rightarrow dx\, dy\, dz\, dt$$
$$= dx'\, dy'\, dz'\, dt'$$

Q39) Q 32

Q40) <Q 18>

No it does not as it was inferred that Earths drags ether along with it and relative velocity $=0$
Einstein's hypothesis finally settled the debate

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें।
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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

OPTICS (Selected Questions)

Tut Sheet ~~7~~A Damped & Forced Oscillations

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q 1) For the weak damping system, or the underdamped system, the response is given by:

$$x = Ae^{-\cancel{2}ct} \sin(\omega_d t - \theta)$$

where $\omega_d = \sqrt{\omega_0^2 - c^2} \approx \omega_0$ (given)

We are given $\omega_0 \tau >> 1$

i.e. $\omega_0 . \frac{1}{2c} >> 1$ (τ: relaxation time)

$$\Rightarrow \omega_0 >> 2c$$

$$\Rightarrow \sqrt{\omega_0^2 - c^2} \approx \omega_0$$

$$\Rightarrow x = Ae^{-\cancel{2}ct} \sin(\omega_0 t - \theta)$$

Rate of energy dissipation or Power of dissipation, is given as:

$$\left(\frac{dE}{dt}\right) = -b.v.v = \underline{-2c\dot{x}^2}$$

$$\cancel{= -2c\left(Ae^{-\cancel{2}ct}\cos(\omega_0 t - \theta)\right)\omega_0}$$

$$\cancel{-2cA\sin(\omega_0 t - \theta)e^{-\cancel{2}ct}(-2c)}$$

$$\cancel{= \left[4c^2 A \sin(\omega_0 t - \theta) - 2c\omega_0 A \cos(\omega_0 t - \theta)\right]e^{-\cancel{2}ct}}$$

(Q 2) lecture 2

(Q 3) lecture 3

(Q 4) lecture 3

(Q 5) lecture 3

(Q 6) For a damped oscillator, the equation

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

of motion is

$$\ddot{x} - b\dot{x} - \cancel{k}x = 0$$

Solution for the underdamped case is

$$x = Ae^{-ct} \sin(\omega_d t + \theta)$$

We are given $\omega_d = 2\pi \times 300\ s^{-1}$

$\Rightarrow$ Time for 3000 oscillations $= 3000\ T_0$

$$= 3000 \cdot \frac{2\pi}{2\pi \times 300} = 10\,s$$

Let initial Amplitude be A

$\Rightarrow$ Amplitude after 10 seconds $= Ae^{-10c}$

We are given,

$$Ae^{-10c} = \frac{A}{10} \Rightarrow \boxed{c = 0.23\ s^{-1}}$$

Damping constant $= b = 2c = \boxed{0.46\ s^{-1}}$

damping constant $= 2c$
$= b$

b = 2mc ??

Energy variation fall off twice as amplitude

i.e. $E = E_0 e^{-2ct} \sin(\omega_d t - \theta)$

$\therefore$ Time for energy to reduce to $1/10$ th of the value is given as

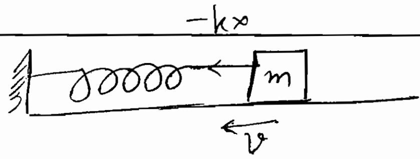
$$\Rightarrow \frac{E_0}{10} = E_0 e^{-2ct}$$

$$\Rightarrow \boxed{t = 5\,s}$$

Q7 lecture 2

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q8)



कृपया इस स्थान में कुछ न लिखें।
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For the spring system, we have,

$$-kx - b\dot{x} = m\ddot{x}$$

$$\Rightarrow \ddot{x} + 2c\dot{x} + \omega_0^2 x = 0 \qquad \text{where } \frac{k}{m} = \omega_0^2, \quad \frac{b}{m} = 2c$$

<derivation>

For a light damping, we have.
Under damped solution

i.e. $x = Ae^{-ct} \sin(\omega_d t - \theta)$

$$\omega_d = \sqrt{\omega_0^2 - c^2}$$

Frequence of oscillation $= \dfrac{1}{2\pi}\sqrt{\omega_0^2 - c^2}$

$$= \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

In the absence of damping i.e. $b = 0$,
we have

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

We are given,

$$\frac{1}{\cancel{2\pi}}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\cancel{2\pi}}\sqrt{\frac{k}{m}}$$

$$\Rightarrow \frac{2k}{m} - \frac{b^2}{2m^2} = \frac{k}{m}$$

$$\Rightarrow \frac{k}{\cancel{m}} = \frac{b^2}{2m^{\cancel{2}}} \Rightarrow b = \sqrt{2km}$$

$$= \sqrt{2 \times 10 \times 0.1} \quad \frac{Ns}{m}$$

$$= \sqrt{2}\ Ns/m$$

Q9) lecture 2

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q10)

The equation for oscillation under external harmonic force is :

$$\ddot{x} + 2c\dot{x} + \omega_0^2 x = F_0 \sin(pt)$$ where $F_0 = \frac{F}{m}$

where $\omega_0^2 = \frac{k}{m}$

$2c = 2b = \beta$

Let $x = A \sin(pt-\theta)$

$$\Rightarrow -Ap^2 \sin(pt-\theta) + 2cAp\cos(pt-\theta) + \omega_0^2 A \sin(pt-\theta) = F_0 \sin(pt-\theta+\theta)$$

$$\Rightarrow (\omega_0^2 A - Ap^2)\sin(pt-\theta) + 2cAp\cos(pt-\theta) = F_0\cos\theta \sin(pt-\theta) + F_0 \sin\theta\cos(pt-\theta)$$

Comparing coefficients, we have

$$\omega_0^2 A - Ap^2 = F_0 \cos\theta$$

$$2cpA = F_0 \sin\theta$$

$$\boxed{\begin{aligned}\Rightarrow \tan\theta &= \left(\frac{2cp}{\omega_0^2 - p^2}\right) \\ A &= F_0 / \sqrt{(\omega_0^2 - p^2)^2 + (2cp)^2}\end{aligned}}$$

For maximum Amplitude,

$$\frac{dA}{dp} = 0 \quad \& \quad \frac{d^2A}{dp^2} < 0$$

$$\Rightarrow \frac{-F_0}{\left[(\omega_0^2-p^2)^2 + (2cp)^2\right]^{3/2}} \cdot \left[2(\omega_0^2 - p^2)(-2p) + 4(2cp)c\right] = 0$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$\Rightarrow 8c^2 \cancel{p} = 4\cancel{p}(\omega_0^2 - p^2)$

$\Rightarrow$ Cancelling the trivial solution $p=0$,

$2c^2 - \omega_0^2 = -p^2$

$\Rightarrow \boxed{p = \sqrt{\omega_0^2 - 2c^2}}$

At $p = \sqrt{\omega_0^2 - 2c^2}$

$$A = \frac{F}{m\sqrt{(2c^2)^2 + (2c)^2(\omega_0^2 - 2c^2)}}$$

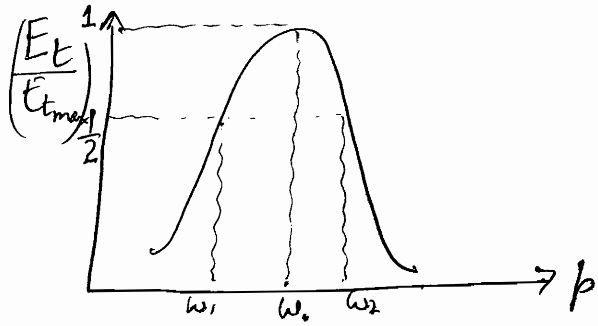
$$= \frac{F}{m\sqrt{4c^4 + 4c^2\omega_0^2 - 8c^4}}$$

$$= \frac{F}{m\sqrt{4c^2\omega_0^2 - 4c^4}}$$

$$= \frac{F}{2cm\sqrt{\omega_0^2 - c^2}}$$

$$\boxed{A = \frac{f}{2c\sqrt{\omega_0^2 - c^2}}}$$

The energy transferred by the external system varies as



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The sharpness of resonance is the measure of the sharpness of this curve. Let the resonance occur at ω_0 and let half power max be transferred at ω_1 and ω_2.

Mathematically, sharpness is defined in terms of Quality factor Q,

$$Q = \left(\frac{\omega_0}{\omega_2 - \omega_1}\right)$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Tut Sheet : 7B → Beats, Stationary Wave

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q1) Mathematically, group velocity v_g is given as,

$$v_g = v_p - \lambda\left(\frac{dv_p}{d\lambda}\right) \quad — ①$$

given $v_p = \left[\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right]^{1/2}$

$$\Rightarrow \frac{dv_p}{d\lambda} = \frac{1}{2}\left[\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right]^{-\frac{1}{2}}\left[\frac{-2\pi T}{\rho\lambda^2} + \frac{g}{2\pi}\right]$$

From ①

$$\Rightarrow v_g = \left[\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right]^{\frac{1}{2}} + \frac{\left[\frac{\pi T}{\rho\lambda} - \frac{g\lambda}{4\pi}\right]}{\left(\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right)^{\frac{1}{2}}}$$

$$= \frac{\left[\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi} + \frac{\pi T}{\rho\lambda} - \frac{g\lambda}{4\pi}\right]}{\left(\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right)^{1/2}}$$

$$= \frac{\left[\frac{3\pi T}{\lambda\rho} + \frac{g\lambda}{4\pi}\right]}{\left(\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right)^{1/2}}$$

$$= \frac{2\pi T}{\lambda\rho v_p} + \frac{1}{2}v_p$$

$$\boxed{v_g = \frac{v_p}{2} + \frac{2\pi T}{\lambda\rho v_p}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For phase velocity to be minimum,

$$\frac{dv_p}{d\lambda} = 0$$

$$\Rightarrow \frac{2\pi T}{\rho\lambda^2} = \frac{g}{2\pi} \Rightarrow \boxed{\lambda = 2\pi\sqrt{\frac{T}{\rho g}}}$$

& $\left(\frac{d^2v_p}{d\lambda^2}\right) > 0$

for $\lambda = 2\pi\sqrt{\frac{T}{\rho g}}$, double derivative is ~~negative~~ positive, hence a minima.

(Q2) lecture 1

(Q3) Given

λ	n
5090	1.647
5340	1.640
5890	1.630

$$\Rightarrow \frac{\Delta n}{\Delta\lambda} = \frac{-2.8\times10^{-5} - 1.8\times10^{+5}}{2}$$

$$= \underline{\underline{-2.3\times10^{+5}}}$$

$$\text{Phase velocity} = v = \frac{c}{n}$$

$$= \frac{3\times10^8}{1.640}\ m/s$$

$$= \underline{\underline{1.82\times10^8\ m/s}}$$

Group velocity is given as

$$v_g = v_p - \lambda\left(\frac{dv_p}{d\lambda}\right)$$

Now $v_p = \frac{c}{n}$

$$\Rightarrow \frac{dv_p}{dn} = \frac{-c}{n^2}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow v_g = v_p - \lambda \frac{dv_p}{dn} \cdot \frac{dn}{d\lambda}$$

$$= v_p + \lambda \cdot \frac{c}{n^2}\left(\frac{dn}{d\lambda}\right)$$

$$\approx v_p + \lambda \cdot \frac{c}{n^2}\left(\frac{\Delta n}{\Delta\lambda}\right)$$

$$= 1.82\times10^{8} + \frac{5340\times10^{-10}\times3\times10^{8}}{(1.64)^2} \cdot 2.3\times10^{+5}$$

$$= (1.82 + 0.137)\times10^{8}$$

$$= \underline{1.683\times10^{8}\ m/s}$$

Q4) lecture 1

Q5) For a string the velocity of a travelling wave v is given as

velocity on string

$$\boxed{v = \sqrt{\frac{T}{\lambda}}}$$

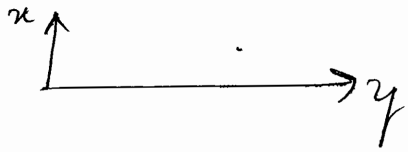
T: Tension
λ: mass per unit length

$$= \sqrt{\frac{25}{0.25}} = \underline{10\ m/s}$$

$\nu_{wave} = 5\ Hz$

$A_{wave} = 0.01\ m$

The equation of wave is given as



$$x = A\sin(\omega t - ky)$$

$$k = \left(\frac{2\pi}{\lambda}\right) \quad \omega = (2\pi\nu)$$

$$\lambda = \frac{v}{\nu} = \frac{10}{5} = \underline{2m}$$

$\Rightarrow$ wave equation : $\underline{\underline{x = 0.01 \sin(10\pi t - \pi y)\ m}}$

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

Q6)

$$v_p = \sqrt{\frac{g}{k}} = \sqrt{\frac{g\lambda}{2\pi}}$$

$$v_g = v_p - \lambda\left(\frac{dv_p}{d\lambda}\right)$$

$$= \sqrt{\frac{g\lambda}{2\pi}} - \lambda \cdot \sqrt{\frac{g}{2\pi}} \cdot \frac{1}{2\sqrt{\lambda}}$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g\lambda}{2\pi}} = \frac{1}{2}\sqrt{\frac{g}{k}} = \left(\frac{v_p}{2}\right)$$

Q7)

$$\psi = 10 \cos(5x + 25t)$$

Comparing with $\psi = A\cos(kx + \omega t)$, we have

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{5} = \underline{\underline{1.256\ m}}$$

$$v = \lambda\nu = \frac{\omega}{k} = \frac{25}{5} = \underline{\underline{5\ m/s}}$$

$$\nu = \frac{\omega}{2\pi} = \frac{25}{2\pi} \approx \underline{\underline{4\ Hz}}$$

direction of propagation : $(-x)$ axis

$$\psi_1 = 10 \cos(5x + 25t)$$
$$\psi_2 = 20 \cos(5x + 25t + \pi/3)$$

$$\psi = \psi_1 + \psi_2$$

$$= 10 \cos(5x + 25t)$$
$$+ 20 \cos(5x + 25t) \cdot \frac{1}{2}$$
$$- 20 \sin(5x + 25t) \cdot \frac{\sqrt{3}}{2}$$

$$= 20 \cos(5x + 25t) - 10\sqrt{3} \sin(5x + 25t)$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$= -\sqrt{700}\ \sin\left[5x + 25t - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)\right]$$

$$= \sqrt{700}\ \cos\left[5x + 25t + \left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)\right]\right]$$

$$\Rightarrow \theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\approx \underline{41^\circ}$$

Remember to convert the form to initial type.....

(Q8)

(Q9)

Q1

$$k = \frac{2\pi}{1} = \underline{2\pi\ m^{-1}}$$

$$v = \frac{\omega}{k} \Rightarrow \omega = vk = 2\pi \times 3 = \underline{6\pi\ s^{-1}}$$

For wave 1

$$y = A \sin(6\pi t - 2\pi x)$$

For wave 2

$$y = A \sin(6\pi t + 2\pi x)$$

$$\Rightarrow y_{total} = y_1 + y_2$$

$$= \underline{2A \sin(6\pi t) \cos(2\pi x)}$$

<u>Nodes</u>

$$y = 0 \Rightarrow 2\pi x = (2n-1)\pi/2$$

$$\Rightarrow \boxed{x = \frac{(2n-1)}{4}} m = \frac{1}{4} m, \frac{3}{4} m, \dots$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

if strings are fixed at both ends, then at x=0, it will be node ALWAYS. hence choose the wave wisely i.e. amplitude of resultant wave should have sinkx term

~~Antinodes~~

$\cancel{y = y_{max}}$

$\cancel{\Rightarrow 2\pi x = n\pi}$

$\cancel{\Rightarrow \boxed{x = \frac{n}{2}\,m} = 0, \frac{1}{2}m, \frac{1}{4}m, \frac{3}{2}m, \ldots}$

कृपया इस स्थान में कुछ न लिखें।
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★ very important to take right type of wave at starting in standing waves

let the two waves be

$y_1 = A\sin(kx - \omega t)$

$y_2 = A\sin(kx + \omega t)$

$\boxed{\begin{array}{l} k = 2\pi \\ \omega = 6\pi \end{array}}$

$\Rightarrow y = A\sin kx \cos \omega t$

$\Rightarrow$ At $x = 0$, $y = 0$: satisfied

Node

$kx = n\pi$

$\Rightarrow x = \frac{n\pi}{k} = \left(\frac{n}{2}\right) m$

Antinode

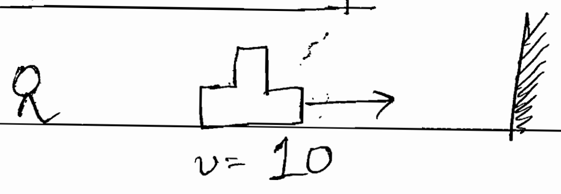
$kx = (2n+1)\frac{\pi}{2}$

$\Rightarrow x = \frac{(2n-1)}{4} m$

Q10) For doppler effect in sound,

$\cancel{\nu = \nu_0 \left(\frac{v - v_o}{v - v_s}\right)}$

No underroot in sound doppler !!



$$\nu_{direct} = \nu_0 \sqrt{\frac{v-0}{v+v_s}} = 900\sqrt{\frac{330}{340}}$$

$$= 873 \text{ Hz}$$

$$\nu_{indirect} = \nu_{wall}$$

$$= \nu_0 \sqrt{\frac{v-0}{v-v_s}} = 900\sqrt{\frac{330}{320}}$$

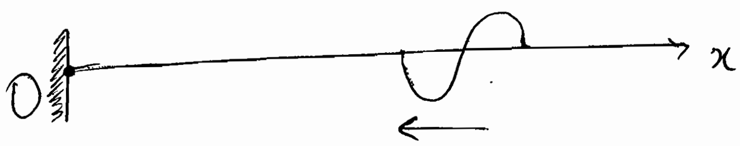
$$= 928.9 \text{ Hz}$$

No. of beats $= \nu_1 - \nu_2$

$= 55$

But person's ear can't decipher more than 10 beats. Hence No beats heard.

Q11)



Let the wave be described as

$y_1 = A \sin(kx + \omega t)$ (travelling in $-x$ direction)

Since the point O is fixed, it will be reflected off; let the reflected wave be:

$y_2 = A \sin(kx - \omega t)$

$\therefore$ the total standing wave will be superposition of y_1 and y_2

$\Rightarrow y_{total} = y_1 + y_2 = 2A \sin kx \cos \omega t$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Now for nodes

$y(x) = 0$

$\Rightarrow \sin kx = 0$

$\Rightarrow kx = n\pi$

$\Rightarrow x = \frac{n\pi}{k} = \frac{n\cancel{\pi}}{2\cancel{\pi}} \cdot \lambda = \left(\frac{n\lambda}{2}\right)$

$\therefore$ Nodes at at $\left[0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots\ldots\right]$

At antinodes,

$\sin kx = 1$

$\Rightarrow y = 2A\cos\omega t$

Now the kinetic energy for a small length dl around the antinode is given by,

$$k.E. = \frac{1}{2} m \dot{y}^2$$

$$= \frac{1}{2} \cdot dl \cdot \rho \cdot (2A\omega)^2 \cdot \cos^2\omega t$$

$$= dl \cdot 2A^2\omega^2\rho \cos^2\omega t$$

$\Rightarrow$ k.E per unit length $= \underline{2A^2\omega^2\rho\cos^2\omega t}$

(Q12) lecture 1

(Q13) The equation for a standing wave fixed at both the ends is given as

$y = 2A\sin kx \cos\omega t$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

At $x=0$ $\quad y=0$

At $x=L$ $\quad y=0 \Rightarrow kL = n\pi$

$\Rightarrow k = \left(\frac{n\pi}{L}\right)$

$$\Rightarrow y = 2A \sin\left(\frac{n\pi x}{L}\right) \cos\omega t$$

Let a consider a different portion dx around the point x

$$\Rightarrow y\big|_x = 2A \sin\left(\frac{n\pi x}{L}\right) \cos\omega t$$

$$\dot{y}\big|_x = -2A\omega \sin\left(\frac{n\pi x}{L}\right) \cdot \sin\omega t$$

The energy density is given as

$$\frac{E}{dl} = \frac{1}{2}\frac{dm\, v^2}{dl}$$

$$= \frac{1}{2}\frac{\rho\, dl}{dl} \cdot 4A^2\omega^2 \sin^2\left(\frac{n\pi x}{L}\right)\sin^2\omega t$$

$\Rightarrow$ Energy per unit length = Energy density (E_l)

$$= 2\rho A^2\omega^2 \sin^2\left(\frac{n\pi x}{L}\right)\sin^2\omega t$$

In a time period $\langle \sin^2\omega t\rangle = \frac{1}{2}$.

$$\Rightarrow \boxed{\langle E_l\rangle = \rho A^2\omega^2 \sin^2\left(\frac{n\pi x}{L}\right)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Now E_e will be maximum where

$$\sin^2\left(\frac{n\pi x}{L}\right) = 1$$

i.e. $\left|\sin\left(\frac{n\pi x}{L}\right)\right| = 1$ i.e. antinodes

& minimum

at $\left|\sin\left(\frac{n\pi x}{L}\right)\right| = 0$ i.e. nodes.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q14) Let a Consider a tube of cross sectional area A. Let PQ and RS be two traverse sections of the tube located at x and $x+\Delta x$ from fixed point O. Let the longitudinal displacement be denoted by $\xi(x)$.

$\therefore$ the displacement of the two planes be $\xi(x)$ and $\xi(x+\Delta x)$

Now the difference between the two planes is

final separation between the two planes

$$[(x+\Delta x) + \xi(x+\Delta x)] - [x + \xi(x)]$$

$$= \Delta x + \Delta x\left(\frac{d\xi}{dx}\right)$$

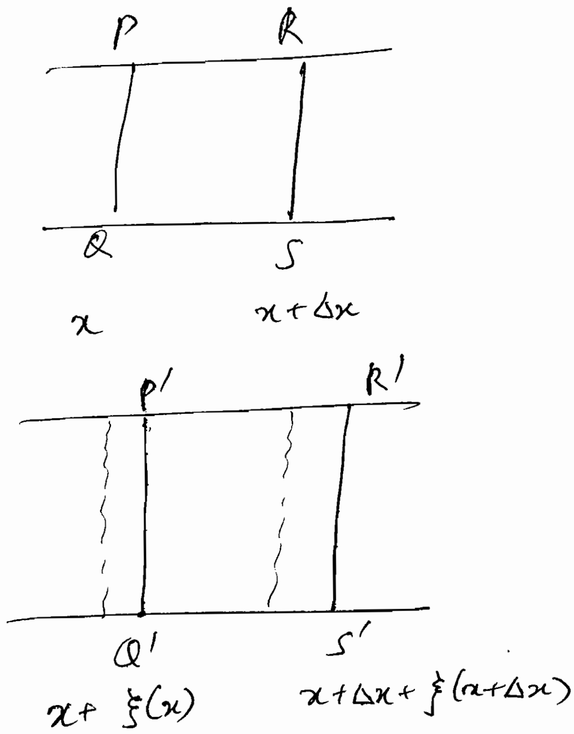
Change in volume $=$ A * change in length

$$= A\,\Delta x\left(\frac{d\xi}{dx}\right)$$

Original volume $= A\Delta x$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow \left(\frac{dV}{V}\right) = \text{strain} = \left(\frac{d\xi}{dx}\right)$$



judge साहब ये derivation note की जाए !!

Now let the change the volume be dV and the final volume be V.

Since mass remains same,

$$\Rightarrow M_1 = M_2$$

$$\Rightarrow \rho_0 [V - dV] = \rho V$$

$$\Rightarrow \rho = \rho_0 \left[\frac{V}{V} - \frac{dV}{V}\right] = \rho_0 \left[1 - \frac{dV}{V}\right]$$

$$\boxed{\rho = \rho_0 \left[1 - \frac{d\xi}{dx}\right]}$$

Or take initial volume as V and binomially we can have same result....

कृपया इस स्थान में कुछ न लिखें।
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कृपया इस स्थान में कुछ न लिखें।
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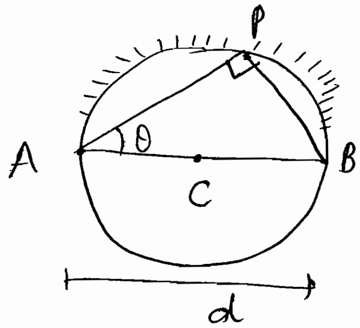
Tut Sheet 8 : Geometrical Optics.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q1) Now we know for a Point that joins 2 diametrically opposite points, an angle of 90° is subtended.

Now the Path length is $\underline{d\cos\theta + d\sin\theta}$



For extremely,

$ds = 0$

$\Rightarrow -d\sin\theta\, d\theta + d\cos\theta\, d\theta = 0$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \boxed{\theta = 45^\circ}$

$\Rightarrow \overline{APB} = 2\cancel{d}.\cos 45^\circ + 2\cancel{d}\sin 45^\circ$

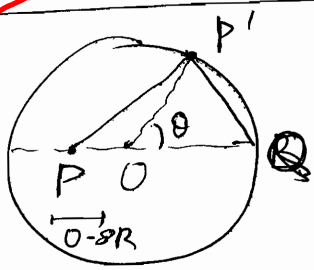
$= \underline{2\sqrt{2}\,\cancel{d}} = \underline{\sqrt{2}\,d}$

$$\frac{d^2S}{d\theta^2} = -d\cos\theta\Big|_{\theta=45^\circ} \mp d\sin\theta\Big|_{\theta=45^\circ}$$

< 0

Hence the path is maximum

Q2) To prove: $\cos\left(\frac{\theta}{2}\right) = \frac{3}{4}$



$PP' = \sqrt{OP^2 + OP'^2 - 2\,OP\,OP'\cos(\pi-\theta)}$

$= \sqrt{(0.8R)^2 + R^2 + 2.\,0.8R^2\cos\theta}$

$= \sqrt{1.64 + 1.6\cos\theta}\;R$

$P'Q = \sqrt{R^2 + R^2 - 2R^2\cos\theta}$

$= R\sqrt{2 - 2\cos\theta} = R.\,2\sin(\theta/2)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\frac{dS}{d\theta} = 0 \quad \text{where } S = R\sqrt{1.64 + 1.6\cos\theta} + 2R\sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow -\frac{1}{2}R \frac{1.6\sin\theta}{\sqrt{1.64+1.6\cos\theta}} + 2R\cos\left(\frac{\theta}{2}\right)\cdot\frac{1}{2} = 0$$

$$\Rightarrow \cos\left(\frac{\theta}{2}\right) = \frac{0.8\sin\theta}{\sqrt{1.64+1.6\cos\theta}}$$

$$\Rightarrow 1.64 + 1.6\cos\theta = \left[1.6\sin\left(\frac{\theta}{2}\right)\right]^2$$

$$\Rightarrow 1.64 + 1.6\left[2\cos^2\frac{\theta}{2} - 1\right] = 2.56\left[1 - \cos^2\frac{\theta}{2}\right]$$

$$\Rightarrow \cos^2\left(\frac{\theta}{2}\right)\left[3.2 + 2.56\right] = 2.56 - 1.64 + 1.6$$

$$\Rightarrow \cos^2\frac{\theta}{2} = \frac{2.52}{5.76}$$

$$\cos\left(\frac{\theta}{2}\right) = \underline{0.4375}$$

ढंग से करो .... आ जाएगा !!

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q3)

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें।
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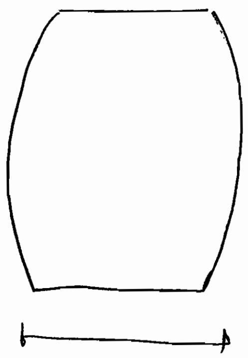


$t = 1$ cm

$R_1 = +4$ cm $R_2 = -4$ cm

Let the system matrix be $\begin{bmatrix} b & -a \\ -d & c \end{bmatrix}$.

It is composition of 3 operations:

$$\begin{bmatrix} 1 & -P_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{d}{n_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & -P_1 \\ 0 & 1 \end{bmatrix}$$

$$P_1 = \frac{n_2 - n_1}{R_1} = \frac{1.5 - 1}{4} = \frac{0.5}{4} = \frac{1}{8}$$

$$P_2 = \frac{n_2 - n_1}{R_2} = \frac{1 - 1.5}{-4} = \frac{1}{8}$$

$$\Rightarrow \begin{bmatrix} b & -a \\ -d & c \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{8} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{8} \\ 0 & 1 \end{bmatrix}$$

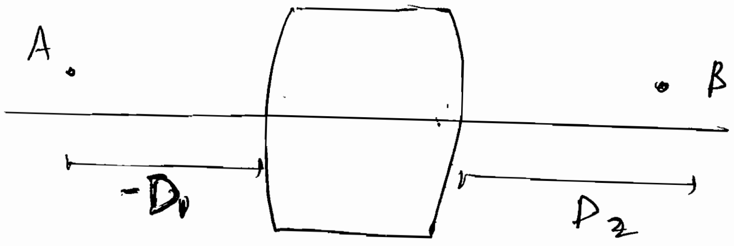
$$= \begin{bmatrix} 1 - \frac{1}{12} & -\frac{1}{8} \\ \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{8} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{12} & -\frac{1}{8} \\ \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{8} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{12} & -\left(\frac{11}{12} + 1\right) \cdot \frac{1}{8} \\ \frac{2}{3} & -\frac{1}{12} + 1 \end{bmatrix} = \begin{bmatrix} \frac{11}{12} & -\frac{23}{96} \\ \frac{2}{3} & \frac{11}{12} \end{bmatrix}$$

$\boxed{\text{Determinant should be 1, not 0}}$

Now consider 2 points D_1 and D_2 distance away



$$\begin{bmatrix}\lambda_2\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ D_2 & 1\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 0\\ -D_1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\ y_1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\lambda_2\\ y_2\end{bmatrix} = \begin{bmatrix}a & b\\ D_2a+c & D_2b+d\end{bmatrix}\begin{bmatrix}1 & 0\\ -D_1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\ y_1\end{bmatrix}$$

$$= \begin{bmatrix}a-bD_1 & b\\ D_2a+c-D_1D_2b-dD_1 & D_2b+d\end{bmatrix}\begin{bmatrix}\lambda_1\\ y_1\end{bmatrix}$$

For $\frac{a}{D_1} - \frac{d}{D_2} = b - \frac{c}{D_1D_2}$

$M = D_2b+d = \frac{D_2a+c}{D_1} = 1$

$\Rightarrow D_2 = \frac{1-d}{b}$

$D_1 =$

$$\begin{bmatrix}\lambda_2\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ D_2 & 1\end{bmatrix}\begin{bmatrix}b & -a\\ -d & c\end{bmatrix}\begin{bmatrix}1 & 0\\ D_1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\ y_1\end{bmatrix}$$

$$= \begin{bmatrix}b & -a\\ D_2b-d & c-D_2a\end{bmatrix}\begin{bmatrix}1 & 0\\ D_1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\ y_1\end{bmatrix}$$

$$= \begin{bmatrix}b-aD_1 & -a\\ D_2b-d+cD_1-aD_1D_2 & c-D_2a\end{bmatrix}$$

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

$$\frac{b}{D_1} + \frac{c}{D_2} = \frac{d}{D_1 D_2} + a$$

$$M = c - D_2 a$$
$$= b - aD_1$$

this comes from Det = 1

Now focal length $= \frac{-1}{a} = \left(\frac{96}{23}\right)$

Nodal ~~Unit~~ points at $D_1 = \frac{b-1}{a} = \frac{\frac{11}{12} - 1}{\left(\frac{23}{96}\right)} = -\frac{1}{12} \times \frac{96}{23}$

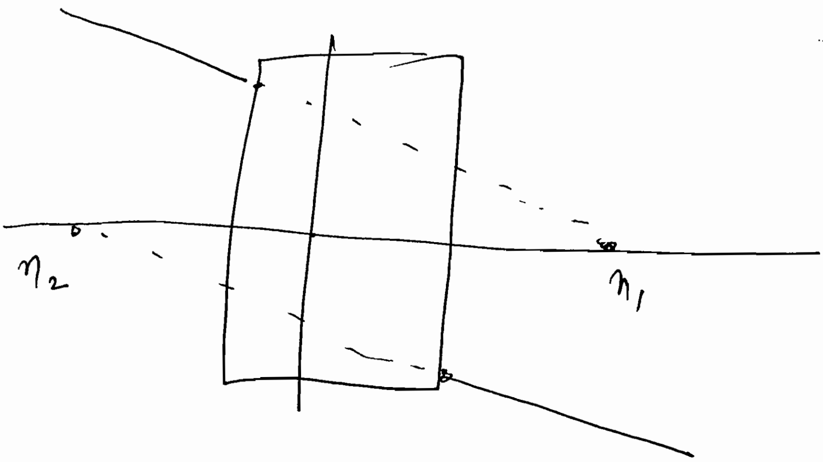
$= -\left(\frac{8}{23}\right)$

$D_2 = \frac{c-1}{a}$

$= \frac{\frac{11}{12} - 1}{\left(\frac{23}{96}\right)} = -\left(\frac{8}{23}\right)$

both negative? One

unit points? same ✓



कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

Q 4)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

~~$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$~~

~~$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} -$~~

Considering the matrix method

$$\begin{bmatrix} 1 & 0 \\ v & 1 \end{bmatrix}\begin{bmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}\begin{bmatrix} 1 & -\frac{1}{f_1} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -u & 1 \end{bmatrix}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

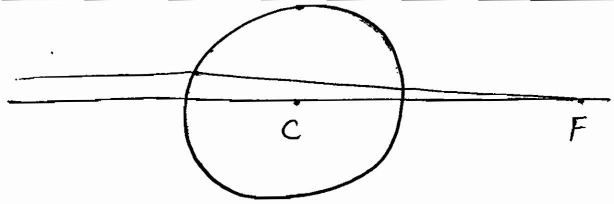
अरे भइया focal lengths तो दे दो !!

Q 5)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$P_1 = \left(\frac{\mu-1}{R}\right) = P$$

$$P_2 = +\left(\frac{\mu-1}{R}\right) = P$$



कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Combined matrix

$$= \begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{2R}{\mu} & 1 \end{bmatrix} \begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix}$$

$R = 20$ cm $= 0.2$ m

$\mu = 1.6$

$P = 3\ m^{-1}$

$$= \begin{bmatrix} 1 - \frac{2PR}{\mu} & -P \\ 2\frac{R}{\mu} & 1 \end{bmatrix} \begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix}$$

$\frac{PR}{\mu} = 0.375$

$$= \begin{bmatrix} 1 - \frac{PR}{\mu} & -P + \frac{P^2R}{\mu} - P \\ 2\frac{R}{\mu} & -\frac{PR}{\mu} + 1 \end{bmatrix}$$

2 से ही गुणा multiply करेगा !!

$$\begin{bmatrix} \lambda \\ y \end{bmatrix} = \begin{bmatrix} 1 - \frac{PR}{\mu} & -2P + \frac{P^2R}{\mu} \\ \frac{R}{\mu} & -\frac{PR}{\mu}+1 \end{bmatrix} \begin{bmatrix} \lambda_i \\ y_i \end{bmatrix}$$

$$\begin{bmatrix} 0.625 & -4.8 \\ 0.125 & 0.62 \end{bmatrix}$$

[redacted] Calculating nodal points

$$\begin{bmatrix} 1 & 0 \\ v & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ u & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ av+c & bv+d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ u & 1 \end{bmatrix} = \begin{bmatrix} a+bu & b \\ av+c+buv+ud & bv+d \end{bmatrix}$$

As such focal length will be "b" but if we need to find position of focal point, we need to find unit planes

$$\boxed{v = \frac{1-d}{b}} \qquad \boxed{u = -\frac{(a-\alpha)}{(b)}}$$

$v = $ [redacted] 0.077

$u = $ [redacted] 0.077

ALS Alternative Learning Systems

$$\begin{bmatrix} \lambda \\ y_0 \end{bmatrix} = \begin{bmatrix} 1 & -4.875 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ y_0 \end{bmatrix}$$

$\Rightarrow \lambda = -4.875 y_0$

$-\tan\theta = -4.875 y_0$

$f = \frac{1}{4.875} = 0.205$

$\Rightarrow$ Position of Paraxial focus $= \boxed{0.282}$ m

interactions A Division of ALS

Q 6)

कृपया इस स्थान में कुछ न लिखें।
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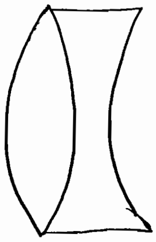

1 : crown
2 ; flint

कृपया इस स्थान में कुछ न लिखें।
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$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{20} \quad \text{— (1)}$$

Also $-\frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} \approx \frac{(\mu_b - \mu_r)/(\mu_{av} - 1)}{(\mu_b' - \mu_r')/(\mu_{av}' - 1)} = \cancel{0.4715}$

$$\cancel{\Rightarrow \frac{1}{f_1} = \frac{-2.1}{f_2}}$$

$$\cancel{\Rightarrow \frac{-2.1}{f_2} + \frac{1}{f_2} = \frac{1}{20}} \quad \cancel{\Rightarrow \boxed{f_2 = -22}}$$

$$\cancel{\boxed{f_1 = 10.47}}$$

$$\boxed{\omega_1 = \frac{\mu_b - \mu_r}{\left(\frac{\mu_b + \mu_r}{2} - 1\right)}} = 0.01546$$

Note the formula

$$\omega_2 = 0.02715$$

$$\boxed{\frac{\omega_1}{\omega_2} = 0.5694} \Rightarrow \frac{1}{f_2} = \frac{-1}{f_1} \cdot 0.5694$$

$$\Rightarrow \frac{1}{f_1} = \frac{-1.75}{f_2}$$

$$\Rightarrow -\frac{0.75}{f_2} = \frac{1}{20} \Rightarrow \boxed{f_2 = -15.1 \text{ cm}}$$

$$\boxed{f_1 = 8.6 \text{ cm}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q7)

$$S = \frac{h^2 \phi(k,n)}{f}$$

Now $\frac{S_2}{S_1} = \frac{h_2^2}{h_1^2} = \left(\frac{2}{6}\right)^2$

$\Rightarrow S_2 = \frac{1}{9} \times 1.8\ cm = 0.2\ cm$

Q8)

$h_1^2 : h_2^2 : h_3^2$

$= 25 : (144-100) : 144$

$= 25 : 44 : 144$

Q9) Lecture 1

Q10) System Matrix for lens is given as

$$\begin{bmatrix} 1 & -\left(\frac{1}{f_1}\right) \\ 0 & 1 \end{bmatrix}$$

The total matrix is, therefore

$$\begin{bmatrix} 1 & 0 \\ v & 1 \end{bmatrix}\begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ +0.2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ v & 1 \end{bmatrix}\begin{bmatrix} 0 & -10 \\ 0.1 & 1 \end{bmatrix}\begin{bmatrix} 0 & -5 \\ 0.2 & 1 \end{bmatrix}$$

In Matrix Method, we transverse from one point to another, so sign will be accordingly!!

Very Important Point: it will be +0.2 ∵ we are going towards right so $\left(\frac{D}{n_1}\right) > 0$. So Sir ji were right in using $+D_1$ and $+D_2$ ....

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$= \begin{bmatrix} 1 & 0 \\ v & 1 \end{bmatrix} \begin{bmatrix} -2 & -10 \\ 0.2 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -10 \\ -2v + 0.2 & -10v + 0.5 \end{bmatrix}$$

Now, $-2v + 0.2 = 0$

$$\Rightarrow \boxed{v = 0.1}$$ ✓

$$\frac{y_2}{y_1} = 0.5 - 1 = -0.5 = -\left(\frac{1}{2}\right)$$

Magnification of $\left(\frac{1}{2}\right)$ and inverted image will be formed. $y_2 = -0.005$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

long way

(1) find u_0, v_0 for unit planes

$u_0 = -0.1$

$v_0 = -0.05$

(2) find $f = 0.1$

(3) $\Rightarrow u_{eff} = -0.3$

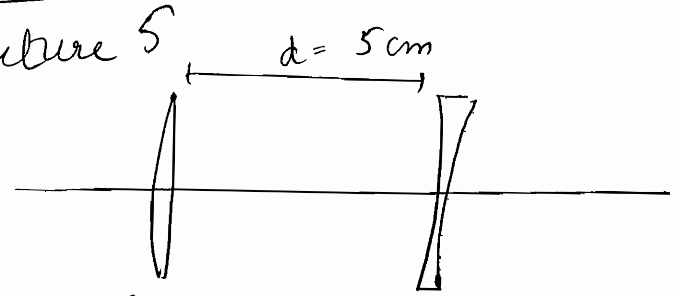
$f = 0.1$

$\Rightarrow v = \left(\frac{3}{20}\right)$

$v_{eff} = 0.1$

Q 11) lecture 5

Q 12)



$f = 10$ $\qquad$ $f = -10$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

$$= \frac{1}{10} - \frac{1}{10} + \frac{5}{10.10} = \left(\frac{1}{20}\right)$$

$$\boxed{f = 20 \text{ cm}}$$

$$\boxed{\text{Power} = \frac{1}{f} = 0.05 = 5 \text{ diopter}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

To find principal points :

Unit point = principal point

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q13) lecture 6

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q14) Fermat's Method : lecture 5

$$\nu = \nu_0 = \frac{c}{\lambda} = \frac{3\times10^8}{600\times10^{-9}} = \underline{\underline{5\times10^{14}\ Hz}}$$

$$v = \frac{c}{n} = \frac{3\times10^8\times2}{3} = \underline{\underline{2\times10^8\ m/s}}$$

$$\lambda = \frac{v}{\nu} = \frac{2\times10^8}{5\times10^{14}} = 4\times10^{-7} = \underline{\underline{400\ nm}}$$

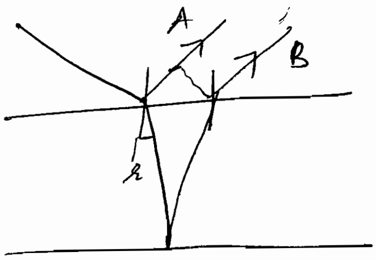
Q15) lecture 5

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Tut Sheet 9 : Interference

Q1)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)



Path difference between ray A and ray B is $2\mu t\cos r - \frac{\lambda}{2}$

Now from snell's law

$\mu_1 \sin i = \mu_2 \sin r$

$\Rightarrow \sin 45^\circ = \mu_2 \sin r = 1.33 \sin r$

$\Rightarrow \sin r = 0.53$

$\Rightarrow \cos r = .84$

For a destructive interference

$\Delta = (2n-1)\frac{\lambda}{2}$

$\Rightarrow 2\mu t\cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$

$\Rightarrow 2\mu t\cos r = n\lambda$

$\Rightarrow t = \left(\frac{n\lambda}{2\mu\cos r}\right)$

For minimum thickness, $n = 1$

$\Rightarrow t = \frac{\lambda}{2\mu\cos r} = \frac{5890\ \text{Å}}{2\times 1.33\times 0.84} = 0.2636\ \mu m$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q2)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For interference, intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

where I_1 and I_2 are intensity due to first & second wave while ϕ is the phase difference.

Now $I_{av} = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle \cos\theta \rangle$

$$= [I_1 + I_2]$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2 = 1.1\,(I_1 + I_2)$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$= (\sqrt{I_1} - \sqrt{I_2})^2 = 0.9\,(I_1 + I_2)$$

$\boxed{\text{1st Method}}$

Let $\frac{I_1}{I_2} = x > 1$

$\Rightarrow$

$$x + 1 + 2\sqrt{x} = 1.1\,(x+1) \quad \text{— (1)}$$

$$x + 1 - 2\sqrt{x} = 0.9\,(x+1)$$

$$4\sqrt{x} = 0.2\,(x+1)$$

$$20\sqrt{x} = x + 1$$

$$400x = x^2 + 1 + 2x$$

$$x^2 - 398x + 1 = 0 \Rightarrow x = \frac{398 \pm \sqrt{398}}{2} \approx 398$$

$(\because x > 1)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

2nd Method

$$\Rightarrow \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = \sqrt{\frac{1.1}{0.9}} = 1.105$$

$$\Rightarrow \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{2.1}{0.1}$$

$$\Rightarrow \frac{I_1}{I_2} = 401.9 \approx \underline{402}$$

remember this

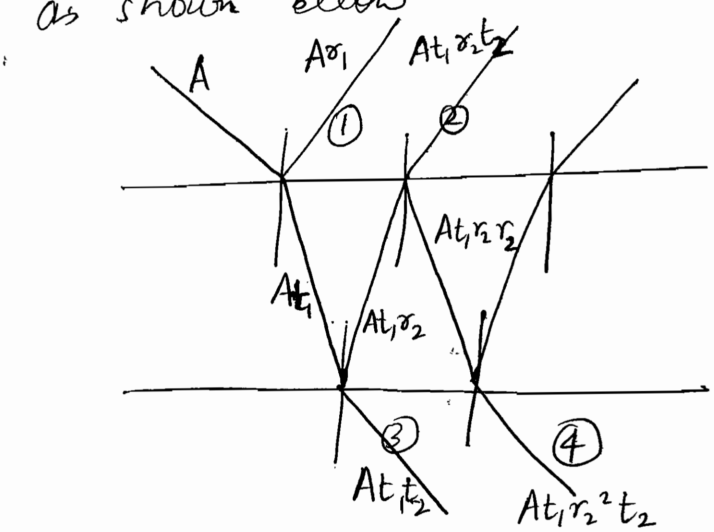
(Q3) Let the index of the fix be $\boxed{1.33}$

air to film

$\Rightarrow$ Transmission Ratio $= \frac{2\mu_1}{\mu_1+\mu_2} = \frac{2}{1+1.33} = 0.858 = t_1$ (say)
$\left(= \frac{A_t}{A_i}\right)$

Reflection ratio $= \frac{A_R}{A_i} = \frac{\mu_1-\mu_2}{\mu_1+\mu_2} = \frac{1-1.33}{1+1.33} = -0.14 = r_1$ (say)

Let the consider the multiple reflections as shown below



Similarly

film to air

$$t_2 = \frac{2\mu_1}{\mu_1+\mu_2} = \frac{2\times 1.33}{2.33} = 1.141$$

$$r_2 = \frac{1.33-1}{1.33+1} = 0.14$$

Consider waves ① and ②

$A_1 = A r_1 = \underline{0.14 A}$

$A_2 = A t_1 t_2 r_2 = A (0.858)(1.141)\, 0.14$

$= 0.137 A \approx \underline{0.14 A}$

Consider wave ③ and ④

$A_3 = A t_1 t_2 = A\ 0.858\ 1.141$

$= \underline{0.97 A}$

$A_4 = A t_1 t_2 r_2^2 = 0.97 * (0.14)^2 A$

$= \underline{0.02 A}$

In transmitted light, the ray 2' differs from ray 1' in amplitude ratio r^2. Therefore the interfering transmitted rays are of very unequal magnitude & hence the contrast of maxima and minima is poor.

→ Rather give the proof of complete darkness

"B.S. Agarwal"

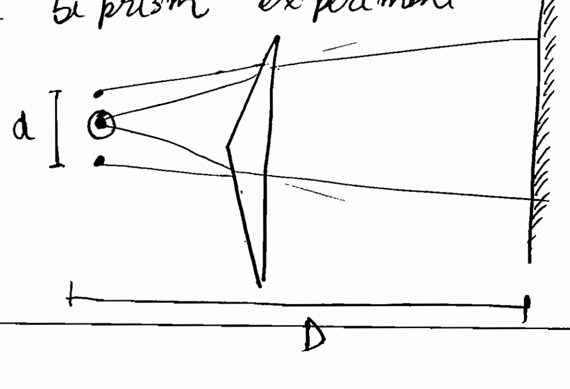
Now since for reflected waves, we have almost similar intensities ⇒ contrast in interference will be more

while

For transmitted waves, difference between intensities is so much that there will be no contrast.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q4) For a biprism experiment



कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$d\sin\theta = n\lambda$ (for constructive interference)

$\Rightarrow d \cdot \frac{y_n}{D} = n\lambda$

$\Rightarrow d\frac{\beta}{D} = \lambda$

$\Rightarrow \boxed{\beta = \left(\frac{D\lambda}{d}\right)}$

When mica sheet introduced, path difference for central fringe

$= d\sin\theta = (\mu-1)t$

$\Rightarrow d \cdot \frac{y_n}{D} = (\mu-1)t$

$\Rightarrow d \cdot \frac{1.89}{D} = 0.59\, t$

$\Rightarrow \frac{\lambda}{\beta} \cdot 1.89 = 0.59\, t$

$\Rightarrow \frac{5900\,\text{Å} \times 1.89}{0.43} = 0.59\, t$

$\Rightarrow \boxed{t = 4.395\ \mu m}$

(Q5) lecture 7

(Q6) **Colours produced in thin films**

When a thin film of oil on water, or a soap bubble, exposed to an extended to a white light (such as sky)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

is observed under reflected light, brilliant colours are seen in the film or the bubble. These colours arise due to the interference of the light waves reflected from the top and bottom surface of the film.

At a particular point of the film, and for a particular position of the eye (i.e. for a particular t and r), rays of only certain wavelengths will have a path difference satisfying the condition of maxima. Hence only these colours will be visible with maximum intensity. The colouration will vary with the thickness of the film as well as with the position of eye. If different points of the film are observed with eye in the same position, a different set of colours will be observed.

In the problem, considering normal incidence,

For appearing dark in reflected light

$$\Delta = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow 2\mu t - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow 2\mu t = n\lambda \Rightarrow t = \left(\frac{n\lambda}{2\mu}\right)$$

$$\boxed{\text{For minimum thickness, } n=1 \Rightarrow t = \frac{\lambda}{2\mu} = 0.2\,\mu m}$$


: eye

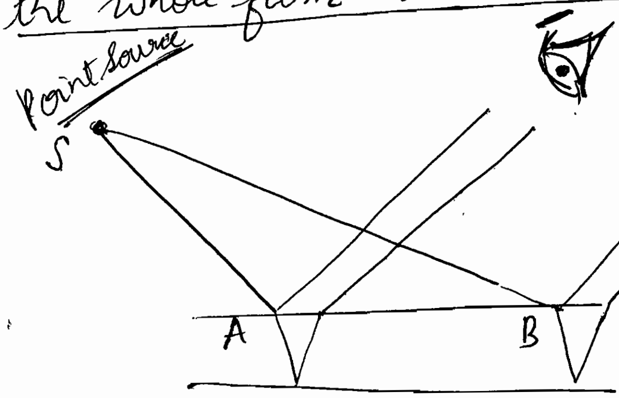
Q7) lecture 5

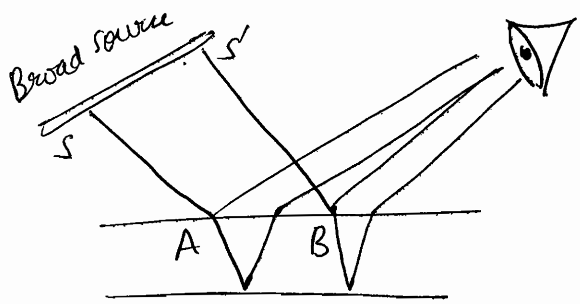
$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = \left(\frac{2+1}{2-1}\right)^2 = \frac{9}{1}$$

Q 8)

REQUIREMENT OF EXTENDED SOURCE

In the case of interference exhibited by thin films (eg. soap film) an extended (broad) source of light is required so that light from each point on film may reach the eye of the observer to facilitate the whole film to be viewed at a glance.





For example, consider a narrow source. SA reaches the eye but SB does not. Thus only limited portion of the film is visible.

Now consider a broader extended source. Both SA and S'B could reach the eye. Therefore a large portion of film is visible and colours can be seen.

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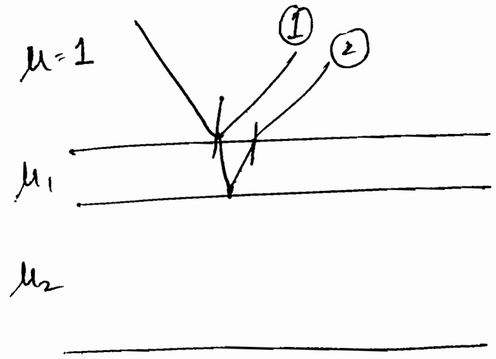
For non reflecting film, a film of lower index will be used



$\mu_1 < \mu_2$

$$\Rightarrow \Delta_{12} = 2\mu t \cos r \approx 2\mu t \quad (\perp \text{ incident})$$

For non reflectivity, destructive interference is required.

$$\Rightarrow 2\mu t = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow t = \frac{(2n-1)\lambda}{4\mu} = (2n-1).\frac{5.5\times10^{-5}}{\frac{5}{4}\times4}$$

$$= (2n-1) * 1.1\times10^{-5}\text{ cm}$$

For minimum thickness, $t = 0.11\ \mu m$

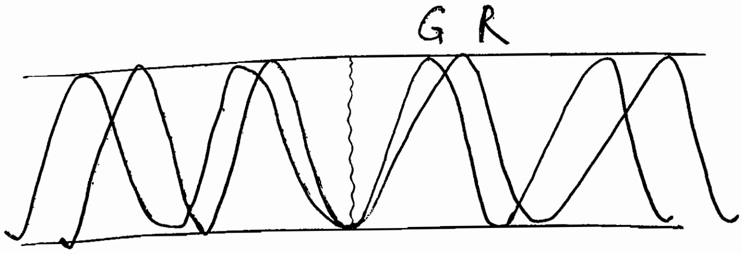
Q9)

WHITE LIGHT IN MICHELSON INTERFERENCE

If a white light source (eg. Incandescent lamp) is used, no fringes will be seen except when the path difference is so small that is less than few wavelengths, only then few curved and coloured localized fringes are obtained.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For a white light source, only a few fringes are observed because light contains all wavelengths between 4000 Å to 7,500 Å. Fringes in different colour will coincide.



Intensity distribution for green light and red light is shown. The central fringe is uncoloured and different colours will begin to separate on either sides.

After about 10 fringes, many colours will tend to overlap at a point resulting in white.

(Q10) Δ central point $= (\mu-1)t$ (Ignoring diffraction effects)

$$\phi = \frac{2\pi}{\lambda}.(\mu-1)t$$

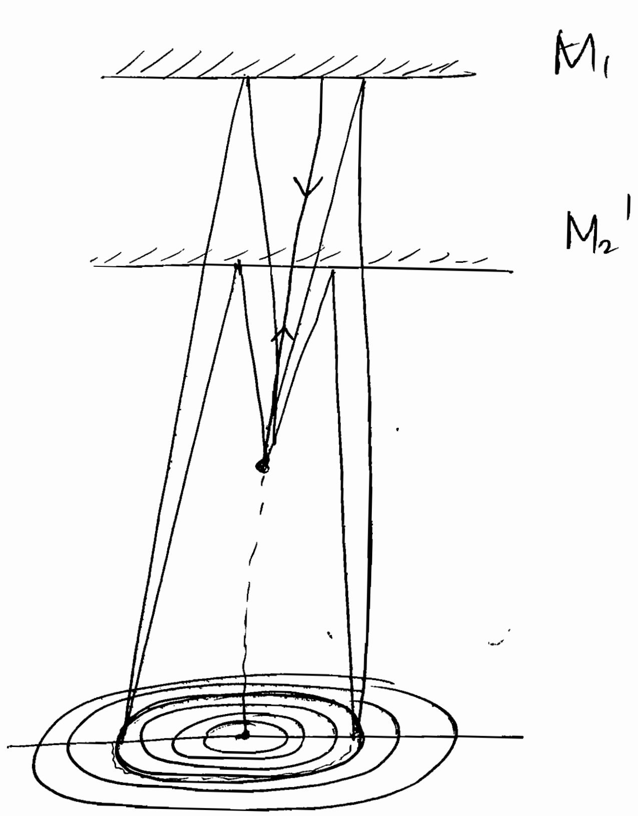
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{2\pi}{\lambda}(\mu-1)t\right)$$

$$= 2I\left[1+\cos\left(\frac{2\pi}{\lambda}(\mu-1)t\right)\right]$$

$$\boxed{I = 4I\cos^2\left(\frac{\pi}{\lambda}(\mu-1)t\right)}$$

[Note that even if we take diffraction into affect $\left(\frac{\sin\alpha}{\alpha}\right) \to 1 \because \theta = 0$ for central point]

Q11)



Circular Fringes

When M_1 and M_2 are exactly perpendicular i.e. M_1 and M_2' are exactly parallel, the film M_1M_2' is of uniform thickness, and we obtain circular fringes localized at infinity.

$$\Delta = 2d\cos r + \frac{\lambda}{2}$$

For maximum intensity $\Delta = n\lambda$

Since d is constant, Haidinger rings of Constant [redacted] inclination are obtained i.e. difference rings for different angles of refraction.

कृपया इस स्थान में कुछ न लिखें।
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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Straight Fringes

When M_1 and M_2 are not $\perp$, we have similar configuration as wedge shaped film. $\therefore$ straight fringes are obtained.

Extended Source

In order to view the whole mirror filled with light we use extended source of light.

For change in distance d,

Change in Δ = $2d$

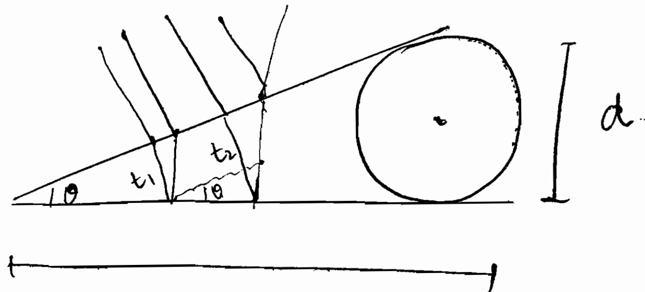
Now if $2d = N\lambda$, N fringes are shifted

(Q12)

$\lambda = 5000\ \text{Å}$

$\beta = 1.25\text{cm}$

$d = ?$



Since intervening medium is air, $\mu = 1$

$\Rightarrow \Delta_1 = 2t_1 = n\lambda$

$\Delta_2 = 2t_2 = (n+1)\lambda$

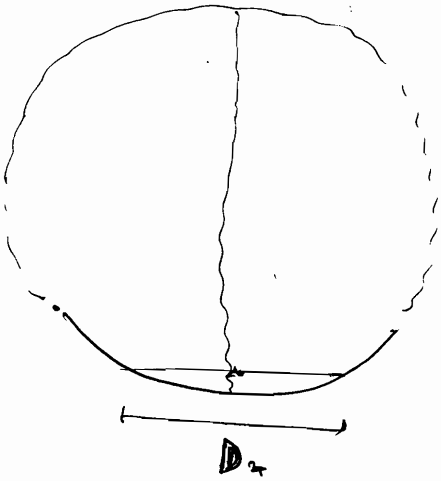
$\Rightarrow t_2 - t_1 = \lambda/2 \Rightarrow \frac{t_2 - t_1}{\beta} = \tan\theta \approx \frac{d}{l}$

$\Rightarrow \beta = \left(\frac{\lambda l}{2d}\right)$ $\Rightarrow d \approx \frac{\lambda l}{2\beta} = 2\ \mu m$

Q 13)

Q 14)

Q-11 lecture 9



$$\left(\frac{D}{2}\right)^2 = t\,(2R-t) \approx 2Rt$$

$$\boxed{D^2 = 8Rt}$$

Now for bright rings

$$\Delta = n\lambda$$

$$\Rightarrow 2\mu t + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow t = \frac{(2n-1)}{2\mu}\frac{\lambda}{2} = (2n-1)\frac{\lambda}{4\mu}$$

$$\Rightarrow \boxed{\frac{D_n^2}{8R} = (2n-1)\frac{\lambda}{4\mu}}$$

$$\Rightarrow \frac{(2.303 \cancel{\times 2})^2 \times 10^{-6}}{8R} = \frac{9\lambda}{4\mu}$$

$$\frac{(4.134 \cancel{\times 2})^2 \times 10^{-6}}{8R} = \frac{29\lambda}{4\mu}$$

$\mu = 1.5$

$\lambda = 5282 \text{ Å}$

$$\Rightarrow \boxed{R = 0.84\,m}$$

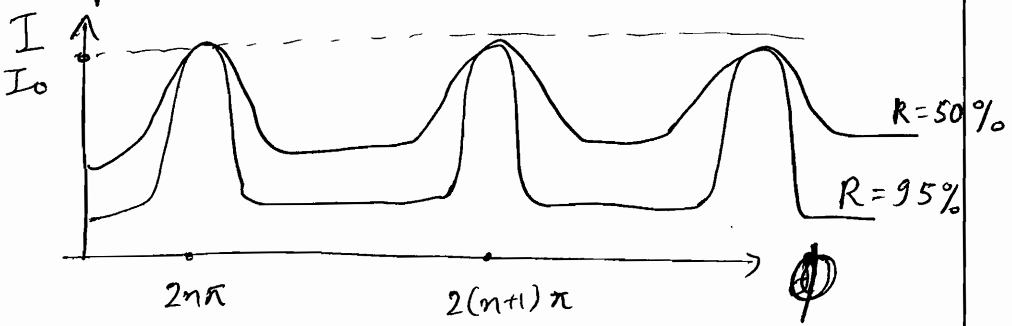
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q15) lecture 9

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

SHARPER FRINGES IN FABRY PEROT

The maxima of the fringes in this case is very sharp & narrow as compared to Michelson.



The reason for this is found in the fact that there are several interfering beams. The intensity I_t is given by

$$I_{total} = \frac{I_0}{1 + \frac{4R}{(1-R)^2}\sin^2\left(\frac{\phi}{2}\right)}$$

where ϕ is the phase difference between successive rays and R is reflectivity.

For maxima, $\delta = 2n\pi \Rightarrow I_{total} = I_0$

When $\frac{\delta}{2} \neq n\pi$, $(1-R) \to 0$ due to large R. and I_{total} drops sharply

When the reflectance is large & the intensity is due to superposition of several beams, the maxima are sharp & narrow.

Q16)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$E_1 = 2\cos\left(\vec{k_1}\cdot\vec{\partial} - \omega t + \frac{\pi}{3}\right) \quad kV/m$$

$$E_2 = 5\cos\left(\vec{k_2}\cdot\vec{\partial} - \omega t + \frac{\pi}{4}\right) \quad kV/m$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$I_1 = \frac{1}{2}\varepsilon_0 E_1^2 c = \frac{1}{2}\varepsilon_0 . 4 . c \times 10^6 \; W/m^2$$

$$I_2 = \frac{1}{2}\varepsilon_0 E_2^2 c = \frac{1}{2} . \varepsilon_0 . 25 c \times 10^6 \; W/m^2$$

$$I_{12} = (\sqrt{I_1} + \sqrt{I_2})^2 = (2+5)^2 . \frac{1}{2}\varepsilon_0 c \times 10^6$$

$$= \frac{49}{2}\varepsilon_0 c \times 10^6 \; W/m^2$$

$$I_{min} = (2-5)^2 \frac{1}{2}\varepsilon_0 c \times 10^6 = \frac{9}{2}\varepsilon_0 c \times 10^6 \; W/m^2$$

$$\Rightarrow V = \frac{49-9}{49+9} = \left(\frac{40}{58}\right)$$

कृपया इस स्थान में कुछ न लिखें।
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कृपया इस स्थान में कुछ न लिखें।
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Q17) lecture 7

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q18) Q3

Q19) Coherence length $= 2d = \dfrac{\lambda^2}{\Delta\lambda}$

$\because$ at this length, the pattern disappears.

For pattern to disappear

$$\frac{2d}{\lambda} - \frac{2d}{\lambda + \frac{\Delta\lambda}{2}} \geq \frac{1}{2}$$

$$\Rightarrow \frac{2d}{\lambda} - \frac{2d}{\lambda}\left(1 - \frac{\Delta\lambda}{2\lambda}\right) \geq \frac{1}{2}$$

$$\Rightarrow \frac{2d}{\lambda} \cdot \frac{\Delta\lambda}{2\lambda} \geq \frac{1}{2}$$

$$\Rightarrow \boxed{2d \geq \left(\frac{\lambda^2}{\Delta\lambda}\right)}$$

$$\Rightarrow l = 2d = \frac{(600)^2 \times 10^{-18}}{(0.01) \times 10^{-9}}$$

$$= \underline{0.036\ m}$$

Q20) $\vec{E_1} = E_1 \sin(kx - \omega t)\,\hat{i}$

$\vec{E_2} = E_2 \sin(kx - \omega t + \phi)\,\hat{j}$

$(\vec{E_1} + \vec{E_2}) = E_1 \sin(\ \ )\hat{i} + E_2 \sin(\ \ + \phi)\,\hat{j}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$I \propto E^2$

Perfect

$I = \langle (\vec{E_1} + \vec{E_2}) \cdot (\vec{E_1} + \vec{E_2}) \rangle$

$= \langle E_1^2 \sin^2(kx - \omega t)$

$+ E_2^2 \sin^2(kx - \omega t + \phi) \rangle$

$\cancel{+ 2E_1E_2 \sin(kx - \omega t) \sin(kx}$

$I = E_1^2 \langle \sin^2(kx - \omega t) \rangle$

$+ E_2^2 \sin$

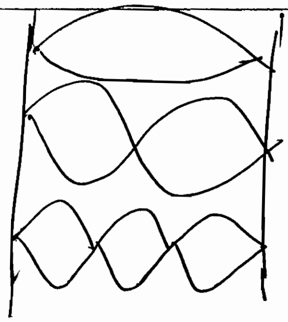
$\vdots$

$= \frac{(E_1^2 + E_2^2)}{2}$

= independent of ϕ

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q21)



For Fabry Perot etalon,

each mode differs by :

$$2d = n\lambda_1$$

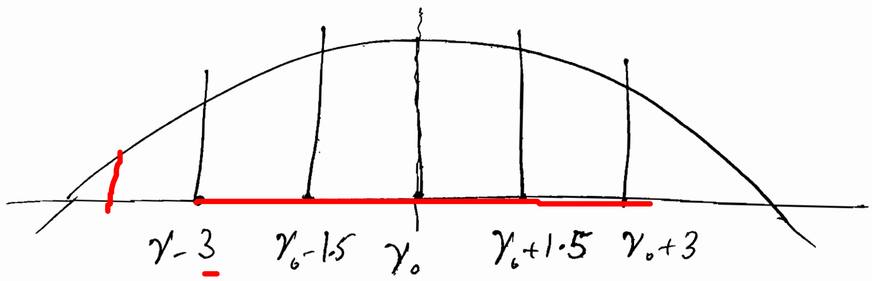
$$2d = (n+1)\lambda_2$$

$$\Rightarrow \cancel{\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2}}$$

$$\frac{2d}{\lambda_2} - \frac{2d}{\lambda_1} = 1$$

$$\frac{2d(\nu_2 - \nu_1)}{c} = 1$$

$$\Rightarrow \boxed{\Delta\nu = \frac{c}{2d}}$$ between 2 successive modes



$$\Delta\nu = \frac{3\times10^8}{2\times100\times10^{-3}} = \boxed{1.5\text{ GHz}}$$

5 modes can be supported.

कृपया इस स्थान में कुछ न लिखें।
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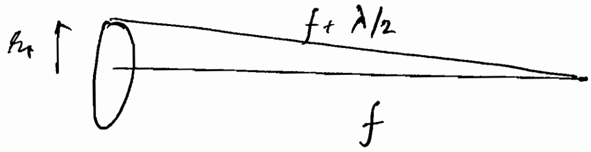
Tut Sheet 10 Diffraction

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q1) Considering the first image as the 1^{st} bright image i.e. when the Central Zone acts as ~~central~~ 1^{st} Half Period Zone, we have

Focal length f is calculated as

1st Way



$$f^2 + r_1^2 = \left(f + \frac{\lambda}{2}\right)^2$$

$$\Rightarrow r_1^2 = \frac{\lambda^2}{4} + 2f\frac{\lambda}{2}$$

Now $f \gg \lambda$

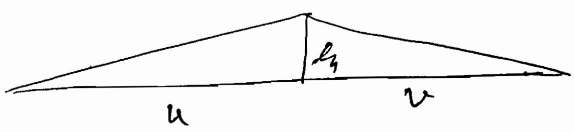
$$\Rightarrow \boxed{\frac{r_1^2}{\lambda} = f}$$

$\Rightarrow$ using

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{6} = \frac{5189.3 \times 10^{-3}}{\left(\frac{2.3}{2}\right)^2}$$

we have $\Rightarrow \boxed{v = 3.58\,m}$

2nd Way



$$\sqrt{u^2+r_1^2} + \sqrt{v^2+r_1^2} - u - v = \frac{\lambda}{2}$$

$$\Rightarrow \frac{u \cdot r^2}{2u^2} + \frac{v\,r^2}{2v^2} = \frac{\lambda}{2}$$

$$\Rightarrow \frac{1}{u} + \frac{1}{v} = \frac{2\lambda}{2r^2} = \frac{\lambda}{r^2} = \frac{1}{\left(\frac{r^2}{\lambda}\right)}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q 2)

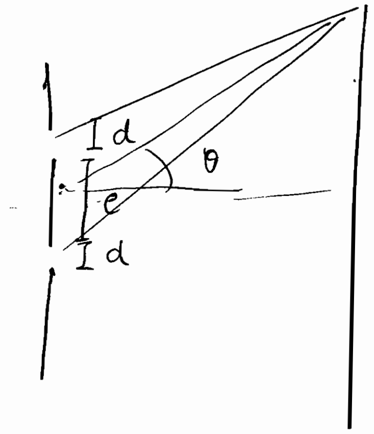
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

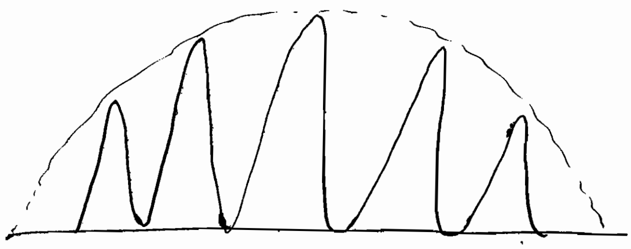
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For double slit fraunhaufer diffraction

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \cdot \cos^2\beta \qquad \alpha = \frac{\pi d \sin\theta}{\lambda}$$

$$\beta = \frac{\pi (e+d) \sin\theta}{\lambda}$$





For interference maxima, we have

$$\beta = n\pi$$

$$\frac{\cancel{\pi}(e+d)\sin\theta}{\lambda} = n\cancel{\pi}$$

$$\boxed{(e+d)\sin\theta = n\lambda}$$

For calculating fringe width (F.W.)

$$\frac{y_{n+1}}{D} = \frac{(n+1)\lambda}{(e+d)} \Rightarrow \frac{F.W.}{D} = \frac{\lambda}{e+d}$$

$$\frac{y_n}{D} = \frac{n\lambda}{e+d}$$

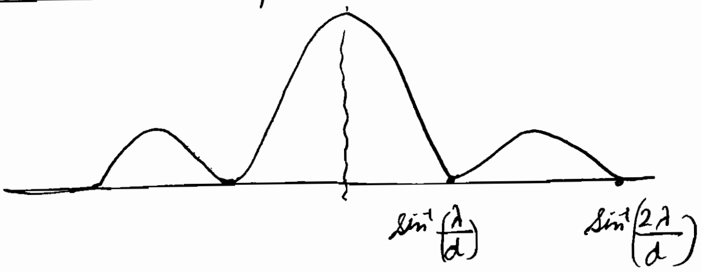
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\Rightarrow F.W = \left(\frac{\lambda D}{e+d}\right) = \frac{4800\,\text{Å} \times 50\text{ cm}}{0.1\text{ mm} + 0.02\text{ mm}}$$

$$= \underline{\underline{2\text{ mm}}}$$

For minima first, $\alpha = \pi \Rightarrow \theta = (\lambda/d) = 24\text{ mm} \Rightarrow y = \theta.D$
$= \underline{12\text{ mm}}$

Q3)



For diffraction pattern

$$I = \frac{I_0 \sin^2\alpha}{\alpha^2} \qquad \alpha = \frac{\pi d \sin\theta}{\lambda}$$

For 1^{st} minima $\alpha = \pi \Rightarrow \theta = \sin^{-1}\left(\frac{\lambda}{d}\right)$

2^{nd} minima $\Rightarrow \theta = \sin^{-1}\left(\frac{2\lambda}{d}\right)$

The position of 2^{nd} dark minima, y_2 is

$$y_2 = D.\sin^{-1}\left(\frac{2\lambda}{d}\right)$$

$$\Rightarrow 2.\sin^{-1}\left(\frac{2\lambda}{d}\right) = 1.6\times10^{-2}$$

$$\Rightarrow \frac{2\lambda}{d} \simeq 0.8\times10^{-2}$$

$$\Rightarrow \lambda = \frac{0.8\times10^{-2}\times 0.14\times10^{-3}}{2}$$

$$= \underline{\underline{5600\,\text{Å}}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q4)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For a 2-slit Fraunhofer diffraction,

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2} \cdot \cos^2 \beta$$

given : $d = 2\lambda$

$e + d = 6\lambda$

$$\alpha = \frac{\pi d \sin\theta}{\lambda}$$

$$\beta = \frac{\pi (e+d) \sin\theta}{\lambda}$$

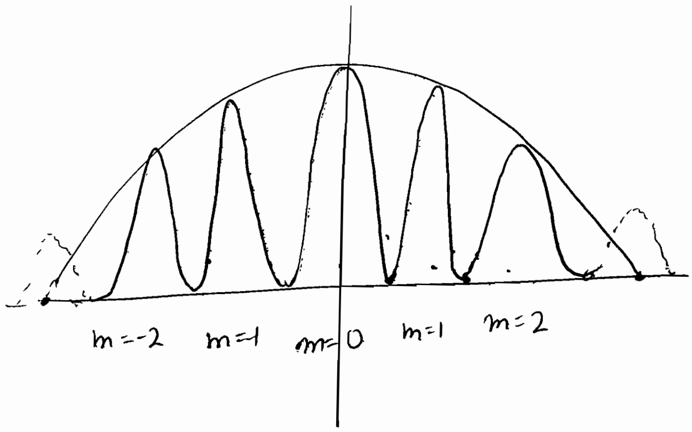
We have for missing order

$\cos^2\beta = 1$ and $\alpha = n\pi$

i.e. $\beta = m\pi$ and $\alpha = n\pi$

$$\Rightarrow \frac{\beta}{\alpha} = \frac{m}{n}$$

$$\Rightarrow \frac{e+d}{d} = \frac{m}{n} \Rightarrow \boxed{\frac{m}{n} = 3}$$



Q5)

lecture 12

Grating equation, for principal maxima of interference

$$\beta = n\pi \Rightarrow \cancel{\pi}(e+d)\sin\theta = n\cancel{\pi}\lambda$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$\Rightarrow (e+d)\sin\theta = n\lambda$

$\Rightarrow (e+d)\cos\theta \cdot d\theta = n \cdot d\lambda$

$\Rightarrow \boxed{\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}}$

Angular Dispersion or dispersive power

Prism Spectrum	Grating Spectrum
(1) Produces only 1 spectrum.	Gives a no. of spectrum on either side of zero order
(2) Spectrum is pure one.	This is impure at higher orders due to overlapping of spectral lines
(3) This will give a bright spectrum since all light is concentrated on 1 spectrum.	Most of the energy goes into zero order where no spectrum is formed. Rest of the energy is distributed among the spectra of various orders
(4) Dispersive power depends on λ. $\Rightarrow$ IRRATIONAL SPECTRUM	Dispersive power depends on order of spectrum n but independent of λ. $\Rightarrow$ RATIONAL SPECTRUM

Q6) lecture 12

Q7) ~~Separation~~ ~~between~~ Given $e = 4a$, $d = a$

For $\frac{e+d}{d} = \frac{m}{n} \Rightarrow \frac{5a}{a} = \frac{m}{n}$

$\Rightarrow m = 5n = \underline{5}$ $(n=1)$

Fringes $= 4+1+4 = 9$ fringes

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q8) lecture 14

कृपया इस स्थान में कुछ न लिखें।
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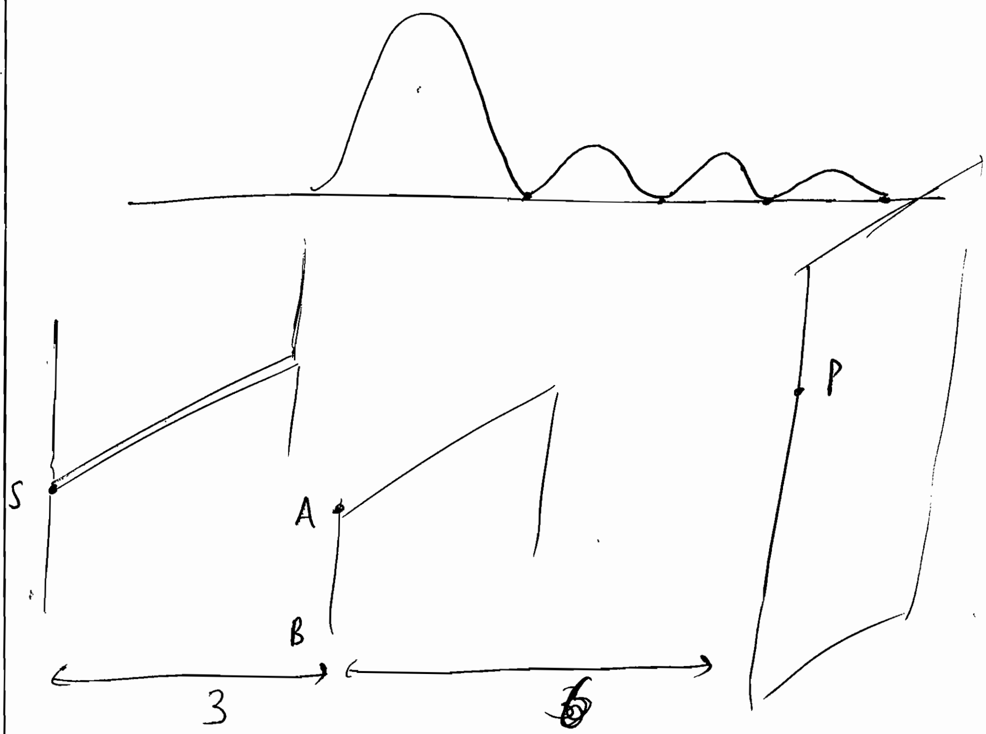
कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

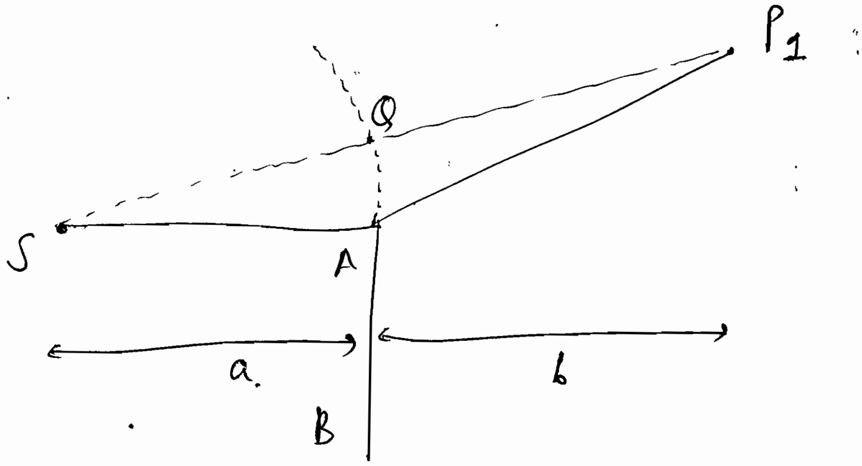
Q9) lecture 14

Q10) X

Q11) X (NO obtainment....) अरे भइया... दिमाग नहीं !!

Q12)





Point P will be dark when even no. of HPZ

$$AS + AP - PS = (2n)\frac{\lambda}{2}$$

$$\Rightarrow AP - QP = (2n)\frac{\lambda}{2}$$

remember this step

Do not write the condition for normal interference

~~For 1st dark band~~

~~$AS + AP_1 - SP_1 = \frac{\lambda}{2}$~~

~~For 4th dark band~~

~~$AS + AP_2 - SP_2 = \frac{7\lambda}{2}$~~

~~$P_1P_2 = ?$~~

$$AP^2 = b^2 + y_1^2 \Rightarrow AP_1 = \sqrt{b^2+y_1^2} = b\left(1 + \frac{y_1^2}{2b^2}\right)$$

$$= b + \frac{y^2}{2b}$$

$$QP_1 = SP - SQ = \sqrt{(a+b)^2 + y_1^2} - a$$

$$= (a+b)\left[1 + \frac{y_1^2}{2(a+b)^2} - a\right]$$

$$= b + \frac{y^2}{2(a+b)}$$

simple pythagoras + binomial

$$\Rightarrow (2n-1)\frac{\lambda}{2} = b + \frac{y_n^2}{2b} - b - \frac{y_n^2}{2(a+b)}$$

$$= \frac{y_n^2}{2}\left[\frac{1}{b} - \frac{1}{a+b}\right] = \frac{y_n^2\, a}{2b(a+b)}$$

$$\Rightarrow y_n = \sqrt{\frac{b(a+b)\lambda(2n-1)}{a}}$$

$$\text{Now } y_1 = \sqrt{\frac{b(a+b)\lambda}{a}}\sqrt{2}$$

$$y_4 = \sqrt{\frac{b(a+b)\lambda}{a}}\sqrt{8}$$

$$\Rightarrow \Delta y = \sqrt{\frac{6\times 9}{3}\times 6400\times 10^{-10}}\left(\sqrt{8}-\sqrt{2}\right)$$

$$= \boxed{4.8\ mm}$$

(Q13)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q14)

lecture 14

(Q1~~8~~)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Tut Sheet 11 — resolving power

(Q1) Resolving Power is primary of instruments with circular aperture

eg. Telescope
Microscope.

So if nothing is given assume circular aperture.....

For a circular aperture,

$$I = I_0 \left[\frac{J(\alpha)}{\alpha}\right]^2 \qquad \alpha = \frac{\pi D \sin\theta}{\lambda}$$

Since $J(\alpha)$ is a damping sine function, for the first ring, assuming $J(\alpha) \Leftrightarrow \sin\alpha$, we have

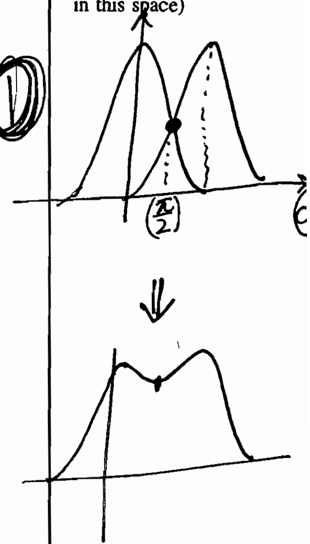
$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \checkmark$$

$$I_{max} = I_0 \qquad [\alpha = 0]$$

$$I_{middle} = I_0 \frac{\sin^2\left(\frac{\pi}{4}\right)}{\left(\frac{\pi}{4}\right)^2} = \frac{8 I_0}{\pi^2}$$

$$\Rightarrow \boxed{\frac{I_{middle}}{I_0} = \frac{8}{\pi^2}}$$

(I)



I_{middle}

$$= 2 I_0 \frac{\sin^2\left(\frac{\pi}{2}\right)}{\frac{\pi^2}{4}}$$

$$= \frac{8 I_0}{\pi^2}$$

$$= \frac{8}{\pi^2} I_{max}$$

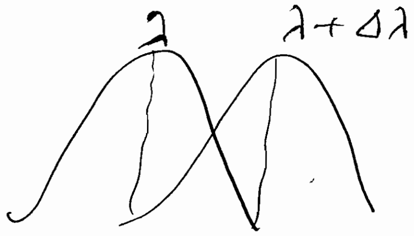
Q 2)

For diffraction grating, for principal maxima

$$\beta = n\pi$$

$$\frac{\cancel{\pi}(e+d)\sin\theta}{\lambda} = n\cancel{\pi}$$

$$\boxed{(e+d)\sin\theta = n\lambda} \quad \text{for maxima}$$



For just resolved,

$$\theta_{\text{Maxima}(\lambda+\Delta\lambda)} = \theta_{\text{Minima}\,\lambda}$$

$\Rightarrow$ $\cancel{(e+d)\sin}$

$$\frac{n(\lambda+\Delta\lambda)}{e+d} = \frac{n\lambda}{e+d} + \frac{\lambda}{N(e+d)}$$

$$\Rightarrow \frac{n\Delta\lambda}{e+d} = \frac{\lambda}{N(e+d)}$$

$$\Rightarrow \boxed{\frac{\lambda}{\Delta\lambda} = nN}$$

Now for $N = 3\times10^4$, $n = 2$, $\lambda = 6000\,\text{Å}$

$$\Delta\lambda = \frac{6000}{2\times3\times10^4} = \underline{\underline{0.1\,\text{Å}}}$$

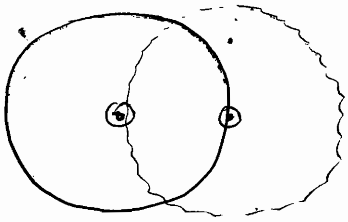
कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q3) Resolving Power : $\frac{\lambda}{\Delta\lambda} = Nn$

Dispersive Power = $\left(\frac{d\theta}{d\lambda}\right) = \frac{n}{(e+d)\cos\theta}$

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

Q4) For resolution in circular aperture,



1 sec = $\frac{1}{60 \times 60}$ degree

1 degree = $\frac{\pi}{180}$ radian

$\alpha = 1.22\pi$

$\Rightarrow \frac{\cancel{\pi} d \sin\theta}{\lambda} = 1.22 \cancel{\pi}$

$\Rightarrow \lambda = \left(\frac{d \sin\theta}{1.22}\right) = \frac{d\theta}{1.22} \Rightarrow d = \frac{1.22\lambda}{\theta}$

$d = \frac{1.22 \times 5500 \times 10^{-10}}{\frac{1}{60 \times 60} \times \frac{\pi}{180}}$

$= 2.41\,mm \times \frac{180}{\pi} = \boxed{13.8\ cm}$

do not forget to convert degree to radians.

Q5) For the grating, the grating equation is

$(e+d)\sin\theta = n\lambda$

$\Rightarrow \theta_1 - \theta_2 = n\frac{(\lambda_1 - \lambda_2)}{e+d}$

$= \frac{2 \times 6 \times 10^{-10} \times 6000}{1 \times 10^{-2}}$

$= \ldots = 0.0412\cancel{7}°$

$= 2.476$ minutes

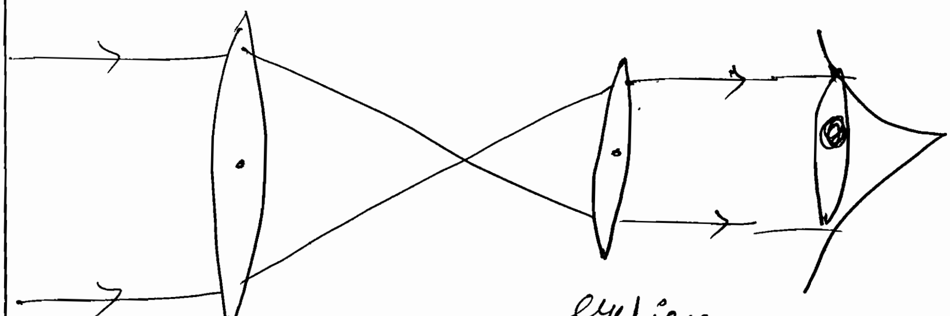
Not Reqd: 2.5 cm

Q6)

Q7)

lecture 12

Magnifying Power of Telescope



Magnification = $\left(\dfrac{\text{focal length of Objective}}{\text{focal length of eyepiece}}\right)$

From the above figure,

Magnification $= \dfrac{f_1}{f_2} = \left(\dfrac{d_1}{d_2}\right)$

$= \left(\dfrac{\text{Diameter of Objective}}{\text{Diameter of Eyepiece}}\right) = \left(\dfrac{D}{d}\right)$

Now diameter of eyepiece = diameter of eye

$d = d_e$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Also, limit of resolution of telescope

$$d\theta_T = \frac{1.22\lambda}{D}$$

Also, limit of resolution of eye

$$d\theta_e = \frac{1.22\lambda}{d_e}$$

$$\Rightarrow \frac{d\theta_e}{d\theta_T} = \frac{1/d_e}{D} \cdot \left(\frac{D}{d_e}\right) = M$$

$\Rightarrow$ Optimum Magnification $= (d\theta_e)\left(\frac{1}{d\theta_T}\right)$

= Product of resolving power of telescope and limit of resolution of eye.

Q8) For grating

$$(e+d)\sin\theta = n\lambda$$

$$\Rightarrow \theta_1 - \theta_2 = \frac{n(\lambda_1-\lambda_2)}{e+d}$$

$$= \frac{1 * 6\ \text{Å}}{1\ \text{cm}} \times 5000$$

= [illegible] = $0.017°$

use a telescope of angular magnification 10 or increase the no. of grating per cm to 50000.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q9)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

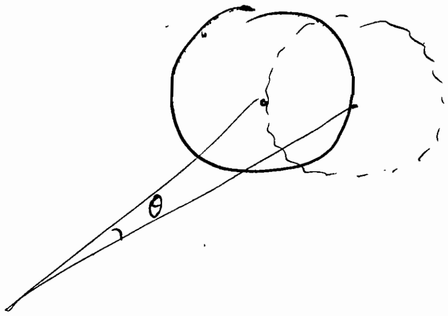
Resolving Power of Telescope $= \left(\frac{D}{1.22\lambda}\right)$

more the D, better the telescope.

Let the distance on the moon that could be resolved be d

$\Rightarrow \theta = \frac{d}{D_{EM}}$ where $D_{E,M} = 384000$ km

Now for the 2 images to be just resolved, using Rayleigh criteria



$\alpha = 1.22\pi$

$\Rightarrow \frac{\cancel{\pi} d_t \sin\theta}{\lambda} = 1.22\cancel{\pi}$

$\Rightarrow \frac{d_t}{\lambda} \cdot \frac{d}{D_{E,M}} = 1.22$

$\Rightarrow d = \frac{1.22 \times \lambda \times D_{E,M}}{d_t}$

$= \frac{1.22 \times 5890 \times 10^{-10} \times 3.84 \times 10^{8}}{5.08}$

$= \underline{\underline{54.31\text{ m}}}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q10) आई कितनी बार करूँ !!

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q11) Resolving Power of ~~an~~ an optical instrument represents its ability to produce distinctly separate spectral lines of light having two or more close wavelengths.

Rayleigh's Criterion of resolution states that two spectral lines of equal intensity are just resolved by an optical instrument when the principal maxima of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other.

$\left(\frac{d\theta}{d\lambda}\right)$ <u>Dispersive power</u> of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wavelength between the two spectral lines.

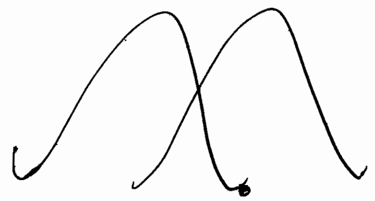
For grating,

$$(e+d)\sin\theta = n\lambda$$

$$\Rightarrow \cancel{(e+d).(\theta_2-\theta_1) = n(\lambda_2-\lambda_1)}$$

$$\Rightarrow \cancel{(e+d) = n/\lambda}.$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)



θ'

$\theta_{Maxima\,(\lambda+\Delta\lambda)} = \theta_{Minima\,\lambda}$

$$\frac{n(\lambda+\Delta\lambda)}{e+d} = \frac{n\lambda}{e+d} + \frac{\lambda}{N(e+d)}$$

$$\Rightarrow \frac{n\Delta\lambda}{\cancel{e+d}} = \frac{\lambda}{N\cancel{(e+d)}}.$$

$$\Rightarrow \boxed{\frac{\lambda}{\Delta\lambda} = Nn}$$

$$\Rightarrow \frac{500}{0.5} = N.2$$

$\Rightarrow N = 500$ lines

$\Rightarrow$ 500 = 250 lines per cm × x cm

$$\Rightarrow \boxed{x = 2\,cm}$$

(Q12) (a) lecture 12

(Q13) (b) Now for grating,

$$(e+d)\sin\theta = m\lambda$$

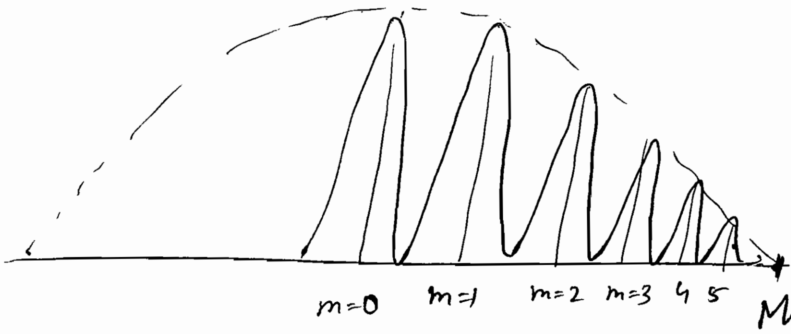
$$\Rightarrow (e+d)\sin 30^\circ = 2\lambda$$

$$\Rightarrow \frac{1\,cm}{4250} \times \frac{1}{2} = 2\lambda \Rightarrow \boxed{\lambda = 5882\,Å}$$

Q13)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$\frac{l+d}{d} = \frac{0.96}{0.16} = 6 \qquad m=6 \text{ : Missing Order}$$



कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें।
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कृपया इस स्थान में कुछ न लिखें।
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Tut Sheet 12 : Polarization

Q1) कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

From linearly polarised light changing to circularly polarized light,

$$\Delta\phi = 90° = \frac{\pi}{2}$$

$$\Rightarrow \Delta = \frac{\lambda}{2\pi} \times \frac{\pi}{2} = \left(\frac{\lambda}{4}\right)$$

We know $\Delta = (\mu_e \sim \mu_o)t$

$$\Rightarrow (\mu_e \sim \mu_o) = \frac{\lambda}{4t} = \frac{6 \times 10^{-7}}{4 \times 3 \times 10^{-5}}$$

$$= \frac{1}{2} \times 10^{-2}$$

$$= \underline{\underline{5 \times 10^{-3}}}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q2) Let us consider 2 linearly polarized light waves:

$$E_1 = E_0 \cos\phi_1 \sin(kz - \omega t)\hat{i} + E_0 \sin\phi_1 \sin(kz - \omega t)\hat{j}$$

$$E_2 = E_0 \cos\phi_2 \sin(kz - \omega t + \theta)\hat{i} + E_0 \sin\phi_2 \sin(kz - \omega t + \theta)\hat{j}$$

Now note that using two beams will be very confusing. Rather than take two linearly polarization beams in $\perp$ directions. Both are equivalent. as can be seen from addition of above 2 waves along x and y axis, we can have

$$E_3 \sin(kz - \omega t + \phi_3)\hat{i},\ E_4 \sin(kz - \omega t + \phi_4)\hat{j}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Let the 2 polarized beams be

$E_x = E_0 \sin(kz - \omega t)\ \hat{i}$

$E_y = E_0 \sin(kz - \omega t + \phi)\ \hat{j}$

NOW

$E_y = E_0 \sin(kz-\omega t)\cos\phi + E_0 \cos(kz-\omega t)\sin\phi$

$\Rightarrow E_y = E_x \cdot \cos\phi + E_0\sqrt{1 - \frac{E_x^2}{E_0^2}} \cdot \sin\phi$

$\Rightarrow (E_y - E_x\cos\phi)^2 = (E_0^2 - E_x^2)\sin^2\phi$

$\Rightarrow E_y^2 + E_x^2\cos^2\phi - 2E_xE_y\cos\phi$
$= E_0^2\sin^2\phi - E_x^2\sin^2\phi$

$\Rightarrow \boxed{E_y^2 + E_x^2 - 2E_xE_y\cos\phi = E_0^2\sin^2\phi}$

For $\phi = (2n-1)\frac{\pi}{2}$, we have

$E_y^2 + E_x^2 - 0 = E_0^2$

$\Rightarrow \boxed{E_y^2 + E_x^2 = E_0^2}$

$E_x = E_0 \sin(kz-\omega t)\ \hat{i}$

$E_y = E_0 \cos(kz-\omega t)\ \hat{j}$

this defines a circularly polarized wave

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

For even n, i.e $\phi = -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}$

We have

$$E_x = E_0 \sin(kz - \omega t)\, \hat{i} \qquad \boxed{LCP}$$
$$E_y = -E_0 \cos(kz - \omega t)$$

For odd n, i.e. $\phi = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

$$E_x = E_0 \sin(kz - \omega t) \qquad \boxed{RCP}$$
$$E_y = E_0 \cos(kz - \omega t)$$

Step 1 Pass the beam through a rotating polaroid. The intensity for both will remain same. ⇒ circular or unpolarized

Step 2 Take a QWP and pass the beam through it. Now perform step 1.

If intensity drops to 0 twice, ⇒ initially it was CPW

If intensity remains same, then initially it was unpolarized.

Q3) For QWP, $\phi = \frac{\pi}{2}$ i.e. $\Delta = \frac{\lambda}{2\pi} \cdot \frac{\pi}{2} = \frac{\lambda}{4}$

$$\Rightarrow (1.658 - 1.486) \cdot t = \frac{5890\,Å}{4}$$

$$\Rightarrow \boxed{t = 0.8561\,\mu m}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q4)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Optical Rotation

When a plane polarized passes through certain substances, the plane of polarization of light is rotated about the direction of propagation of light through a certain angle. This phenomenon is called OPTICAL ROTATION or ROTATORY POLARIZATION.

The substances which rotate the plane of polarization are called "Optically active" and the property is called "Optical Activity".

Fresnel theory of Optical Rotation

Its based upon the principle that dynamics of a linear vibration may be described as the resultant of two opposite circular motions of the same frequency.

The incident plane-polarized light on entering a substance is broken up into two circularly-polarized waves, one clockwise and the other anti-clockwise. In an optically active substance, the two waves travel with different velocities, (in leavorotatory, $v_{anticlockwise}$ is more, in dextrorotatory, $v_{clockwise}$ is more)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Hence a phase difference is developed between them as they transverse the substance.

On emergence, the two circular components, recombine to form plane polarized light by an angle depending upon the phase difference between them.

Let the vibration in incident light be represented by $y = a\cos\omega t$

These vibrations, upon entering the crystal, are broken down into two equal & opposite circular motions.

$$\left.\begin{aligned} x_1 &= \frac{a}{2}\sin(\omega t) \\ y_1 &= \frac{a}{2}\cos(\omega t) \end{aligned}\right\} \text{RCP (Clockwise)}$$

$$\left.\begin{aligned} x_2 &= -\frac{a}{2}\sin(\omega t) \\ y_2 &= \frac{a}{2}\cos(\omega t) \end{aligned}\right\} \text{RCP (anticlockwise)}$$

These circular components are propaged through the quartz crystal with different velocities. Therefore, when they emerge, they have a phase difference δ between them.

$$\left.\begin{aligned} x_1 &= \frac{a}{2}\sin(\omega t+\delta) \\ y_1 &= \frac{a}{2}\cos(\omega t+\delta) \end{aligned}\right\} \qquad \left.\begin{aligned} x_2 &= -\frac{a}{2}\sin(\omega t) \\ y_2 &= \frac{a}{2}\cos(\omega t) \end{aligned}\right\}$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

The resultant displacement along the two axis are :

$$x = \frac{a}{2}\left[\sin(\omega t+\delta) - \sin(\omega t)\right]$$

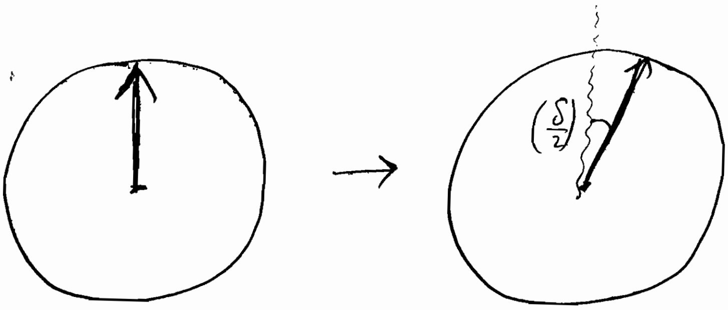
$$= \frac{a}{2}\left[2\sin\left(\frac{\delta}{2}\right)\cos\left(\omega t+\frac{\delta}{2}\right)\right]$$

$$= a\sin\left(\frac{\delta}{2}\right)\cos\left(\omega t+\frac{\delta}{2}\right)$$

$$y = \frac{a}{2}\left[\cos(\omega t+\delta) + \cos\omega t\right]$$

$$= \frac{a}{2}\left[2\cos\left(\omega t+\frac{\delta}{2}\right)\cos\left(\frac{\delta}{2}\right)\right]$$

$$= a\cos\left(\frac{\delta}{2}\right)\cdot\cos\left(\omega t+\frac{\delta}{2}\right)$$



$\Rightarrow$ rotation $\theta = \left(\frac{\delta}{2}\right)$

$$\theta = \frac{\delta}{2} = \frac{(\mu_A - \mu_B)\,t}{2\lambda}\cdot 2\pi = \frac{\pi t}{\lambda}(\mu_A - \mu_B)$$

$$\boxed{\theta \propto \frac{1}{\lambda}}$$

$$\theta = \frac{\pi \times 10^{-3}}{4500\times10^{-10}} \times 12\times10^{-5} = 48^\circ$$

Remember to convert radians to degrees

RCP & LCP

Q4)

~~different velocity~~

Chirality

different velocities of ⇒ dextro rotatory ~~Or~~ leavo rotatory motion are produced.

$$\frac{(\mu_o \sim \mu_e)\, t}{\lambda} \cdot 2\pi = \frac{\pi}{2}$$

$$\Rightarrow (\mu_o \sim \mu_e) = \frac{\lambda}{4t} = \frac{589 \times 10^{-5}}{4 * 2.945 \times 10^{-5}}$$

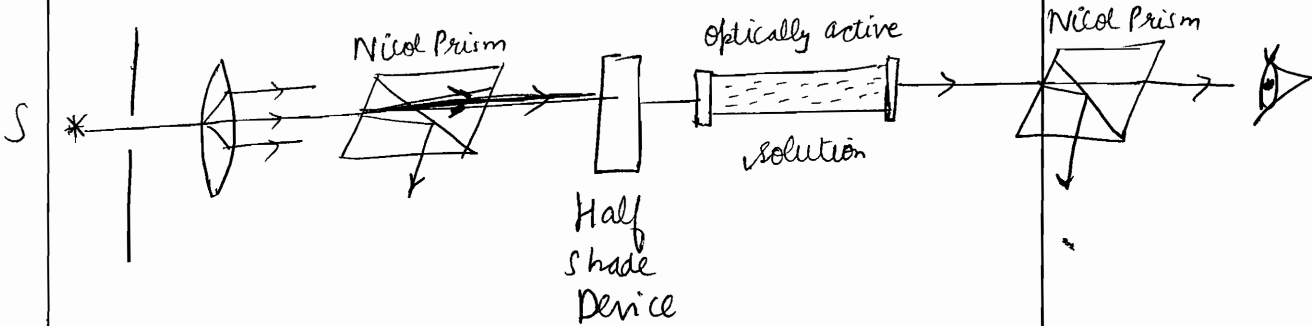
$$= \underline{\underline{0.005}}$$

Q5)

a) Q4

b) $\boxed{\text{Half Shade Device}}$

Its an instrument to measure the optical rotation of certain substances.



In the absence of Half Shade Device, in order to measure the angle θ of optical rotation, the precise measurement of complete darkness has to be observed. but eye cannot judge position of complete darkness accurately and field appears dark over a considerable range of rotation.

Half shade devices overcomes this

कृपया इस स्थान में कुछ न लिखें।
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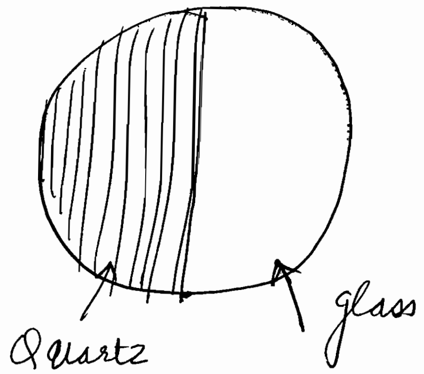
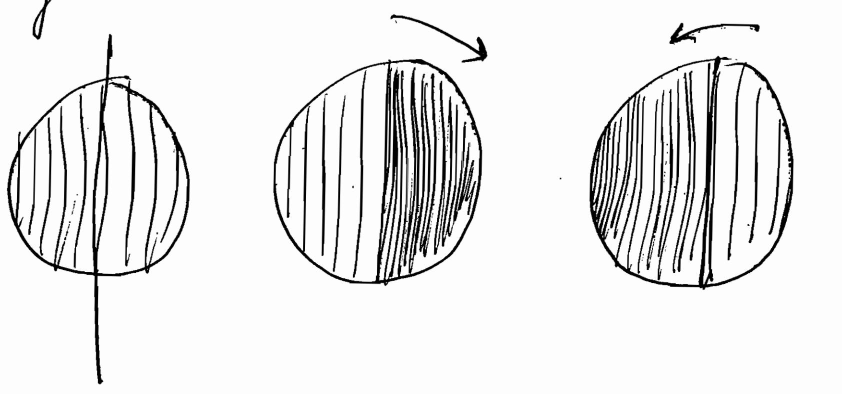
problem



Half shade device is combination of 2 semi circular plates, one of glass and other of quartz cut along the optic axis. Thickness of plate is such that it introduces a path difference of $\left(\frac{\lambda}{2}\right)$. (Note that Half shade device is constructed for a particular λ)

Half wave

Due to such a structure, slight rotation left or right of the total darkness angle will lead to half illumination, and therefore the rotation angle could be accurately measured.



(Q7) $\Delta\phi = \frac{2\pi}{\lambda} \cdot \Delta = \frac{2\pi}{\lambda} \cdot t(\mu_o \sim \mu_e)$

$= \frac{2\pi}{6000\times10^{-10}} \times 2\cdot3\times10^{-5} \times 9\cdot4\times10^{-3}$

$= \underline{\underline{0\cdot72\pi}}$

Elliptically Polarized Beam will come out with $\Delta\phi = \frac{\pi}{2} + 0\cdot72\pi$

$= \underline{\underline{1\cdot2\pi}}$

(Q8) Numerical

For QWP : $\Delta\phi = (2n-1)\frac{\pi}{2}$

$\Rightarrow \Delta = \frac{\lambda}{2\cancel{\pi}} \cdot (2n-1)\frac{\cancel{\pi}}{2} = (2n-1)\left(\frac{\lambda}{4}\right)$

Also $(\mu_o - \mu_e)t = (2n-1) \cdot \frac{\lambda}{4}$

$\Rightarrow t = \frac{(2n-1)}{(\mu_o \sim \mu_e)} \cdot \frac{\lambda}{4}$

$= \frac{5472\times10^{-10}(2n-1)}{4\times0\cdot171}$

$= 0\cdot8\ \mu m\ (2n-1)$

$\boxed{n=20}$ $t = 0\cdot8(39)\ \mu m$

$= \underline{\underline{31\cdot2\ \mu m}}$

Q9)

QWP

$$(1.5533 - 1.5443)\, 0.1436\times 10^{-3} = \frac{\lambda}{4}(2n-1)$$

$$\frac{51696}{2n-1}\,\text{Å} = \lambda$$

For λ should lie between 4000 Å & 7500 Å

Odd multiples of $\left(\frac{\lambda}{4}\right)$

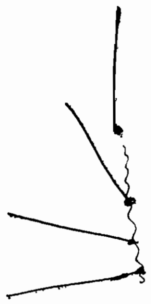
$\lambda = 7385.14$ Å $n = 4$

$\lambda = 5744$ Å $n = 5$

$\lambda = 4699$ Å $n = 6$

$\lambda = 3976.6$ Å $n = 7$

Q10)



$$I = I_1(\cos^2 30^\circ)^3$$

$$= I_1 \cdot \left(\frac{\sqrt{3}}{2}\right)^6$$

$$= I_1\left(\frac{3\sqrt{3}}{8}\right)^2 = 0.42\, I_1$$

Crossed ⊥

Now $I_1 = I_0 \langle \cos^2\theta \rangle$ (∵ initial light is unpolarized)

do not miss this step ⇒ $= \left(\frac{I_0}{2}\right)$

$\Rightarrow I = I_0 * 0.21$

around 21% of intensity is transmitted ....

कृपया इस स्थान में कुछ न लिखें।
(Please don't write anything in this space)

(Q11) After Polarizer, intensity $= \left(\frac{I_0}{2}\right)$

After analyzer, intensity $= \frac{I_0}{2}\cos^2\theta$

Now we want,

$$\frac{I_0}{2}\cos^2\theta = \frac{I_0}{4}$$

$$\Rightarrow \boxed{\theta = 45^\circ}$$

(Q12) If we take 2 polarizers and put them in series with their optical axis $\perp$ to each other, we get 0 output. This proves that light is transverse.

(Q13) अरे भाई μ तो दे दो

(Q14) Calcite : $\ominus$ve Quartz : $\oplus$ve

(Q15) Chiral Molecule has non-superimposable mirror image. Optical Activity is rotational of linearly polarized light as it travels through a solution of chiral molecules. If the solution is composed of molecules of opposite chirality (called enantiomers), the rotation in opposite direction takes place.

Also rotation is proportional to concentration of solution i.e. more the molecules more the rotation !!

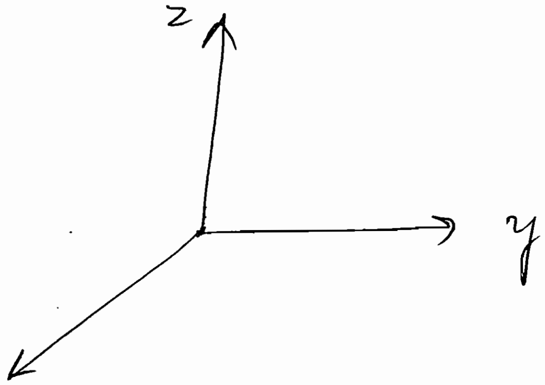
$$\boxed{\theta = \theta_c\, c\, l}$$

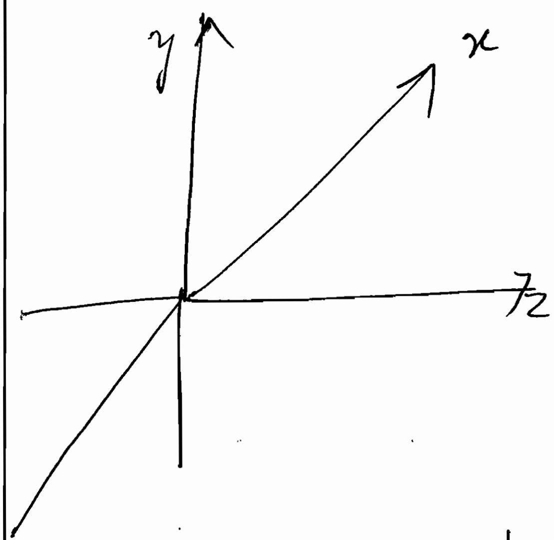
(Q16) $\theta = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}(1.33) = \underline{53^\circ}$ with vertical

<Q2> With horizon, $\angle = 36.9^\circ$

$= \underline{36^\circ 56'}$

(Q17)





For circulator polarization,

1st condition $\underline{a = b = A \text{ (say)}}$ <derive the trajectory equation> & then conclude !!

$\Rightarrow \quad E_y = A\cos(kx - \omega t)\,\hat{y}$

$E_z = A\cos(kx - \omega t + \phi)\,\hat{z}$

Take $\phi = \left(\frac{\pi}{2}\right)$

$\Rightarrow \quad E_y = A\cos(kx - \omega t)$

$E_z = -A\sin(kx - \omega t)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

This is RCP i.e. Clockwise from Point of view of Observer travelling with the wave.

Take $\phi = -\frac{\pi}{2}$

$$\Rightarrow \quad E_y = A\cos(kx - \omega t)\ \hat{y}$$
$$E_z = A\ \cancel{\sin}\ (kx - \omega t)\ \hat{z}$$

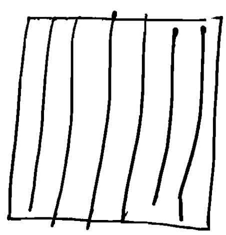
This is LCP i.e. anticlockwise from Point of view of observer goes along the wave.

Q18) (i) lecture 16

Quartz (+)ve crystal

(ii) For Quartz

$$\boxed{\begin{aligned}\mu_e &= 1.554 \\ \mu_o &= 1.544\end{aligned}}$$



remember this !!

Now $\Delta = (1.554 - 1.544)\, t$

$$\phi = \frac{2\pi}{\lambda}\Delta = \frac{\pi}{3}$$

$$\Rightarrow \Delta = \left(\frac{\lambda}{6}\right)$$

$$\Rightarrow 0.010\ t = \frac{\lambda}{6}$$

$$\Rightarrow \boxed{t = \frac{100\lambda}{6}}$$

कृपया इस स्थान में कुछ न लिखें (Please don't write anything in this space)

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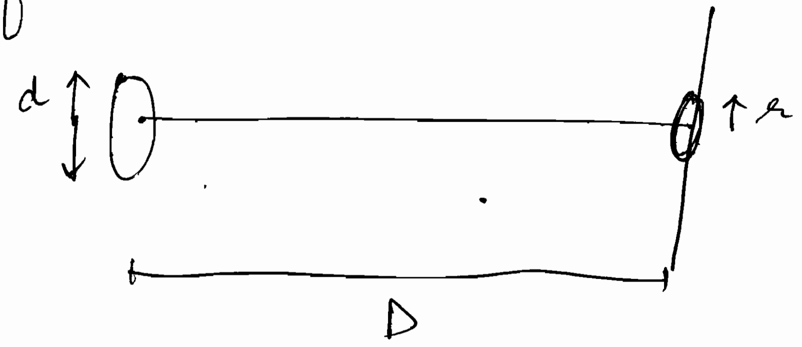
Tut 13 · LASERS

(Q1) Laser is diffraction limited, due to diffraction on airy pattern is formed.



For large distance

Put $\alpha = 2$

$\frac{\pi d \sin\theta}{\lambda} = 2$

For the first ring of airy pattern

$\alpha = 1.22\pi$ where $\alpha = \frac{\pi d \sin\theta}{\lambda}$

$\Rightarrow \frac{d \sin\theta}{\lambda} = 1.22$

$\Rightarrow \frac{d}{\lambda} \cdot \frac{r}{D} = 1.22$

$\Rightarrow r = \frac{1.22 \lambda D}{d}$

$\theta = \left(\frac{\lambda}{\pi a_0}\right)$

$\Rightarrow$ diameter $= 2.44 \times \frac{\lambda D}{d}$

$\frac{r_f}{d_{se}} = \left(\frac{\lambda}{\pi a_0}\right)$

$r_f = \left(d_{se} \frac{\lambda}{\pi a_0}\right)$

$= \frac{2.44 \times 6943 \times 10^{-10} \times 1000 \times 10^3}{10^{-2}}$

$\boxed{44.2\ m}$

$= \underline{\underline{169.4\ m}}$ घंटा intensity आएगी इतने बड़े area में !!

(Q2) lecture 17

Let a consider a time of $1 s$

Energy emitted $= 10\,mJ = \underline{10^{-2}\,J}$

कृपया इस स्थान में कुछ न लिखें (Please don't write anything in this space)

For focussing of laser via a lens: we have [NOT CIRCULAR APERTURE]

radius of focussed spot $= \left(\dfrac{\lambda_0 f}{r_{lens}}\right) = r_0$

$\boxed{\text{Intensity} = P/\pi r_0^2}$

$\boxed{\alpha = 2\pi}$

$\dfrac{\cancel{\pi} d \sin\theta}{\lambda} = 2\cancel{\pi}$

$\dfrac{d \cdot r_0}{f\lambda} = 2$

$r_0 = \dfrac{2f\lambda}{d}$ (3)

$= \dfrac{f\lambda}{r_{lens}}$

No. of pulses emitted = 500

Energy of 1 pulse $= \dfrac{10^{-2}}{500}$ J

Power of 1 pulse $= \dfrac{10^{-2}}{500 \times 3} \times 10^9$

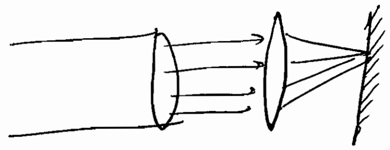
$= \underline{6666.6 \text{ W}}$

$\lambda = 6000$ Å

$d_{laser} = 1$ cm

$P_{laser} = 0.2$ W

$r_0 = \dfrac{\lambda_0 f}{r} = \dfrac{6000 \times 10^{-10} \times 1}{\frac{1}{200} \times 10}$



$f = 10$ cm

~~Note that the diameter of the Airy pattern will be more dependent upon where the image is formed.~~

For diffraction

$\alpha = 1.22\pi$

$\dfrac{\cancel{\pi} d \cdot \sin\theta}{\lambda} = 1.22\cancel{\pi}$

$\boxed{\sin\theta = 1.22 \dfrac{\lambda}{d}}$

It is not Airy disk

$\dfrac{r}{f} = \dfrac{1.22\lambda}{d}$ $r_0 = \boxed{1.2 \times 10^{-5}}$

$r = \dfrac{1.22\lambda f}{d} = \underline{7.32 \times 10^{-6} \text{ m}}$

$A = \pi r^2 = 1.68 \times 10^{-10} \text{ m}^2$ $I = \dfrac{P}{\pi r_0^2} =$

$I = \dfrac{0.84 P}{A} = \underline{9.98 \times 10^8 \text{ W/m}^2}$ 4.42×10^8

Q4) **lecture 17**

$$P = \frac{3J}{1 \times 10^{-5}} = \underline{\underline{3 \times 10^{9} W}}$$

Q5) Ruby laser

Q6) Now we are talking !!

P (stimulated emission) $= B_{21}\, u(\nu)$

Rate of stimulated emission $= N_2\, B_{21}\, u(\nu)$

P (spontaneous emission) $= A_{21}$

Rate of spontaneous emission $= A_{21} . N_2$

Now $\dfrac{P(St.E.)}{P(Sp.E)} = \left[\dfrac{B_{21}\, u(\nu)}{A_{21}}\right]$

Prob. Vs Rate

Now let Probability of stimulate absorption $P_{(St.A.)} = B_{12} . u(\nu)$

Rate of stimulated absorption $= B_{12}\, u(\nu)\, N_1$

Now at equilibrium

rate of absorption = rate of emission

$\Rightarrow B_{12} N_1 u(\nu) = B_{21} N_2 u(\nu) + A_{21} N_2$

$$\Rightarrow u(\nu) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$= \frac{\left(\frac{A_{21}}{B_{21}}\right)}{\left(\frac{B_{12}}{B_{21}}\right) e^{\frac{h\nu}{kT}} - 1} \quad \text{——— (1)}$$

$$\frac{8\pi p^2 dp}{h^3}$$

$$p = \frac{h\nu}{c}$$

$$\frac{8\pi}{h^3} \cdot \frac{h^2\nu^2}{c^2} \cdot \frac{h\,d\nu}{c}$$

$$h\,\frac{8\pi\nu^3 d\nu}{c^3\left(e^{\frac{h\nu}{kT}}-1\right)}$$

Now we also know,

$$u(\nu) = \frac{8\pi h}{c^3}\nu^3 \cdot \frac{1}{\left(e^{\frac{h\nu}{kT}}-1\right)} \quad \text{—(2)}$$

Comparing (1) and (2), we get

$$\frac{B_{12}}{B_{21}} = 1 \Rightarrow \boxed{B_{12} = B_{21}}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

$$\frac{P(st.e)}{P(sp.e)} = \frac{B_{21}\,u(\nu)}{A_{21}}$$

$$= \frac{u(\nu)}{\frac{8\pi h\nu^3}{c^3}} = \left[\frac{1}{e^{\frac{h\nu}{kT}}-1}\right]$$

For $\boxed{\begin{array}{l}\lambda = 6000\,\text{Å} \\ T = 10^3\,\text{K}\end{array}}$

$$\frac{h\nu}{kT} = \left(\frac{hc}{\lambda kT}\right) = 25$$

$$\Rightarrow \frac{P(st.e)}{P(sp.e)} = \frac{1}{e^{25}} \approx 10^{-10}$$

Hence Probability of stimulated emission is negligible.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q7)

E : Bohr

$\Rightarrow \omega$: ✓

$\Rightarrow \nu$: ✓

$\Rightarrow u(\nu)$: ✓

Prob : $B_{21}\, u$

Rate : $B_{21}\, u\, N_2$

A_{21} : given

$\frac{B_{21}}{A_{21}}$: known

$\Rightarrow B_{21}$: ✓

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

 $Area = \pi\left(\frac{\lambda f}{r_{lens}}\right)^2$ Intensity $= \left(\frac{P}{Area}\right)$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q8) We know that laser is diffraction limit

$\alpha = 1.22\pi$ (for central zone of airy pattern or Airy disk)

$$\frac{\cancel{\pi} d\sin\theta}{\lambda} = 1.22\cancel{\pi}$$

$$\sin\theta = \left(\frac{1.22\lambda}{d}\right)$$

$$\frac{r}{f} = \frac{1.22\lambda}{d}$$

$$r = \frac{f\,1.22\lambda}{d} \Rightarrow Area = \pi\left(\frac{1.22 f\lambda}{d}\right)^2 = \underline{1.168\times10^{-10}\,m^2}$$

$$\boxed{I = \frac{0.84\,P}{Area}} = \frac{0.84 * 3\times10^6}{\pi\left(\frac{1.22 f\lambda}{d}\right)^2}$$

$$= \frac{0.84\times3\times10^6\times10^{-4}}{\pi(1.22)^2\times(5\times10^{-2})^2\times(10000\times10^{-10})^2}$$

$$= \underline{2.156\times10^{16}}\ W/m^2$$

(Q9) ✓ $N_1 = N_0 e^{-\frac{\Delta E}{kT}}$ $N_1 > N_0 \Rightarrow \frac{\Delta E}{kT} < 0 \Rightarrow \boxed{T<0}$

(Q10) $d \simeq \lambda$

$\alpha = 1.22\pi \Rightarrow d\sin\theta = 1.22\lambda$

$\Rightarrow \sin\theta = 1.22(\lambda/d)$

$\Rightarrow \frac{r}{f} = 1.22\frac{\lambda}{d} \Rightarrow r = 1.22 f\frac{\lambda}{d} \simeq \underline{1.22\lambda}$

$Area = \pi(1.22\times6328\times10^{-10})^2 = \underline{1.87\times10^{-12}}\ m^2$

* Focussing of lens is not Airy disk, so do not use 84%. To calculate r_0, use

$\alpha = 1.22\pi$ Or $r_0 = \lambda f/r_{lens}$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q 11)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

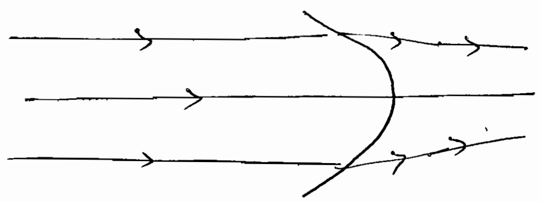
Q 12)

lecture 17

Self Focussing of Laser Beam

A non-linear optical phenomenon due to the variation of refractive index due to high intensity of laser as a result of which medium acts a converging lens. The beam is said to undergo self focussing.

Now $n^2 = n_0^2\left[1 - \left(\frac{r}{\alpha}\right)^2\right]$



The medium acts as converging lens of focal length $= \frac{\pi}{2}\alpha$

Therefore, the spreading of the beam due to diffraction will be balanced by self focussing, due to high directionality of LASERs can be explained.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

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Q13)
Q14)
Q15)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Q16)

lecture 17

For rate of stimulated emission to be equal to rate of spontaneous emission

$$B_{21}\, u(\nu)\, \cancel{N_2} = A_{21} . \cancel{N_2}$$

$$\Rightarrow u(\nu) = \frac{A_{21}}{B_{21}}$$

$$\Rightarrow e^{\frac{h\nu}{kT}} = 2$$

$$\Rightarrow \frac{hc}{\lambda kT} = \ln 2$$

$$\Rightarrow T = \frac{hc}{\ln 2\, \lambda k} = 41.28 \times 10^3\, K$$

(ii) copy

(iii) Fiber attenuation $= 10 \log_{10}\left(\frac{P_{in}}{P_{out}}\right)$ dB

$= \frac{10}{100} \log_{10} \cdot \left(\frac{1}{0.3}\right)$ per unit length

$= 0.052$ dB/m

Q17) $\frac{1}{2} \varepsilon_0 E_0^2 c$ = Intensity = $\frac{\text{Power}}{\pi r_0^2}$

$$\Rightarrow E_0 = \sqrt{\frac{2\,\text{Power}}{\varepsilon_0 c \pi r_0^2}}$$

$r_0 = \left(\frac{\lambda f}{a}\right)$

$k = \left(\frac{2\pi}{\lambda}\right)$ $\omega = \left(\frac{2\pi c}{\lambda}\right)$

$E_0 = E_0 \sin(kz - \omega t)$

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Tut Sheet : 14 Special Topics

(Q 1) lecture 18 (Q5) (Q 7) (Q 8)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

(Q6) Not in Syllabus

(Q 2)

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Coherence Length

A wave which appears to be a pure sine wave for an infinitely large period of time, or in an infinitely extended space, is said to be a perfectly coherent wave. In such a wave, there is a definite relationship between the phase of the wave at a give time and at a certain time later, or at a given point and at a certain distance away. No actual light source, however, emits a perfectly coherent wave. Light waves that are coherent only for a limited of[redacted] time or for a limited [redacted] space are called "partially coherent waves".

The average time-interval for which the field remains sinusoidal is known as "coherence time" or "temporal coherence". Its denoted by τ.

The distance L for which field is sinusoidal is called coherence length

$$L = \tau c$$

(*) For Young slit : Spatial Coherence is reqd.
For Michelson : temporal coherence is reqd.

In interference experiments, distance after which pattern disappear is called coherence length.

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

$$L = \frac{\lambda^2}{\Delta\lambda}$$

$$= \frac{623.8 \times 623.8 \times 10^{-18}}{2 \times 10^{-12}}$$

$$= \underline{0.194\, m}$$

Proof For disappearance of pattern,

we have : $\dfrac{2d}{\lambda} - \dfrac{2d}{\lambda + \frac{\Delta\lambda}{2}} = \dfrac{1}{2}$

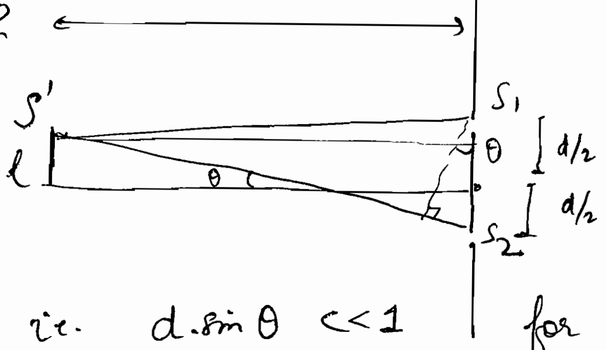
$$\frac{2d\Delta\lambda}{2(\lambda)(\lambda + \frac{\Delta\lambda}{2})} = \frac{1}{2}$$

$$\frac{2d\Delta\lambda}{\lambda^2} = 1 \Rightarrow \boxed{2d = \frac{\lambda^2}{\Delta\lambda}}$$

(Q3) Q2

(Q4) (a) Q2

(b)



$\Delta << 1$ i.e. $d.\sin\theta << 1$ for good coherence

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कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

Therefore for an extended incoherent source, interference fringes of good contrast will be formed when

$$l << \frac{\lambda a}{d}$$

P. 17.5

$$S'S_2 = \left[a^2 + \left(\frac{d}{2} + l\right)^2\right]$$

$$S'S_1 = \left[a^2 + \left(\frac{d}{2} - l\right)^2\right]$$

$$\Rightarrow S'S_2 - S'S_1 \approx \frac{1}{2a} \cdot 2l \cdot d = \left(\frac{dl}{a}\right)$$

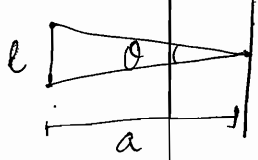
For fringes to disappear,

$$S'S_2 - S'S_1 = (\lambda/2) \Rightarrow l = \left(\frac{\lambda a}{2d}\right)$$

$\therefore$ If we have a linear source of dimension $\frac{\lambda a}{d}$, for every point on the source, there exists a point at distance $\left(\frac{\lambda a}{2d}\right)$ which produces fringes shifted by λ.

$\therefore$ for good pattern $l << \frac{\lambda a}{d}$

Now consider,



Now $\theta \approx \left(\frac{l}{a}\right)$

$$\Rightarrow \boxed{d << \left(\frac{\lambda}{\theta}\right)}$$

PTO

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)

थोड़ी आसान भाषा में कहें तो,

The path difference between the rays reaching the slits, should be much less than λ

$$\Delta x << \lambda$$

similar to Δx being $d \sin\theta$ in Young double slit, here Δx is $l \sin\theta$

$$\Rightarrow l \sin\theta << \lambda$$

$$\Rightarrow \frac{dl}{a} << \frac{\lambda}{\theta}$$

$$\Rightarrow l << \frac{\lambda a}{d}$$

$$\theta = \left(\frac{l}{a}\right)$$

कृपया इस स्थान में कुछ न लिखें। (Please don't write anything in this space)