Paper-1 - PYQ's [2013-22] (selective) ①

[2020 Q2(b) - Torsion]

DSM pg 729:

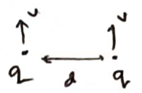
$W = \tau \times \theta$ ~ total angle revolution

$\tau = \frac{8000}{5\,\text{rev/sec}} = \frac{8000}{5 \times 2\pi}$

Now $\phi = \frac{TL}{GJ}$ — Torque; $J \to \pi R^4/2$; G — modulus of rigidity

relative shift b/w ends $= L \times \phi$

[2022] via NGP Lec 3



$F = q\vec{v} \times B$

(note: $E \propto \frac{1}{r^2}$ only for static, coulomb law not for moving charges)

we can't use $\frac{\mu_0 i}{2\pi r}$ ~ this is for ∞ wire.

here not ∞. Thus use Biot-Savart law

Now $B = \frac{\mu_0}{4\pi} \frac{q\vec{v}}{d^2}$

$F_{21} = qvB = -\frac{\mu_0 q^2 v^2}{4\pi d^2} \hat{i}$

[2022] string EMF

$\mathcal{E} = Blv$

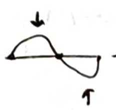
$y = 2a\sin kx \cos\omega t$

$v = \dot{y} = 2a \sin kx \sin\omega t$

$a \sim 3$ mm (max amplitude)

Now integral $\int d\mathcal{E} = \int Bv\,dx$

For 2nd overtone:



~ Thus $\mathcal{E} = 0$

since —|⊢|— center

$\mathcal{E} = 0$ also verified by outthink YT channel.

[2022 Q3(a)]

$V = \frac{1}{3}(\vec{a} \times \vec{b}) \cdot c$ ~ $V_{cone} = \frac{1}{3}\pi r^2 h$

$V_{pyramid} = \frac{1}{3}$(Area of Base)×(height)

(But soln of 3(a) is long & impossible !!)

[2022 Q7)

chegg ✓ verified my soln

$(H_1 - H_2) \times \hat{a}_n = \vec{k}$ ~ simple to get from $\oint B \cdot dl = \mu_0 I$ ✓ $(I_3 - I_2)$

⊗ current sheet flowing inwards

use right hand thumb rule for dir^n of $\vec{H}$ & $\vec{B}$

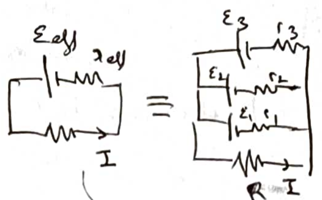
Now $\oint H \cdot dl = I_{enc} \to (H_1 - H_2) l = kl \to H_1 - H_2 = k$

also $\nabla \cdot B = 0 \to (B_1)_n = (B_2)_n \sim \mu_{r_1}\mu_0 H_1 = \mu_{r_2}\mu_0 H_2$

simple soln: $H_1 = 5.5\hat{a}_x + 6\hat{a}_y$ A/m

[2022 Q4)

proof of $\frac{\mathcal{E}_{eff}}{r_{eff}} = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} + \frac{\mathcal{E}_3}{r_3} \dots$ ~ $\mathcal{E}_{eff} = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}$



V ~ potential across R $(V = IR)$

$V = \mathcal{E}_3 - i_3 r_3 = \mathcal{E}_2 - i_2 r_2 = \mathcal{E}_1 - i_1 r_1 \dots$

$i_1 = \frac{\mathcal{E}_3 - V}{r_3}, \quad i_2 = \frac{\mathcal{E}_2 - V}{r_2} \dots$

$I = \frac{V}{R} = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} + \frac{\mathcal{E}_3}{r_3} \dots - V\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \dots\right)$

$V = \mathcal{E}_{eff} - I r_{eff} \to I = \frac{\mathcal{E}_{eff}}{r_{eff}} - \frac{V}{r_{eff}}$ — we compare both eq^n

[2022) Q6

Soln verified by chegg. use long derivation to get α & β.

take $k = \alpha + i\beta \to \nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0$

Both are -ve sign ★

$\nabla E \sim j\vec{k}, \quad \frac{\partial E}{\partial t} \to -j\omega$

Thus $-k^2 - \mu\sigma(-j\omega) - \mu\varepsilon(-\omega^2) = 0$

Thus $k^2 = \mu\varepsilon\omega^2 + j(\mu\sigma\omega)$

$= (\alpha^2 - \beta^2) + 2i\alpha\beta$

we solve for α & β ... using Quadratic

Note for both α & β use formula → dont use $\beta = \frac{\omega}{v}$ + Bp (although answer is almost same only)

Now $v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$ and $H = \frac{k \times E}{\omega\mu}$

$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$ <Note in all places $\mu = \mu_0\mu_r$>

η - intrinsic impedence: $\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| e^{i\theta_n}$ → $\boxed{|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}}$ ★ same as $\eta = \sqrt{\mu/\varepsilon}$

and $\tan 2\theta_n = \frac{\sigma}{\omega\varepsilon}$

Thermo (2022) PYQ – Q7

(Q1) simple – $dP = \rho g\, dh$

and $P = \rho M / RT$

$\rho = \rho_0 e^{-\frac{mgh}{RT}} \Rightarrow \rho = \rho_0 e^{-\frac{mgh}{RT}}$

(Q2) T_f remains fixed since reservoir is large. ~ Thus Final Temp is T_f for Body. $(T_i \to T_f)$

$\Delta S = C_p \ln \frac{T_f}{T_i} + \frac{Q}{T_f}$

Now $Q = C_p(T_f - T_i)$ ~ Total heat taken by Body = heat given by reservoir

Thus $\Delta S = C_p \ln \frac{T_f}{T_i} - \frac{C_p(T_f - T_i)}{T_f}$

since opposite value of Q!

~~let $\frac{T_f}{T_i}$ ...~~

~~let $\frac{T_f - T_i}{T_f} = x$ ... $\Delta S = C_p[\ln x ...]$~~

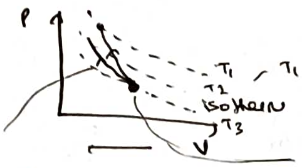
~~...~~

~~let $\frac{T_f - T_i}{T_f}$ ~ $\frac{T_f - T_i}{T_i}$ ... $\Delta S = C_p[...]$~~

~~assume $x = \frac{T_f}{T_i} - 1$ ... Now $|x| < 1$~~

(Q4) simple: use adiabat more steep ~ Thus more temp rise – overheat

$\gamma \uparrow$ ~ slope $\uparrow$



$T_1 > T_2 > T_3$

compress means volume reduced ✗

starting P & V are same as given

(Q5) easy ✗ ~ given in Q7 DPP soln (easy)

[2020 Q2]

(easy Q4 soln in DPP easy)

[2019 Q4]

DPP soln – (1) & (2) : POF = 1

(3) : POF = 2

(4) : POF = 0

GPT: restrictions for –ve Temp to be meaningful:

(1) Two-state or limited energy levels

(2) Bounded energy levels (i.e. there must be a max energy cutoff)

(3) population inversion

(4) limited particle count (6) non – equilibrium conditions

(5) short time scales

Thus $\Delta S = C_p\left[\ln\frac{T_f}{T_i} - \left(\frac{T_f - T_i}{T_f}\right)\right]$. Now given $\left|\frac{T_f - T_i}{T_f}\right| < 1$

(1) assume $T_f > T_i$ ~ ~~...~~ ~ let $x = \frac{T_f}{T_i}$ ~ Thus $x > 1$

Now $\Delta S = C_p\left[\ln x - \left(1 - \frac{1}{x}\right)\right]$

at $x = 1$ ~ $\Delta S = 0$ ~ Now $\frac{\partial}{\partial x}\Delta S = \frac{1}{x} - \frac{1}{x^2} > 0$ for $x > 1$

thus +ve slope of $\frac{\partial \Delta S}{\partial x}$ ~ ΔS is +ve for $x > 1$

(2) assume $T_f < T_i$ ~ Then $\Delta S = C_p\left[+\ln\frac{T_f}{T_i} + \left(\frac{T_f - T_i}{T_f}\right)\right]$

Thus $x < 1$ $\left(x = \frac{T_f}{T_i}\right)$

This sign same because $\ln\frac{T_f}{T_i}$ will automatically become –ve

This sign reversed cos we manually had to fix sign of Q.

Thus $\Delta S = C_p\left(\ln x + \left(1 - \frac{1}{x}\right)\right)$

at $x = 1$ ~ $\Delta S = 0$

$\frac{d\Delta S}{dx} = \frac{1}{x} + \frac{1}{x^2} > 0$ (for $x < 1$)

Thus in Both cases $\Delta S > 0$.

[extra info: $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$] & as $x \ll 0 \to \log(1+x) \approx x$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$]

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$ $\quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

(Q6) use 1) ✓ conserve Mole : $n_1 + n_2 = n$

$\frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2} = \frac{PV}{RT}$

2) ✓ conserve energy :

$\frac{3}{2} n_1 R T_1 + \frac{3}{2} n_2 R T_2 = \frac{3}{2} n R T$

$\Rightarrow P_1 V_1 + P_2 V_2 = PV$

$\longrightarrow P = \frac{P_1 + P_2}{2}$

(Q7) –ve Temp ~ GBG A4 append ~ energy in magnetic field

→ popⁿ inversion

~ hole ✗ –ve mass (Bragg diffraction)

various restrictions

To make it easier for using x ~ $x = \frac{T_f - T_i}{T_f}$

write $C_p \ln\frac{T_f}{T_i} = -C_p \ln\frac{T_i}{T_f}$ ~ Now: $-\ln(1-x) - x$

$= x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots - x$

$= \frac{x^2}{2} + \frac{x^3}{3} \cdots > 0$

since $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \cdots$

$-\ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} \cdots$

soln in my physics TG folder

3

[2018 Q6]

Chegg soln : $dV = \left[\left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP\right]$

Thus $W = PdV = -PV\beta \cancel{dT}^{0} + PVk\,dP$

(since isothermal)

Thus $W = kV\left(\frac{P_2^2 - P_1^2}{2}\right)$

[2016 Q9(ii)]

viscosity in liquid ~ friction b/w layers

" " gas → molecular diffusion that transports momentum b/w layers of flow

($\eta = \frac{1}{3}mn\bar{v}\lambda$ (Transport theorem))

[2015 Q3]

$\ln\frac{P_2}{P_1} = \frac{L}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

[2015 Q5]

coefficient of cubical expansion β

$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$ (GBT 8 Q4 sir verified...)

[volume expansion coefficient α]

[2014 Q3]

Throttling is const enthalpy

Now $dQ = 0 \to dU = -dW$

$dW = P_2\int dV_2 - P_1\int dV_1$

$= P_1V_1 - P_2V_2$

Thus $dU = U_2 - U_1 = P_1V_1 - P_2V_2$

$H_1 = U_1 + P_1V_1 = U_2 + P_2V_2 = H_2$

[2017 Q3(iii)]

$T^4 = \frac{300^4 + 200^4}{2}$

use $\sigma A(T_2^4 - T_1^4) = \sigma A(T_1^4 - T_0^4)$

< emitted) <absorbed

T_0 & T_2 are external Temp (and) T_1 is body Temp

[2019 Q7] ~ GBG pg 13.8 / 12.34 / 13.2

$Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$ - to find $\langle KE \rangle$ and C_V

proof : $\ln\Omega_{MB} = \ln N! - \sum \ln n_i! + \sum n_i \ln g_i$

$= N\ln N - \cancel{N} - \sum n_i \ln n_i + \cancel{\sum n_i} + \sum n_i \ln g_i$

[N is total particles as in MB derivation]

$= N\ln N + \sum n_i \ln\frac{g_i}{n_i}$

$= N\ln N + \sum n_i \ln \frac{Z}{N}e^{\beta E_i}$

since $\frac{n_i}{N} = \frac{g_i e^{-\beta E_i}}{Z}$

so $\frac{g_i}{n_i}$ value put in

$= \cancel{N\ln N} + \sum n_i \ln Z - \cancel{\sum n_i \ln N} + \sum n_i E_i \beta$

$= \sum n_i \ln Z + \beta \sum E_i n_i$

$= N\ln Z + \beta E$

Now $S = k_B \ln\Omega_{MB} = Nk_B \ln Z + \beta k_B E$

Thus $S = Nk_B\ln Z + \beta k_B E = \boxed{Nk_B \ln Z + \frac{U}{T}}$

Now $F = U - TS$ ~ $\boxed{F = -Nk_BT\ln Z}$

And Now $dF = \cancel{TdS} - PdV - \cancel{TdS} - SdT$

$= -PdV - SdT$

Thus $S = -\left(\frac{\partial F}{\partial T}\right)_V$

$= -\frac{\partial}{\partial T}(-Nk_BT\ln Z)$

$= Nk_B \ln Z + Nk_BT\frac{\partial}{\partial T}(\ln Z)$

IMP substitution

$S = -\frac{F}{T} + Nk_BT\frac{\partial}{\partial T}(\ln Z)$

use $F = U - TS$ ✓

Thus $\boxed{U = Nk_BT^2\frac{\partial}{\partial T}(\ln Z)}$

for P ~ $P = \left(-\frac{\partial F}{\partial V}\right)_S = Nk_BT\frac{\partial}{\partial V}(\ln Z)$

$= Nk_BT \times \left(\frac{1}{Z}\frac{\partial Z}{\partial V}\right)_T$

$P = \frac{Nk_BT}{Z}\left(\frac{\partial Z}{\partial V}\right)_T$

$\mu = \frac{\partial F}{\partial N} = \frac{\partial}{\partial N}(-Nk_BT\ln Z)$

$\mu = -k_BT\ln Z$

Now with $Z = \frac{V}{h^3}(2m\pi kT)^{3/2}$ ~ $U = k_BT^2\frac{\partial}{\partial T}\ln Z$

$= \frac{3}{2}Nk_BT$

$C_V = \underbrace{\frac{3}{2}Nk_B}_{} = \frac{3}{2}nR$

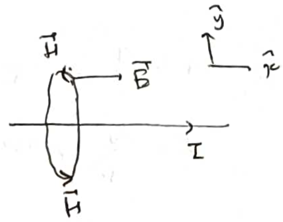
[N is total particles]

[2019 Q6(c) & 2013 7(b)]

Poynting in a wire

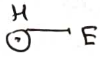
(Toppr.com & physicsHub YT) — same for both cases

$H = \frac{I}{2\pi r}$ and $E = \frac{V}{L} = \frac{IR}{L}$

here 

here $\vec{E}$ is along wire since we used $V = E\times L$

Thus $S = EH = \frac{I^2 R}{2\pi a L}$ ✓ (This is correct)

thus 

$S = \vec{E}\times\vec{H}$ is in $-\hat{y}$ dirn

Thus $P = S\times Area$
$= S\times 2\pi a L$

since these are along $\vec{S}$ using $S = \vec{E}\times\vec{H}$

Then Power $= \underline{VI}$

(Wikipedia)

using Gauss law: (assume ⊙ symmetry)

$E(r) = \frac{W}{r}$ ~ Now $V = -\int_{R_1}^{R_2} E\cdot dl = W\ln\frac{R_2}{R_1}$

also $H = \frac{I}{2\pi r}$ — S is Intensity

Thus $S = EH = \frac{W}{r}\cdot\frac{I}{2\pi r} = \frac{WI}{2\pi r^2}$ ✓

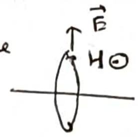
Now $P_{total} = \int S\times dA = \int S\times 2\pi r\,dr$ (don't use πr^2 directly)

$= \int_{R_1}^{R_2} \frac{WI}{r}\,dr$

$= WI\ln\frac{R_2}{R_1} = \frac{V}{\ln\frac{R_2}{R_1}}\times\ln\frac{R_2}{R_1}$

$= \underline{VI}$ ✓

"W" comes out bcoz $\ln R_2/R_1$ is fixed constant. R_2 & R_1 were used as fixed to find V b/w 2 points to get value of W

Here 

we use radial $\vec{E}$ and hence $\hat{S}$ along $-\hat{x}$ dirn

thus Area $= 2\pi r\,dr$ (don't use πr^2 directly)

Thus $P = \int S\,dA$
$= \underline{VI}$

[2022 Q4(a)]

$h_1 = h_2$ in Bernoulli ~ same height

$\Delta p = \rho g h$ (h is diff in U-tube)

$v_1^2 - v_2^2 = 2gh$ & $Q = A_1 v_1 = A_2 v_2$ (eqn of continuity)

[2022 Q2(b)] — verify this Q4

don't use internet method of Toppr.com & Saadhak since they only use θ & not have θ & φ as given in question.

my method: $\tan\theta = \frac{\sin\theta'}{\frac{m_1}{m_2} + \cos\theta'}$

Now $\phi = 90 - \frac{\theta'}{2} \to \theta' = 180 - 2\phi$

Thus $\tan\theta = \frac{\sin 2\phi}{\frac{m_1}{m_2} - \cos 2\phi}$ — correct since if we put $m_1 = m_2$ in this, we get $\tan\theta = \cot\phi = \tan(90-\phi)$. Thus $\theta + \phi = 90$

Now assume m_1 & m_2 are fixed.

θ & φ can vary in whatever way (v_1 & v_2 will accordingly adjust value to make sure $\vec{p}$ is conserved in x & y dirn

eg: take $m_1 = m_2$ ~ for any value of φ, the ball transfers its ~~equal~~ momentum to 2nd ball in the direction & ~~itself~~ move ⊥ since full momentum in original dirn is transferred

also v_1, v_2, θ, ϕ are variable (4) in 3 eqn ~ Thus there is freedom to shift.

Thus we want max θ.

$\frac{d}{d\phi}\tan\theta = 0$ ~ gives $\cos 2\phi = \frac{m_2}{m_1}$

putting value of φ → $\tan\theta_{max} = \frac{\sqrt{1 - \frac{m_2^2}{m_1^2}}}{\frac{m_1}{m_2} - \frac{m_2}{m_1}}$

case 2: $m_2 \le m_1$ ~ $\cos 2\phi = \frac{m_2}{m_1}$ applicable.

now head-on gives θ = 0 

so we need θ_{max} with $\phi \ne 0$

then formula is applicable. ~ $\tan\theta_{max} = \frac{m_2}{\sqrt{m_1^2 - m_2^2}}$

eg: take $m_2 = m_1$ → $\tan\theta = \infty$

Thus $\theta = 90°$

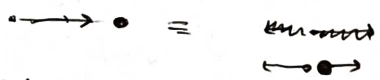
This is correct 

(full momn transferred)

θ_{max} more than 90° not possible for $m_1 = m_2$.

Now case 1 → $m_1 < m_2$ ~ $\cos 2\phi = \frac{m_2}{m_1}$ not applicable

then θ_{max} anyway 180°



[$v_{app} = v_{sep}$.

such that for elastic collision $e = \frac{\text{rel. velocity sep}}{\text{rel. velocity approach}} = 1$ and $\vec{p}$ & E are conserved ✓

My answer is correct, but see pg6 for same, but simpler. (Basically we don't need to put $\theta' \to \phi$, doing $\frac{d}{d\phi}$ is then same as $\frac{d}{d\theta'}$ only....)

But actual easy way to get $\theta + \phi = 90$ is $\theta' = \frac{\theta}{2}$ when $m_1 = m_2$

and then $\phi = 90 - \frac{\theta'}{2}$

$\theta + \phi = 90$

easy proof for $\theta + \phi = 90°$

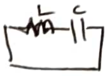
[2022] Q5 ESM sheet (verify Thu Q4)

$Q = \frac{\omega_0 L}{R}$ where $\omega_0 = \frac{1}{\sqrt{LC}}$

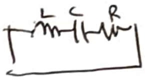
since we get $c > \omega_0$ in given circuit,
Thus overdamped.

No oscillation! Thus no resonant frequency.

in underdamped – oscillation exists & we have Q & ω_0 applicable



~ pure oscillator $\omega = \frac{1}{\sqrt{LC}}$



→ damped oscillation

$Q = A_1 e^{-(c \pm \sqrt{c^2-\omega_0^2})t}$

for $c > \omega_0$ ~ overdamped ~ only exponential terms (no oscillation)

for $c < \omega_0$ ~ underdamped ~ we get sine terms & oscillations

$\omega_d = \sqrt{\omega_0^2 - c^2}$

outhinks YT channel:
Also resonance occurs when $\omega_{external} = \omega_0$

in DC → $\omega = 0$ ~ Thus no resonant frequency

< Resonance always needs AC >

[2013 Q7] ~ inhomogenous (verify Q7)

(a) $\vec{D} = \epsilon \vec{E} = \varepsilon_0 n^2(x,y,z) \vec{E}$

$\nabla \cdot D = \nabla \cdot (\underbrace{n^2}_{\text{scalar}} \underbrace{\vec{E}}_{\text{vector}}) = 0 \quad (\nabla \cdot D = 0)$

$\nabla \cdot D$ is $= 0$ not $\nabla \cdot E$. (don't put $\nabla \cdot E = 0$)

Thus $n^2(\nabla \cdot E) + \nabla(n^2) \cdot E = 0$

Thus $\nabla \cdot E = -1/n^2 (\vec{\nabla} n^2) \cdot \vec{E}$

Now $\nabla \times (\nabla \times E) = \nabla \times \left(-\frac{\partial B}{\partial t}\right) = -\mu_0 \frac{\partial^2 D}{\partial t^2}$

Thus $-\nabla^2 E + \nabla(\nabla \cdot E) = -\mu_0 \frac{\partial^2 D}{\partial t^2}$

$\nabla^2 E + \nabla\left(\frac{1}{n^2}\nabla(n^2) \cdot E\right) - \mu_0 \varepsilon_0 n^2 \frac{\partial^2 E}{\partial t^2} = 0$

(b) $\nabla \times H = \frac{\partial D}{\partial t} = \varepsilon_0 n^2 \frac{\partial E}{\partial t}$ (since n^2 not fn of time)

Now $\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = \nabla(\cancel{\nabla \cdot H}) - \nabla^2 H = \varepsilon_0 \nabla \times \left(n^2 \frac{\partial \vec{E}}{\partial t}\right)$

Now $\nabla \cdot (f\vec{A}) = f(\nabla \cdot A) + A \cdot (\nabla f)$
$\nabla \times (f\vec{A}) = f(\nabla \times \vec{A}) - \vec{A} \times (\nabla f)$

Thus $-\nabla^2 H = \varepsilon_0 \left[n^2 \left(\nabla \times \frac{\partial \vec{E}}{\partial t}\right) - \frac{\partial \vec{E}}{\partial t} \times (\nabla n^2) \right]$

$= +\varepsilon_0 \left[-n^2 \frac{\partial^2 B}{\partial t^2} - \frac{\partial}{\partial t}\left[\frac{D}{\varepsilon_0 n^2}\right] \times \nabla(n^2) \right]$

$-\nabla^2 H = \varepsilon_0 \left[-n^2 \frac{\partial^2 B}{\partial t^2} - \underbrace{\frac{\partial D}{\partial t} \times \frac{1}{\varepsilon_0 n^2} \nabla(n^2)}\right]$ we took n^2 to other side...

put $\nabla \times H = \frac{\partial D}{\partial t}$

Thus $\frac{1}{\varepsilon_0} (\nabla \times H) \times \left(\frac{1}{n^2} \nabla(n^2)\right) = \vec{H}\left(\cancel{\nabla \cdot \left(\frac{1}{n^2}\nabla(n^2)\right)}\right) - \nabla\left(\vec{H} \cdot \left(\frac{1}{n^2}\nabla n^2\right)\right)$

↳ put zero – I don't know why

Thus we get $\nabla^2 H = n^2 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} - \nabla\left(\frac{1}{n^2}\nabla n^2 \cdot H\right)$

Now $B = \mu_0 H$

Thus $\nabla^2 H = +\nabla\left(\frac{1}{n^2}\nabla n^2 \cdot \vec{H}\right) - \mu_0 \varepsilon_0 n^2 \frac{\partial^2 H}{\partial t^2}$

[2021 Q6(a), ESM Sheet]
(exact question solved in Saaltak.com & Topper)

Wikipedia → $E_\parallel' = E_\parallel$
$B_\parallel' = B_\parallel$
$E_\perp' = \gamma(E_\perp + \vec{v} \times \vec{B})$
$B_\perp' = \gamma\left(B_\perp - \frac{1}{c^2}\vec{v} \times \vec{E}\right)$

~ remember, as ~~even~~ for normal force: using "inverse"
$F_\parallel' = F_\parallel$ and $F_\perp' = \gamma F_\perp$

~ here we have extra $\vec{v} \times \vec{B}$ and $-\frac{1}{c^2}\vec{v} \times \vec{E}$ < MUG!! >

$\vec{B}$ & $\vec{E}$ are full here (no components)

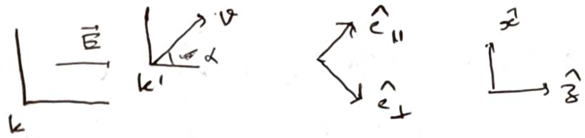
MUG!! write in formula sheet

we can also write as:

$E_x' = E_x \quad E_y' = \gamma(E_y - vB_z) \quad E_z' = \gamma(E_z + vB_y)$
$B_x' = B_x \quad B_y' = \gamma\left(B_y + \frac{v}{c^2}E_z\right) \quad B_z' = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$

Now in question:

$E_\parallel' = E_\parallel, \quad B_\parallel' = 0, \quad E_\perp' = \frac{E_\perp}{\sqrt{1-\frac{v^2}{c^2}}}, \quad B_\perp' = \frac{-\vec{v} \times \vec{E}/c^2}{\sqrt{1-v^2/c^2}}$



let $\vec{E} = (0, 0, E)$

Thus $\vec{v} = (v\sin\alpha, 0, v\cos\alpha)$

Now $\hat{e}_\parallel = (\sin\alpha, 0, \cos\alpha)$ (unit direction vector of $\parallel$)
This is always along $\vec{v}$

(a) Now $E_\parallel' = E\cos\alpha$ ✓

$E_\perp' = \frac{E\sin\alpha}{\sqrt{1-v^2/c^2}}$ ✓

so $|E'| = E\sqrt{\frac{1-\beta^2\cos^2\alpha}{1-\beta^2}}$ and $\tan\alpha' = \frac{\tan\alpha}{\sqrt{1-v^2/c^2}}$

(b) $\vec{B}_\parallel' = 0$ and $\vec{B}_\perp' = \frac{\vec{v} \times \vec{E}/c^2}{\sqrt{1-v^2/c^2}}$ Full $\vec{B}$ & $\vec{v}$ (no components)

$= \frac{Ev\sin\alpha}{c^2\sqrt{1-v^2/c^2}}$

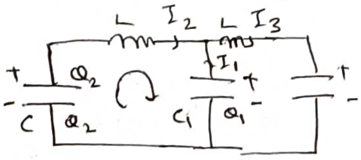
< derivation of Field transformations in A4 append ~~sheet~~ >

Thermal Neutron $\to$ 3 dof.

but use $v_{mp} = \sqrt{\frac{2kT}{m}}$

Thus $\lambda = \frac{h}{\sqrt{2mkT}}$

[2013]



$(I_3 = I_1 + I_2)$

$k_1 = \frac{1}{LC_1}$ $\quad k = \frac{1}{LC}$

Loop 1 $-\frac{Q_2}{C} - L\frac{dI_2}{dt} - \frac{Q_1}{C_1} = 0$

$\frac{dI_2}{dt} = \frac{Q_2}{LC} - \frac{Q_1}{LC_1}$

$\frac{d^2 I_2}{dt^2} = \frac{1}{LC} I_2 - \frac{1}{LC_1} I_1 = \frac{1}{LC} I_2 - \frac{1}{LC_1}$

$= -kI_2 + k_1(I_3 - I_2)$

Now symmetric $- I_2 = I_3 \sim \frac{d^2 I_2}{dt^2} = -kI_2 \Rightarrow \omega_s = \sqrt{k}$

Asymmetric $\to I_2 = -I_3 \sim \frac{d^2 I_2}{dt^2} = -(k+2k_1)I_2 \to \omega_a = \sqrt{k+2k_1}$

Thus $\frac{\omega_a}{\omega_s} = \sqrt{\frac{k+2k_1}{k}}$ ~ But easier soln in my PIAS (3) Notes

take $\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$

symmetric case

$z_1 = z_2 = \omega L + \frac{1}{\omega C}$

$z_3 = \frac{1}{\omega C_1}$

now put $g + jb$, $b = 0 \ldots$

symmetric

$\omega_s' = \sqrt{2k'^2 + k^2}$

now just put

$\omega_a = \frac{1}{\sqrt{LC}} = k$

Thus $\frac{\omega_a}{\omega_s} = \frac{k}{\sqrt{2k'^2 + k^2}}$

[2022 Q2(c)]

system matrix of thick lens

$$\begin{pmatrix} 1 - \frac{P_2 t}{n} & -P_1 - P_2 + \frac{tP_1P_2}{n} \\ t/n & 1 - \frac{P_1 t}{n} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Now for abcd - system matrix and D_1, D_2 - basically u & v, we have

$bD_2 + aD_1D_2 - cD_1 - d = 0$

$\left(1 - \frac{P_2 t}{n}\right)D_2 + \left(P_1 + P_2 - \frac{P_1 P_2 t}{n}\right)D_1 D_2 - \left(1 - \frac{P_2 t}{n}\right)D_1 - \frac{t}{n} = 0$

↳ This is required lens formula for thick lens (D_1 & D_2 is v & u)

for thin: lens formula is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

[2022 Q2(b)]

$\tan\theta = \frac{\sin\theta'}{\cos\theta' + \frac{m_1}{m_2}}$

$\frac{d\theta}{d\theta'} = 0 \to \cos\theta' = -\frac{m_2}{m_1} \sim \sin\theta' = \sqrt{1 - \frac{m_2^2}{m_1^2}}$

$\cos^2\theta' + \cos\theta'\left(\frac{m_1}{m_2}\right) + \sin^2\theta' = 0$

Thus $\tan\theta_{max} = \frac{m_2}{\sqrt{m_1^2 - m_2^2}} \sim \sin\theta_{max} = \frac{m_2}{m_1}$

This answer is verified by my Pf 4 soln & Toppr.com & Q4

alternate soln

$\vec{P_1} = \vec{P_1}' + \vec{P_2}' \sim \vec{P_2} = (\vec{P_1} - \vec{P_1}')$

so $P_2'^2 = P_1^2 + P_1'^2 - 2P_1P_1'\cos\theta$

also $\therefore \frac{P_1^2}{2m_1} = \frac{P_1'^2}{2m_1} + \frac{P_2'^2}{2m_2}$

eliminating P_2' we get:

$0 = P_1'^2\left(1 + \frac{m_2}{m_1}\right) - 2P_1'P_1\cos\theta_1 + P_1^2\left(1 - \frac{m_2}{m_1}\right)$

This quadratic in P_1' has real soln if:

$4\cos^2\theta_1 \geq 4\left(1 - \frac{m_2^2}{m_1^2}\right) \sim \sin\theta_{1\,max} = \frac{m_2}{m_1}$

[2014 Q3] → relativistic Mirror

Now $\nu' = \frac{\sqrt{1 + \frac{v}{c}}}{\sqrt{1 - \frac{v}{c}}}\nu_0$ (soln from website electron6.phys)

photon (from $x = \infty$)

(i) Then $\nu'' = \nu' \times \sqrt{\frac{(1 + v/c)}{(1 - v/c)}} = \nu_0\left(\frac{1 + v/c}{1 - v/c}\right) = \nu_0\frac{(c+v)}{(c-v)}$

(Same like sound ↳ do doppler twice for reflection)

(ii) Now Energy = Intensity

Intensity $\propto E_\perp^2$

Now $E_\perp' = \gamma[E_\perp + \vec{v} \times \vec{B}]$

$= \gamma E_{0\perp}(1 + v/c)$

now $|B| = \frac{E}{c}$ and in EM wave $= \frac{E_\perp}{c}$ ~ all 3 are ⊥!

Now $E_\perp'' = \gamma[E' - v \times B']_\perp$

$= \gamma E_{0\perp}'(1 + v/c)$

'plus' since v in opp. direction

$= \gamma^2 E_{0\perp}(1 + v/c)^2$ ~ Both are thus $(1 + v/c)$!

don't miss this square

$E_\perp'' = E_{0\perp} \times \frac{(1 + v/c)^2}{(1 - v^2/c^2)} = E_{0\perp}\left(\frac{c+v}{c-v}\right)$

Now Intensity $\propto E_\perp^2 \Rightarrow$ Thus $I/I_0 = \left(\frac{c+v}{c-v}\right)^2$

EM theory of double refraction:
(BJS Pg 501/502)

Anisotropy in diff drn

$D_x = \varepsilon_{11} E_x$, $D_y = \varepsilon_{22} E_y$, $D_z = \varepsilon_{22} E_z$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$, etc...

Now $n_x = \sqrt{\frac{\varepsilon_{11}}{\varepsilon_0}}$, $n_y = \sqrt{\frac{\varepsilon_{22}}{\varepsilon_0}}$, $n_z = \sqrt{\frac{\varepsilon_{33}}{\varepsilon_0}}$

Guard rings - Electric field shielding &
- capacitance reduction
- noise redn
- isolation of components
- improving accuracy & reliability of measurements.

Guard ring → It is a conducting ring placed around a sensitive component to control electric field and prevent unwanted currents.

They reduce leakage current & parasitic capacitance (when C spills over to other components)

They are mainly used in IC's

- conducting ring of copper with low impedance.

Metal detector (minelab website)

- detector transmits EM field from search coil. any metal object within EM field gets energized and retransmits EM field of it's own which is detected by coil

AC voltage - oscillating magnetic field

- works on principle of EMF induction

eddy current in metal object create 2ndary magnetic field which opposes original magnetic field .. This change is detected by coil.

Artificial dielectric ~ Metamaterials created: Nano-structures

- work on principle of interaction of EM wave & Nano-structure
  - this creates novel EM properties which not in Natural material

properties & applications

- → Negative refractive Index ~ use in invisibility cloaks
- → superlensing → resolve objects smaller than λ light
- waveguides & Antenna's ~ high efficiency & low loss
- Absorbers of EM → stealth technology, EMI shielding
- medical imaging → MRI/PET

artificial dielectric manipulates EM waves at sub-wavelength ($nm \sim 10^{-9}$) scales for new properties &