QM Paper-2 PYQ's (2013-22) (selective)

[2022 Q5]

no r — since $\hat{n} = \vec{r}/r$ — since $z = r\cos\theta$

$\hat{n} = (\sin\theta\cos\phi)\hat{i} + (\sin\theta\sin\phi)\hat{j} + \cos\theta\,\hat{k}$

$\hat{n}\cdot\vec{S} = \frac{\hbar}{2}\sin\theta\cos\phi\begin{bmatrix}0&1\\1&0\end{bmatrix} + \frac{\hbar}{2}\sin\theta\sin\phi\begin{bmatrix}0&-i\\i&0\end{bmatrix} + \frac{\hbar}{2}\cos\theta\begin{bmatrix}1&0\\0&-1\end{bmatrix}$

$= \frac{\hbar}{2}\begin{pmatrix}\cos\theta & \sin\theta(\cos\phi - i\sin\phi)\\ \sin\theta(\cos\phi + i\sin\phi) & -\cos\theta\end{pmatrix}$

$= \frac{\hbar}{2}\begin{bmatrix}\cos\theta & e^{-i\phi}\sin\theta\\ e^{i\phi}\sin\theta & -\cos\theta\end{bmatrix}$ (mug, don't show maths)

determinant of this gives −1, just like other Pauli Matrices.

mug top term: $\cos\theta$; now we can get 2nd row since $|ad| = 1$; $e^{-i\phi}\sin\theta$

I can derive upto here easily

Now solving normally using $\begin{pmatrix}a\\b\end{pmatrix}$, we get $\lambda = \pm\frac{\hbar}{2}$ (mug don't show maths)

Mug

$|\psi_{+1/2}\rangle = \cos\frac{\theta}{2}|\chi_{1/2}\rangle + e^{i\phi}\sin\frac{\theta}{2}|\chi_{-1/2}\rangle$ (for $\lambda = +\frac{\hbar}{2}$)

$|\psi_{-1/2}\rangle = -\sin\frac{\theta}{2}|\chi_{1/2}\rangle + e^{i\phi}\cos\frac{\theta}{2}|\chi_{-1/2}\rangle$ (for $\lambda = -\frac{\hbar}{2}$)

1st $\chi_{+1/2}$

both have $e^{+i\phi}$, only −ve sign for $-\sin\theta/2$

ϕ isn't $\phi/2$, only θ because $\theta/2$

here put $\theta \to \theta/2$ & add $e^{+i\phi}$ to Both 2nd terms ★ (no $-i\phi$!) Both are $+e^{i\phi}$

euler angle

remember as

$\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & \end{bmatrix}$

+ + + −

just like ⟨S⟩ first row

Thus $\chi_{+1/2}$ 1st & $\chi_{-1/2}$ is 2nd

$\begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix}$

determinant = 1 ★

Thus we can find out where −ve sign come

[2020 Q5]

Given: z-component is $+\frac{\hbar}{2} \to \chi_{+1/2}$

Now probability z' has $+\frac{\hbar}{2}$ $(\psi_{1/2})$: $|\langle\chi_{1/2}|\psi_{1/2}\rangle|^2 = \cos^2\frac{\theta}{2}$

probability z' has $-\frac{\hbar}{2}$ $(\psi_{-1/2})$: $|\langle\chi_{1/2}|\psi_{-1/2}\rangle|^2 = \sin^2\theta/2$

Now Avg value $\langle S_{z'}\rangle = \left(+\frac{\hbar}{2}\times|C_1|^2\right) + \left(-\frac{\hbar}{2}\times|C_2|^2\right)$

$= \frac{\hbar}{2}\cos^2\frac{\theta}{2} + \left(-\frac{\hbar}{2}\sin^2\frac{\theta}{2}\right)$

$= \frac{\hbar}{2}(\cos^2\theta/2 - \sin^2\theta/2)$

$= \frac{\hbar}{2}\cos\theta$

[2019] Q7

Best and easiest way is to follow RJk method

[2014 Q6]

To get TDSE ~ take $\psi(x,t) = Ae^{i(kx-\omega t)}$

Now work backwards and use $H = \frac{p^2}{2m} + V$

and use $p = \hbar k$, $E = \hbar\omega$

To get TISE from TDSE, split $\psi(x,t) = \psi(x)T(t)$ and $T(t) = e^{-iEt/\hbar}$ ~ we get TISE in terms of $\psi(x)$ (no "t" term)

Note: TDSE was in terms of $\psi(x,t)$

[2017 Q1]

No need for $d\sin\theta = \lambda$ step, we can add it, but it won't be used in the calculation.

$\Delta x = r$

$\Delta p_x = p\sin\theta = \frac{pr}{d}$

$\Delta x\,\Delta p_x \geq \hbar/2$ ~ use basics: $\hbar/2$ (don't assume ~ħ, etc)

$\frac{r^2 p}{d} \geq \hbar/2$

$r \approx \sqrt{\frac{\hbar d}{2p}}$

AMP

[2022 Q3(ii)]

↳ 3 optical e^-,

Thus config: N → $1s^2 2s^2 2p^3$

P : $1s^2 2s^2 2p^6 3s^2 3p^3$

[Note]

always use $d\nu = \frac{c}{\lambda^2} d\lambda$

and always remember $\bar{\omega} = \frac{1}{\lambda} = \frac{\nu}{c}$

thus $d\nu = c\, d\bar{\omega}$ (linear)

in PPP7 ~ Q4 : $d\nu = c\,d\bar{\omega}$ (simple linear)

Q8 : use $d\nu = \frac{c}{\lambda^2} d\lambda$

[Note] for $k = \sqrt{\frac{k}{\mu}}$ ~ use reduced mass

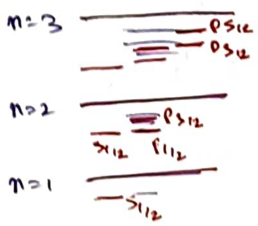
[2022 Q7(ii)]

$J = \sqrt{J(J+1)}\,\hbar$

and $\vec{J} = I \times \vec{\omega}$

[~~2020~~ 2021 Q5]

For spin-orbit coupling in H ~~(...)~~



~ H_α : 7 line (from 5+)

L_α : 2 (Doublet)

Lyman-α line from n=2 → n=1 ~ doublet fine structure

$2P_{3/2} \to 2S_{1/2}$

$2P_{1/2}$

(always $\Delta l = \pm 1$ for spin-orbit coupling)

(diagram: MIT chapter 2: H Fine structure)

[~~2020~~ 2021 Q7] and [2019 Q4] (Zeeman in H-atom)

Diagram in Wikipedia and PPP-7 soln (same)

Just like 3s & 3p ~ D_1 & D_2 Zeeman in Sodium,

For H ~ take ~~ss~~ splitting of L_α line (Wikipedia)

same as ~ $2p_{1/2}, 2p_{3/2}$ & $2s_{1/2}$

sodium D_1 & D_2 line

[but sodium was n=3 ~ 3s & 3p

here H_α zeeman n=2 → n=1]

we take 2p & 1s since spin orbit coupling requires $\Delta l = \pm 1$

Then for the 2 lines: $2p_{3/2} - 2s_{1/2}$ & $2p_{1/2} - 2s_{1/2}$

we further split for Zeeman like Sodium D_1 & D_2

[2020 Q1]

Bohr correspondence principle ~ at large n — classical conditions.

$\omega = v/r$

$\nu_{classical} = \nu_{orb} = \frac{\omega}{2\pi} = \frac{v}{2\pi r} = \frac{1}{2\pi}\left(\frac{e^2}{4\pi\varepsilon_0 m r^3}\right)^{1/2} = \frac{me^4}{4\varepsilon_0^2 h^3} \cdot \frac{1}{n^3}$

get v and r using $mvr = \frac{nh}{2\pi}$ and $\frac{kq^2}{r^2} = \frac{mv^2}{r}$

use QM (irony) → technically this is also QM!

Now $\nu_{Bohr} = \frac{E_0}{h}\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \approx \frac{2E_0}{nh^3} = \frac{me^4}{4\varepsilon^2 h^3} \cdot \frac{1}{n^3}$

Thus $\nu_{Bohr} = \nu_{classical}$

[2020 Q5]

Zeeman effect: classical e^- theory (physics Gulati YT)

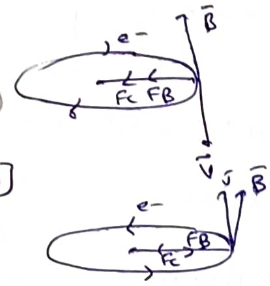
$F_c = m\omega^2 r$

$F_B = evB$

equilibrium: $F_c \pm F_B = mr(\omega + d\omega)^2$ ~ i.e. same in both

$\cancel{m\omega^2 r} \pm evB = mr[\cancel{\omega^2} + 2\omega d\omega]$

(neglect $d\omega^2$)



Thus $\pm evB = 2m(r\omega)d\omega$

$= 2m(v)d\omega$

$\pm eB = 2m\,d\omega$

Now $d\omega = 2\pi\,d\nu$

Thus $d\nu = \frac{\pm eB}{4\pi m}$

in Quantum: $F = \mu_B B g_J m_j$

take $S=0$ ~ $g_l = m_l = 1$

$E = \mu_B B m_l = \frac{eh}{4\pi m} B m_l$

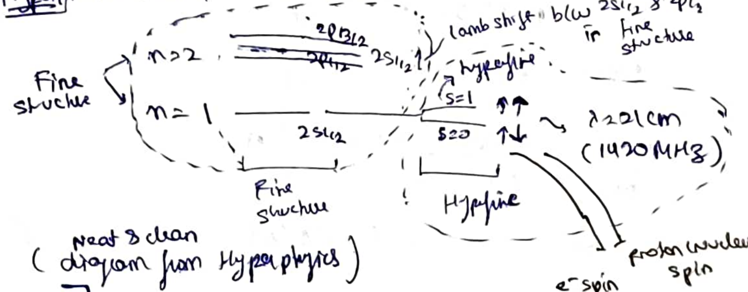
Now ΔE ~ $\Delta m_l = \pm 1$

$\Delta E = \frac{ehB}{4\pi m}$

$\Delta \nu = \frac{eB}{4\pi m}$

[2018 Q5]

Hyperfine structure ~ $L + S + I = F$



(Neat & clean diagram from Hyperphysics)

[2016 Q2]

we don't need m_{Ag} because it cancels out from $\mu_B = \frac{e\hbar}{2m}$

[2019 Q1]

(i) we need metastable state

$\Delta t \approx 10^{-7}$

$\Delta t = 10^{-3}$

(ii) $B_{12} = B_{21}$ ~ no effective pop^n inversion

(iii) v. v. inefficient pumping

eg → putting water of waterfall back at height

(iv) would only produce pulsed laser action

eg: spiking of lasers

[2019 Q2 and 2013 Q4]

why Raman and IR are complementary:

(MSC unit 3.8 and 3.9 + chatGPT)

- for a Band in IR ~ must produce change in dipole moment during vibration

for a Band in Raman ~ must be change in polarizability of molecule during vibration

distortion ↙

eg: if molecule no change in dipole moment but change in polarizability

└ vibration is IR inactive but raman active

Raman and IR of CO_2 [O=C=O]

Mode of vibration	Raman	IR	
ν_1 : symmetric stretching	✓	X	★ complementary to each other
ν_2 : Bending	X	✓	
ν_3 : asymmetric stretching	X	✓	

✓ ~ active

X ~ inactive

ν_1 - no change in dipole coz symmetric - Thus IR X

ν_2, ν_3 - change in dipole. (Thus IR ✓

[2017 Q1] (DPP soln & Q6)

$$N_J = N_0 e^{\frac{-BJ(J+1)}{kT}} \times (2J+1)$$

$$= N_0(2J+1)e^{-\frac{(E_J - E_0)}{kT}}$$

$$N_{J+1} = N_0(2J+3)e^{-\frac{(E_{J+1}-E_0)}{kT}}$$

$$\frac{N_{J+1}}{N_J} = \frac{2J+3}{2J+1} e^{-(E_{J+1}-E_J)/kT}$$

Now $\Delta E = E_{J+1} - E_J << kT$

[2017 Q4]

why some lines in Raman spectra are plain polarized to different extents even though exciting radiation is unpolarized.

(due to Anisotropy ★ – refer 2-pg Q4 Raman Handout ★ PDF

[2014 Q5]

A	Z	I
Even	even	0
odd	even	1/2, 3/2, 5/2 ..
even	odd	1/2, 3/2, 5/2 ..
~~even~~ odd	odd	1, 2, 3 ... (no 0)

→ $I = 0^x$

[2013 Q3]

ESR of H atom – MSC unit 6

\ LB pg 14-17

[2014 Q6]

Relaxation defn (SBP) → average lifetime of nuclei in the non-relaxed state (SBP pg 296-297)

L13 pg 13 : proper defn & spin-spin as well as spin-lattice

(refer L13 pg 13 ★ (only 1 page Nice ✓)

[2017 Q6]

example of Nuclei for NMR: ^1H, ^{13}C, ^{19}F, ^{31}P, $_7N^{14}$

(L13 pg 6)

NMR construction : L13 pg 12 (bottom) or SBP easy

NMR applications : L13 pg 10

L13 pg 9 ~ even for $I = 1$ – $m_I = -1, 0, 1$

still same, $\nu = \frac{\mu_B B g}{h}$ ~ because $E_{(m_I = 1)} - E_{m_I = 0}$

ΔE is ~~same~~ same

Major inference from NMR: (SBP) + (chatGPT)

- ✓ measurement of proton magnetic moment
- ✓ determination of Nuclear magnetic moment
- ✓ chemical shift in organic chemistry
- verification of Zeeman
- " " doppler effect
- study of hyperfine structure
- ✓ measuring coupling constant g_J ★
- ✓ Intensity gives → concentration of components
  → relative abundance

AMP (Extra)

chemical shift : $B_A = B_0(1-\sigma_A)$
$B_B = B_0(1-\sigma_B)$

also $B \propto \nu$ (proton NMR)

δ = chemical shift $= \sigma_A - \sigma_B = \frac{B_A - B_B}{B_0}$

$$\delta = \frac{\Delta\nu}{\nu_0}$$

$$\delta = \frac{\Delta\nu}{\nu_0} \times 10^6 \text{ ppm}$$

$\Delta\nu \to$ shift in Hz from TMS ~ TMS is reference value
$\nu_0 \to$ spectrometer frequency in MHz

or $\delta_{sample} = \frac{\nu_{sample} - \nu_{reference}}{\nu_{reference}} \times 10^6$

ESR of H atom – MSc unit 6
– L13 pg 14–17

H atom ~ $S = 1/2$ (electron) ~ $m_S = +1/2, -1/2$
$I = 1/2$ (proton) ~ $m_I = +1/2, -1/2$

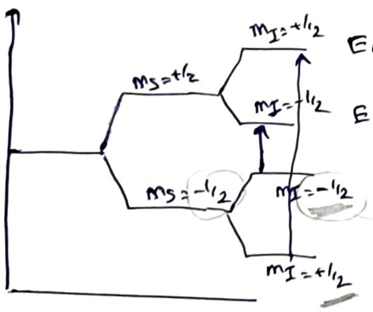
selection rule for Hyperfine structure

just like $\Delta m_j = \pm 1$ for zeeman and NMR ↪ $\Delta m_S = \pm 1$, $\Delta m_I = 0$ ★

$E_{m_S m_I} = g \mu_B B m_S + a\, m_S m_I$ — "a" ~ hyperfine coupling const / splitting constant

(normal similar to $E = \mu_B B g_J m_j$) but take m_S since ESR ~ spin

Zeeman is $\Delta m_j = 0, \pm 1$



$E_{1/2, 1/2} = \frac{1}{2} g \mu_B B + \frac{a}{4}$
$E_{1/2, -1/2} = +\frac{1}{2} g \mu_B B - \frac{a}{4}$
$E_{-1/2, -1/2} = -\frac{1}{2} g \mu_B B + \frac{a}{4}$ since $-1/2 \times -1/2 = +1/4$
$E_{-1/2, +1/2} = -\frac{1}{2} g \mu_B B - \frac{a}{4}$

even Zeeman has $\Delta m_j = \pm 1$ → $\Delta m_S = \pm 1$, $\Delta m_I = 0$ ~ $\Delta E_1 = \mu_B B g - a/2$
$\Delta E_2 = \mu_B B g + a/2$ → $\Delta E = \Delta E_1 - \Delta E_2 = a$ (hyperfine splitting constant)

<we get 2 lines>

L13 – pg 17 ~ shown for e^- with Nucleus of spin $I = 1$

thus $S = 1/2$ – $m_S = +1/2, -1/2$ (S is e^- spin)
$I = 1$ – $m_I = +1, 0, -1$ (I is Nucleus spin)

first m_S split, then m_I

(diagram on pg 17 L13 for $I = 1$) ~ $\Delta E_1 = \mu_B B g - a$
$\Delta E_2 = \mu_B B g$
$\Delta E_3 = \mu_B B g + a$
} MSc unit 6 pg 12/13

NPP

[2019 Q3]

from L14 pg 14

self energy of uniform charge sphere : $\frac{3}{5} \cdot \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R}$

Thus $E_c = \frac{3}{5} \cdot \frac{1}{4\pi\epsilon_0} \frac{Z^2 e^2}{R_0 A^{1/3}} = a_c \frac{Z^2}{A^{1/3}}$

Thus $a_c = \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 R_0}$

Now method to find Radius: (L14 beginning) and mmmut.ac.in

(1) Back-scattering of α-particle
(2) lifetime of α-emitter in α-decay
(3) Muonic X-ray method
(4) High energetic e^--scattering
(5) Isotope effect in line spectra
(6) Mirror Nuclei method
(7) Neutron scattering

Mirror Nuclei method (mmmut.ac.in slides):

in β-decay ~ we get mirror Nuclei

Now Δ(Binding Mass) ~ all terms cancel except a_c term ☆ (only a_3 survives)

Reaction: $Z \to Z+1$ Nuclei

$\Delta E_c = \frac{1}{4\pi\epsilon_0} \frac{3}{5} \frac{e^2}{R} [(Z+1)^2 - Z^2]$

$= \frac{1}{4\pi\epsilon_0} \frac{3}{5} \frac{e^2}{R} (2Z+1)$

$\Delta E_c = \frac{1}{4\pi\epsilon_0} \frac{3}{5} \frac{e^2}{R} (A)$ $[A = Z + Z + 1]$

since A and ΔE_c is known,
we can find Radius R.

we can also get value of R_0 using
$R = R_0 A^{1/3}$ since A is known

$R_0 = 1.2$ Fermi

[2021 Q5 and 2016 Q4] ~ (Q1) use DCT pg 366 just my intermediate steps.
(or think 4T channel):

parabolic potential $V(r)_{\text{interaction}} = -V_0(1 - \frac{r^2}{R^2})$

$V_{\text{modified}}(r) = \underset{\text{potential}}{V_{\text{Nucleus}}(r)} + \underset{\text{potential}}{V_{\text{interaction}}(r)}$

$= \frac{kQ}{R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right] - V_0(1 - \frac{r^2}{R^2})$

$= \underbrace{r^2 \left[\frac{-Ze}{8\pi\epsilon_0 R^3} + \frac{V_0}{R^2}\right]}_{\text{Harmonic potential}} + \left[-V_0 + \frac{3Ze}{8\pi\epsilon_0 R}\right]$

Thus $\frac{1}{2} m\omega^2 r^2 = r^2 \left[\cancel{\frac{-Ze}{8\pi\epsilon_0 R^3}}^{0} + \frac{V_0}{R^2}\right] \approx \frac{V_0}{R^2} r^2$

since $\frac{1}{R^3} \ll$ small

Thus $\omega = \sqrt{\frac{2V_0}{mR^2}} \to \hbar\omega \approx 41 A^{-1/3}$

put $R_0 = 1.3$ fm
$\hbar c = 200$ MeV fm
$mc^2 = 938$ MeV, $\boxed{V_0 = 33 \text{ MeV}}$ ~ my

[2021 Q1]

simple soln ~ Just use concept of strong and weak interaction

[2021 Q6 and 2015 Q4]

Neutrino mass ~ measured by kurie plot

flavour → ν_e, ν_τ, ν_μ

mass state – ν_1, ν_2, ν_3

flavour of neutrino is combination of mass state

3 discrete mass eigenstates
- ν_1 ~ e^- (mostly)
- ν_2 → even blend of e^-, μ^-, τ^-
- ν_3 → mostly μ^- and τ^-

only difference of squares of 3 mass values are known as of know (rms values)

also: sum of 3 neutrino masses must be less than 10^{-6} of mass of e^-

$|\Delta m_{12}|^2_{rms}$, $|\Delta m_{23}|^2_{rms}$ and $|\Delta m_{13}|^2_{rms}$

by solar and Atmospheric
neutrino oscillation, neutrino oscillations

[2013 Q3(a)]

decay processes in which parity violation is demonstrated
- β^- decay (eg: Cobalt Nucleus)
- β^+ decay
- μ^- decay: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ ✓
- τ^- decay: $\tau^- \to e^- + \bar{\nu}_e + \nu_\tau$ ✓
- Pion decay: $\pi^- \to \mu^- + \bar{\nu}_\mu$ ☆ ✓
- Kaon decay → $k^+ \to \pi^+ + \pi^0$, $k^- \to \pi^- + \pi^0$

[2016 Q8] ~ (wikipedia)

Z_0 Boson: Massive, electrically neutral particle in standard model
- Mediates weak force ~~[illegible]~~
- Mass: 91 GeV/c^2 and is chargeless and spin = 1
- Imp for electroweak & spontaneous symmetry breaking
- unstable due to short lifetime
- Z^0 is it's own anti-particle

$W^\pm$ ~ can change charge of emitting particle by 1
(eg β^- decay by W^- and β^+ by W^+)

Feynmann diagram in SBP

$W^\pm$ ~ can change type of quark (eg: $s \to u$) and also change spin.

Z^0 ~ cannot change charge: only changes spin, momentum and energy

PTO

2

Z^0 ~ mediates transfer of spin, $\vec{p}$ and Energy when neutrino's scatter elastically from matter (charge is conserved)

In β^- decay: $n \to p + e^- + \bar{\nu}_e$ (2 steps)

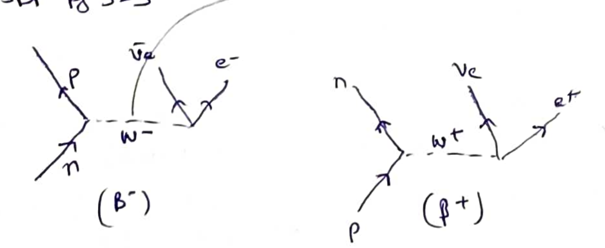
(i) $\overset{Q=-1/3}{d} \to \overset{Q=+2/3}{u} + \overset{-1}{W^-}$ (udd → uud)

(ii) $W^- \to e^- + \bar{\nu}_e$ (W^- decay) instantaneously

Z^0 → provides only known mechanism for elastic scattering of neutrino's in matter

SBP pg 325

step 1: $n \to p + W^-$ (charge conserved)



[2016 Q1]

Deuteron excited state: L15 pg 11

$kob \approx \frac{3\pi}{2}$ in $x\tan(\frac{\pi}{2}+x) = y$

(there is no $x\tan x = y$ for deuteron since only $\sin x$ term)

$\frac{3\pi}{2} \to V_{0\,min} = 225$ MeV ($9 \times 25 = 225$; 25 MeV came by putting $R = r_0 = 4.2$ F)

↳ too large potential

Thus deuteron don't exist in excited states

[2017 Q6(i)] (L23 pg 2 and 3)

CC ~ it is operation under which we convert particle to anti-particle and vice-versa

$\pi^- \to \mu^- + \bar{\nu}_\mu \xrightarrow{C.C.} \pi^+ \to \mu^+ + \nu_\mu$

P ⇒ space parity

	W	EM	S
C: CC or C-parity	×	✓	✓
P: P or space parity/inversion	×	✓	✓
CP: combined inversion (Both)	×	✓	✓
T: Time Reversal $\psi^*(x,-t) = T\psi(x,t)$ eg: $\gamma + \gamma \to e^- + e^+$	✓	✓	✓
CPT is combined C, P and T ★	✓	✓	✓

[2017 Q7]

schematic of single particle energy levels with spin-orbit coupling

↳ our normal spdf but diff

2 examples – take ^7_3Li and $^{13}_6C$

($I = \frac{3}{2}^-$) ($I = \frac{1}{2}^-$)

1 example of violation → predicted $^{23}_{11}Na$ → $5/2$

observed " → $3/2$

[2019 Q1]

↳ B/A ~ awesome explanation in L14 pg 12

[2018 Q5]

(assume rest mass Neutrino ≃ 0

Now simple $p_1^2c^2 = p_2^2c^2$ → .... follow steps DIAS(2) pg 10

we get $E_B = \left(\frac{m_0^2 + m_{1,0}^2 - m_{2,0}^2}{2m_0}\right)c^2$

Now $KE_B = E_B - m_{01}c^2$

we get $KE_\mu = \frac{(m_\pi - m_\mu)^2 c^2}{2m_\pi}$ (taking $m_{\bar{\nu}_\mu} = 0$)

[2017 Q4] DCT ~ pg 324/325

$\vec{\mu} = \mu_p\vec{\sigma}_p + \mu_n\vec{\sigma}_n + \vec{l}$

Now $\vec{\sigma}_n - \vec{\sigma}_p = 0$ (Triplet state – spins are parallel (↑↑)

and ★ $\vec{\sigma}_n + \vec{\sigma}_p = 2\vec{S} = 2(\vec{I} - \vec{L})$

(Deuteron has I=1 ~ spin (S) = 1 → ½ + ½ (↑↑); spin = 0 not for Deuteron! as then L = 1 ×) deuteron is always Triplet (↑↑) (since S = 1 ★)

Thus $\bar{\mu} = \frac{1}{2}(\mu_n + \mu_p)(\underbrace{\sigma_n + \sigma_p}_{2(I-L)★}) + \frac{1}{2}(\mu_n - \mu_p)(\underbrace{\sigma_n - \sigma_p}_{0}) + \frac{L}{2}$

Thus $\bar{\mu} = (\mu_n + \mu_p)\vec{I} - (\mu_n + \mu_p - \frac{1}{2})\vec{L}$ — (1)

Now $\langle I \rangle = 1$ for Deuteron (★ is given I = 1 for deuteron)

to get $\langle L \rangle$ ~ $L_3 = \frac{L \cdot I}{I^2} I_3 = \frac{I(I+1) + L(L+1) - S(S+1)}{2I(I+1)} I_3$ — (2)

since $I = L + S$ ~ $S = I - L$ → $I \cdot L = \frac{I^2 + L^2 - S^2}{2}$

Now for deuteron: I = 1, S = 1 always

for s-state: L = 0 ~ $\langle L \rangle = 0$
for d-state: L = 2 ~ $\langle L \rangle = 3/2$
] from eq (2)

L15 pg 13

we can also directly put $\langle L_3 \rangle = \frac{3}{2}P_D$

$\langle I \rangle = 1$ always

Basically L_3 comes to be $\langle L_3 \rangle = \frac{L(L+1)}{4}$ (mod) ★ $= 0 \times P_S + \frac{3}{2}P_D = \frac{3}{2}P_D$

L = 2 for D-state

(Don't show math, just put answer)

Now $\mu_d = P_S \times \langle \psi_S | \mu | \psi_S \rangle + P_D \langle \psi_D | \mu | \psi_D \rangle$

$= P_S(\mu_n + \mu_p) + P_D((\mu_n + \mu_p) - \frac{3}{2}(\mu_n + \mu_p - \frac{1}{2}))$

put value of $\langle I \rangle$ and $\langle L \rangle$ in (1)

) we put $P_S + P_D = 1$

$\mu_d = (\mu_n + \mu_p) - \frac{3}{2}P_D(\mu_n + \mu_p - \frac{1}{2})$ — (3)

↳ putting value of μ_n, μ_p, μ_d ~ we get P_D

$P_D \simeq 3.93\%$

(Don't show math, just put answer)

for: μ_S-state – put $P_D = 0$ in (3)

Deuteron magnetic moment: μ_D-state – put $P_D = 1$ in (3)

[2020 Q2]

from DCT pg 324/325

$$\mu = (\mu_n + \mu_p) I - (\mu_n + \mu_p - \tfrac{1}{2}) L$$

Now for $^3S_1$ → $S=1$, $I=1$, $L=0$ → S-state of deuteron

$\langle L \rangle = 0$, $\langle I \rangle = 1$ } showed on prev. page.

Thus $\mu = \mu_n + \mu_p$ (correct, even chatgpt verified this)

[2020 Q3]

Basic weak interaction processes in Nuclei: (from chatGPT):

- β^- decay
- β^+ decay
- e^- capture (eg: k-e^- capture) ($p + e^- \to n + \nu_e$)
- Muon capture ($p + \mu^- \to n + \nu_\mu$) — same as e^- capture.
- Neutrino-less double β-decay
  - $2n \to 2p + 2e^-$

[2019 Q2]

chatgpt (same as wikipedia*)

$$\mu_q = \frac{e_q \hbar}{2m_q}$$

$\mu_u = +\frac{2}{3}\frac{\hbar}{2m_q}$, $\mu_d = -\frac{1}{3}\hbar/2m_q$

similar to μ_B or μ_N (Nuclear Bohr magneton)

also, we usually assume m_q is $1/3^{rd}$ of mass of Nucleon but actual mass of quark are only 1% of that of Nucleon (wikipedia)

$$\boxed{\mu_{proton} : \frac{4}{3}\mu_u - \frac{1}{3}\mu_d}$$
$$= \frac{4}{3}\times\frac{2}{3} - \left(-\frac{1}{3}\times\frac{1}{3}\right)$$
$$= 3x$$

< Note! take $e = \pm$ values > $+\frac{2}{3}$ & $-\frac{1}{3}$ — charge of quarks

$$\boxed{\mu_{neutron} : \frac{4}{3}\mu_d - \frac{1}{3}\mu_u}$$
$$= \frac{4}{3}\times-\frac{1}{3} - \left(\frac{1}{3}\times\frac{2}{3}\right)$$
$$= -2x$$

Thus $\frac{\mu_n}{\mu_p} = -\frac{2}{3}$ — matches with ratio in wikipedia table (and chatgpt soln)

[2019 Q5]

- cannot detect parity violation by using only β decay rate since helicity of neutrino changes
- concept of pseudovector and pseudoscalar

[2017 Q2]

DCT pg 347 → $F_{pp} = F_{nn}$ (charge symmetric)
$F_{pp} \simeq F_{nn} \simeq F_{pn}$ (charge-independent)

proof of charge independence (DCT pg 321/322) → Mirror Nuclei
→ similar to diproton, even dineutron don't exist in nature

ChatGPT:

charge independence → Nuclear Force/interaction b/w Nucleons is same regardless of electric charge (eg: Binding-energy of mirror Nuclei $^{6}C^{12}$ and N^{12} are same)

charge symmetry → specific condition of charge independence related to Nucleons with pp or nn.

means Nuclear force is invariant under exchange of a proton with neutron or vice-versa

eg: $n + p \to p + n$ (scattering) (they are exchanged)

[2018 Q6]

for odd & even parabola
- need to mug the diagrams given in SBP!

[2022 Q4]

from L14 pg 18

using $Q = [M_x - m_y - m_\alpha]c^2$

use B.E. formula $M(Z,A)$,

we get for α-decay: $Q > 0 \Rightarrow A > 160$

also, other reason why U^{238} is α-emitter and not β-emitter

- → α-decay is more energetically favourable than β process
- → U238 has so many protons, so not prefer β emission which would give another proton and increase instability
- → strong nuclear force b/w nucleons inside nucleus is able to overcome the coulomb ~~barrier and~~ repulsion between protons allowing the α-particle to tunnel through potential Barrier
- → Large nucleus favours α-decay for stability.
- → α-particle is very stable due to doubly magic configuration

[2020 Q5]

exact question in MIT problem 3 F2020

Energy released $Q = (m_d - m_p - m_n)c^2 = 2.234$ MeV

Now $Q = E_{photon} + KE_{deuteron}$

since $Q = (\text{Rest mass})_{pdt} - (\text{Rest mass energy})_{reactants}$

$= \underline{(KE)_{final} - (KE)_{initial}}$ *

Now $E_{photon} = pc$, $KE_{deut} = p^2/2m$

(both have same p (momentum conserved)

Thus $pc + \frac{p^2}{2m_d} = Q$

rearrange: $(pc)^2 + 2mc^2(pc) - 2mc^2 Q = 0$

$$\left.\begin{aligned} pc &= mc^2\left(-1 + \sqrt{1 + \frac{2Q}{mc^2}}\right) \\ &= mc^2\left(-1 + 1 + \frac{Q}{mc^2}\right) \end{aligned}\right\} \text{Binomial}$$

$= \cancel{Q}\ Q$

Thus $KE_d = \frac{p^2}{2m} = \frac{Q^2}{2mc^2} = 1.33 \times 10^{-3}$ MeV

$E_\gamma = pc = Q = 2.234$ MeV

$\frac{KE_d}{E_\gamma} \approx 10^{-4} \ll 1$ — Thus deuteron recoil doesn't affect energy of emitted photon

NPP (extra)

L15 pg 14 & DCT pg 323 (potential future question)

$V(r) = V_1(r) + V_2(r)\bar{\sigma}_1 \cdot \bar{\sigma}_2 + V_3(r)\hat{S}_{12}$ — Tensor force operator

Nuclear potential — Central Force

$\sigma_1 \cdot \sigma_2 = 1$ (~~singlet~~ triplet)

$\sigma_1 \cdot \sigma_2 = -3$ (singlet)

$\hat{S}_{12} = \dfrac{3(\bar{\sigma}_1 \cdot \bar{r})(\bar{\sigma}_2 \cdot \bar{r})}{r^2} - \sigma_1 \cdot \sigma_2$

for singlet $\rightarrow \hat{S}_{12} = -3-(-3) = 0$

$\vec{S} = \frac{1}{2}(\bar{\sigma}_1 + \bar{\sigma}_2) = 0$

Thus $\sigma_1 = -\sigma_2$

singlet has spin $\frac{1}{2}-\frac{1}{2}=0$ $\uparrow\downarrow$

Thus in singlet states, the nuclear forces are central forces. ☆

L16 pg 4/5: $E = (\Lambda + \frac{3}{2})\hbar\omega$, $\Lambda = 0,1,2,3,\ldots$

where $\Lambda = 2n+l-2$

$n = 1,2,3,4,\ldots$

$l = 0,1,2,\ldots$ ~ we can have 1p, 1d, 1f... etc.

zero point Energy → $\frac{3}{2}\hbar\omega$

Use $2(2l+1)$ for each l available (or)

also $N_\Lambda = (\Lambda+1)(\Lambda+2)$

Λ	E	(n,l)		No. of Nucleons filling shell	Total ~ $\frac{1}{3}(\Lambda+1)(\Lambda+2)(\Lambda+3)$
0	$3/2\,\hbar\omega$	1,0	1s	2	2 ✓
1	$5/2\,\hbar\omega$	(1,1)	1p	6	8 ✓
2	$7/2\,\hbar\omega$	(1,2) (2,0)	1d 2s	12	20 ✓
3	$9/2\,\hbar\omega$	(1,3) (2,1)	1f 2p	20	40 ✗
4	$11/2\,\hbar\omega$	(3,0) (2,2) (1,4)	3s, 2d, 1g	30	70 ✗

not corresponding to magic number in high potential due to consideration of Central potential.

Take spin-orbit interaction ~ nL_I ~ $I = l+s$ (inverted LS coupling)

introducing non-central potential $V_{ls} = -\phi(r)\,\bar{l}\cdot\vec{S}$

L16 pg 6: (and DCT pg 367) $\langle V_{ls}\rangle = -\bar{l}\cdot\bar{s}\,\langle\phi(r)\rangle$

Now $\bar{l}\cdot\bar{s} = \dfrac{j^2 - l^2 - s^2}{2} \sim \dfrac{j(j+1) - l(l+1) - s(s+1)}{2}$

and put $s = 1/2$ in both

for $j = l + 1/2 \rightarrow l\cdot s = l/2$

for $j = l - 1/2 \rightarrow l\cdot s = -\dfrac{(l+1)}{2}$

Thus: $j = l + 1/2 \Rightarrow E = -\frac{1}{2}l\,\langle\phi(r)\rangle$

$j = l - 1/2 \Rightarrow E = \frac{l+1}{2}\,\langle\phi(r)\rangle$

$E_{l-1/2} > E_{l+1/2}$ ~ $d_{3/2} > d_{5/2}$ ★

$\Delta E = (l+\frac{1}{2})\langle\phi(r)\rangle$

$= 10(2l+1)A^{-2/3}$ MeV

L17 pg 10

Quadrupole moment from shell model

for $I=0$, $Q \approx 0$

for magic number: $Q \approx 0$

for odd A and odd Z Nuclei

$Q_{proton} =$ ~~[scribble]~~ $-\frac{3}{5}\left(\frac{2I-1}{2I+2}\right)R^2$

$Q_{proton} = -\frac{3}{5}\left(\frac{2I-1}{2I+2}\right)R_0^2 A^{2/3}$

$-\frac{3}{5}\left(\frac{2I-1}{2I+2}\right)R_0^2 A^{2/3}$

$Q<0$ ~ oblate

$Q>0$ ~ prolate (positive Q)

$Q_{\text{single Nucleon}} = \dfrac{Z}{A^2}Q_{proton}$

Q-value of β^- decay: $[M(A,Z) - M(A,Z+1)]c^2$ (reactant, pdt)

Q-value of β^+ decay: $[M(A,Z) - M(A,Z-1) - 2m_e]c^2$ (reactant, pdt)

always Q = (Rest mass) reactant − pdt

= (KE) pdt − reactant

LI7 pg 11/13 (DCT pg 375 ~~[illegible]~~)

MIT PDF same as ~~[illegible]~~... DCT pg 375 ($J \equiv I$)

dont follow LI7 because - calculation mistake in final answer sign

Note: $J \equiv I$ ★

Now DCT pg 375 ~ Nuclear magnetic moment + schmidt lines

$$\vec{\mu} = \vec{\mu}_s + \vec{\mu}_l = \frac{\mu_N}{\hbar}(g_l \vec{l} + g_s \vec{s})$$ ★

follow DCT steps:

since $\vec{l} = \sqrt{l(l+1)}\hbar$

$$\mu = \left(g_l\sqrt{l(l+1)}\cos(\vec{l}\cdot\vec{j}) + g_s\sqrt{s(s+1)}\cos(\vec{s}\cdot\vec{j})\right)\frac{\mu_N}{\hbar}$$

Nuclear (not μ_B!)

$$= g_l\sqrt{l(l+1)}\,\frac{l(l+1)+j(j+1)-s(s+1)}{2\sqrt{l(l+1)}\sqrt{j(j+1)}} + \ldots$$

since $\cos(l \cdot j) \rightarrow$

$$j = l + s$$
$$j - l = s$$
$$j^2 + l^2 - 2j\cdot l = s^2$$
$$j \cdot l = \frac{j^2+l^2-s^2}{2}$$

$j \cdot l = |j||l|\cos(j-l)$

Thus $\cos(j \cdot l) = \dfrac{j^2+l^2-s^2}{2|j||l|}$

Now
$$\mu = \left[\frac{j(j+1)+l(l+1)-s(s+1)}{2\sqrt{j(j+1)}} \times g_l + \frac{j(j+1)-s(s+1)-l(l+1)}{2\sqrt{j(j+1)}} \times g_s\right]\mu_N$$

for $s = 1/2$ ~ $I = j = l + 1/2$ (stretch) $\uparrow : l + 1/2$

~ $I = j = l - 1/2$ (Jackknife) cut: $l - 1/2$

knife is -ve so $l - 1/2$

our final answer is in terms of I (or j). Thus write l in terms of I/j and put!

put $s = 1/2$

$$\mu = \left[(j - 1/2)g_l + 1/2\, g_s\right]\mu_N \text{ [stretch case)}$$

$$\mu = \left[\frac{j}{j+1}\left\{(j+3/2)g_l - \frac{g_s}{2}\right\}\right]\mu_N \text{ (jackknife case)}$$

j & I can be inter-changed (Both are same)

↳ Thus we get μ vs I ~ Schmidt lines

proton	neutron
$g_l = 1$	$g_l = 0$
$g_s = 5.58$	$g_s = -3.82$

} putting values, we get μ for odd proton or odd neutron for both stretch & jackknife cases

6

Derivation of $41 A^{-1/3}$ ~ proper DCT pg 366 method ★

$V = \frac{1}{2}kr^2 \rightarrow -\frac{\hbar^2}{2m}\nabla^2\psi + \left(\frac{1}{2}kr^2\right)\psi = E\psi$

Now $\nabla^2 = -\frac{L^2}{\hbar^2 r^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)$ ~ put $R(r) = \frac{u(r)}{r}$:

$-\frac{\hbar^2}{2m}\frac{\partial^2 u}{\partial r^2} + \left(\frac{1}{2}kr^2 + \frac{l(l+1)\hbar^2}{2mr^2}\right)\psi = 0$

solving, we get

$E_n = (2n + l - 1/2)\hbar\omega$

$\Lambda = 2n + l - 2$

Now $2(2l+1)$ values per state

eg 1s ~ 2 number

1p ~ 6 number

using all combination of (n, l) for given eigenvalue E_Λ for given Λ

just like H-problem ★

number ← $N_\Lambda = (\Lambda+1)(\Lambda+2)$ of neutron or protons for a given Λ

Total max particles upto Λ_{max}

$$N_{max} = \Sigma N_\Lambda = \frac{1}{3}(\Lambda_m+1)(\Lambda_m+2)(\Lambda_m+3)$$

$$N_{max} \approx \frac{1}{3}(\Lambda_m+2)^3$$

similar

Now $\langle \frac{1}{2}m\omega^2 r^2\rangle = \frac{1}{2}(\Lambda + \frac{3}{2})\hbar\omega$ take this extra!

$$\langle r^2\rangle = \frac{\hbar}{m\omega}(\Lambda + 3/2)$$

Now $\langle R^2\rangle = \dfrac{2}{A}\sum_{\Lambda=0}^{\Lambda=\Lambda_{max}} N_\Lambda \langle r^2\rangle$

2 is added for both neutron & proton (since $A = 2N_{max}$) (we want $\frac{1}{N}\Sigma N_\Lambda \ldots$)

total nucleon

$$= \frac{2}{A}\Sigma(\Lambda+1)(\Lambda+2)\frac{\hbar}{m\omega}\langle\Lambda + 3/2\rangle$$

$$\langle R^2\rangle = \frac{2}{A}\frac{\hbar}{m\omega}\frac{1}{4}(\Lambda_m+2)^4 \quad \text{(imp)}$$

Thus $\hbar\omega = \dfrac{1}{A}\cdot\dfrac{\hbar^2}{m\langle R^2\rangle}\cdot\dfrac{1}{2}\langle\Lambda_m+2\rangle^4$ — ①

but we need to take $2/A$ because later $\Lambda+2 = (\frac{3}{2}A)^{1/3}$ '2' won't get cancelled!

Now $A = 2N_{max} = 2 \times \frac{1}{3}(\Lambda_m+2)^3$ (since $N_{max} \approx \frac{1}{3}(\Lambda+2)^3$)

Thus ~~$\hbar\omega = \ldots$~~

put $(\Lambda+2) = \left(\frac{3}{2}A\right)^{1/3}$ in ①

also $\langle R\rangle^2 = \frac{3}{5}R_0^2 A^{2/3}$ add this

Thus $\hbar\omega = \dfrac{1}{A}\cdot\dfrac{\hbar}{m \times \frac{3}{5}R_0^2A^{2/3}} \times \dfrac{1}{2}\times\left(\dfrac{3A}{2}\right)^{4/3}$

$$= \frac{1}{A}\cdot\frac{\hbar}{m}\cdot\frac{5}{3}\cdot\frac{1}{R_0^2A^{2/3}}\cdot\frac{1}{2}\cdot\frac{3A}{2}\cdot\left(\frac{3A}{2}\right)^{1/3}$$

$$= \frac{5}{4}\left(\frac{3}{2}\right)^{1/3}\frac{\hbar^2}{mR_0^2}A^{-1/3}$$

$$\hbar\omega \approx 41\,A^{-1/3}\ \text{MeV}$$ ★

) no need for math, just put final result

pending — nn & pp scattering in DCT ~ it has been asked now in 2022 and thus based on every year precedent of being repeated next year, the topic must be done by me from DCT read-around those pages only. V.V properly.

critical Mass → smallest amount of fissile material for a sustained nuclear chain reaction ($k=1$)
└ depends on: density, shape, enrichment, purity, temperature, nuclear properties

critical size – min size of nuclear reactor core & reflector that $k=1$ (atleast 1 neutron is positively left behind)

Subcritical : $k<1$ → Nuclear fission not possible if fissionable mass < critical mass
Supercritical: $k>1$ – $m >$ critical mass

[2002 P(2) Q2(c)]

classical particle is analogous to wave packet.
since $v_g = v$, etc...

construction of wave packet is superposition of plane waves:

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) e^{i(kx-\omega t)} dk$$ (general form of wave packet as a superposition of plane waves)

we can find $A(k)$ which is amplitude / coefficients of the linear superpositions of the plane wave solutions using Inverse Fourier.

$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x,0) e^{-ikx} dx$$

To get $v_g = \frac{d\omega}{dk}$ ~ take 2 plane waves (ω, k) & $(\omega+\Delta\omega, k+\Delta k)$

Invariance of Maxwell eqn – no need for $\frac{\partial F}{\partial \lambda}$ wala
└ $\Box^2 A_\mu = -\mu_0 J_\mu$ (I checked internet also & physics Galaxy)
(wala simple proof
this is also eqn of maxwell eqn coupled in term of scalar & vector potential)

[P40] $A^3=1$ proof. (Q4 TG – in my physics stuff folder)

$A|\psi\rangle = \alpha|\psi\rangle$

$A^3|\psi\rangle = A^2\alpha|\psi\rangle = \alpha A^2|\psi\rangle = \alpha^2 A|\psi\rangle = \alpha^3|\psi\rangle$

Now $\alpha^3 = 1$ ~ $\alpha = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i, 1$

$x^3 = 1$ ~ $(x-1)(x^2+x+1) = 0$

Now Hermitian A → real eigenvalues only
Thus $\alpha = 1$ only.

[2004 P(1) Q2(a)]

Moment of Inertia – $\int x^2 dm$
Product of Inertia – I_{xy}, I_{xz} ... etc (cross terms)

[2010 P(2) Q7(a)]

Nearly free electron theory approximation – Gives rise to Bands
└ same as Bloch theorem & Kronig penny model, etc... all same
$u(x) = u(x+a)$

→ Also add: $\pm\frac{\pi}{a}$, Bragg reflection at Boundary of Brillouin Zone
$k_x = k_y = \pm\pi/a$

deadbeat Galvanometer
→ overdamped – quickly comes to rest
→ used to measure small currents

Ballistic Galvanometer
→ underdamped – oscillatory motion
→ used to measure total charge over long period.

Now use: $I\alpha = C\theta$ ~ $\alpha = \ddot{\theta} = \frac{C}{I}\theta$ (restoring couple) — MoI, coupling const

(from DSM) also add mechanical damping $\gamma\dot{\theta}$ & EM damping → $\frac{\Lambda}{R}\dot{\theta}$ ~ resistance

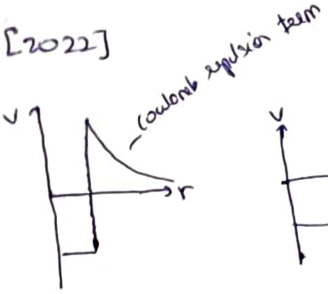
Thus $\frac{d^2\theta}{dt^2} + \left(\frac{\gamma + \Lambda/R}{I}\right)\dot{\theta} + \frac{C}{I}\theta = 0$ ~ for $k>\omega_0$ – deadbeat Galvanometer
for $k<\omega_0$ – Ballistic Galvanometer

[2012 P(1) Q7(c)]

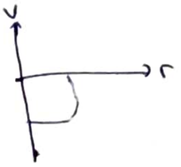
use DIAS T.S. proof: QM → classical
$e^{-\alpha} \ggg 1$ ~ $\left(\frac{V}{N}\right)\left(\frac{2\pi mkT}{h^2}\right)^{3/2} \ggg 1$ (High Temp)

[2019 P(2) Q8(a)(iv)] extrinsic (Ei ≠ EF)
use $n = n_i e^{-\frac{(E_i - E_F)}{kT}}$ or $p = n_i e^{-\frac{E_F - E_i}{kT}}$
where $pn = n_i^2$ and $E_F > E_i$ ~ $n > n_i$ ~ N-type.

[2022]



p-p scattering

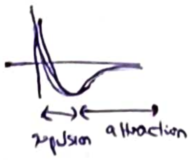


p-n scattering

why no bound state for n-n :

- pauli exclusion prohibits 2 identical Nucleons in same spin state
- also there is lack of EM force to counteract their mutual repulsion

for 2 atoms :



(there is short-range repulsion to prevent both from collapsing)

- Info on Nucleon-Nucleon Force:

similarly Nuclear Force has combination of Attractive & repulsive components

If strong Force were only attractive, then Nucleus would collapse. when Nucleons get too close : there is repulsive force.

Thus strong force / Nucleon-Nucleon force → Both attractive & repulsive at short range distances.

[2019 P(2) Q8(b)]

$I_{ph} = eAG_0[L]$ — active region length

A: Area exposed

G_0: generation rate of e^--hole pairs at surface

(from DIAS(Test 4)2021 Q4(a) Aniket soln)

(extra info) Not reqd here

$L_n = \sqrt{D_n \tau_n}$

L_n: diffusion length; D_n: diffusion coefficient; τ_n: lifetime

and $\frac{D}{\mu} = \frac{kT}{e}$ (einstein reln)

[2002 P(1) Q4(c)(i)]

why MM interferometer require extended source of light (even Fabry-Perot Etalon is extended) — and coherent & monochromatic ★

→ to illuminate larger area for exposing larger field of view (point source will illuminate fringe on a specific point only)

→ Also, amplitude will reduce in subsequent reflection & therefore to ensure high intensity

→ Parallel beam may not produce fringes — an extended source can remedy this problem since interference only occurs for light incident on plate at an angle θ.

[2006 P(1) Q4(a)]

MM proper ray diagram — refer A4 print

[2007 P(1) Q4(a)]

why FP fringes sharper than MM

→ FWHM $\delta\delta = \frac{4}{\sqrt{F}} = \frac{2(1-R)}{\sqrt{R}}$ — at $R \approx 1$ — $F\uparrow$ — transmission resonance and fringe becomes sharp

FP has adjustable Finesse

F: coefficient of Finesse

→ High R.P. since multiple reflections & $R \approx 1$

→ FP involves interference of many reflected beams (MM - only two-beam interference)

(FP: multiple beam AND multiple reflections)

multiple reflections also increases effective path length which gives finer resolution & sharper fringes

[2017 P(1) Q3(a)]

Advantage of Multiple Beam over 2-beam interferometry

Multiple Beam: FP & transmitted light

2-beam: MM & reflected light

- High accuracy / precision due to sharp fringes
- larger resolving power (R.P.)
- High immunity to environmental Noise (MM is more sensitive to envt factors like temperature & vibration)
- multiple beam → wider range of measurement

Given $n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$ and $\omega = \sqrt{2}\,\omega_p$

Now $v_p = \frac{c}{n} = c\sqrt{2}$ $\left(n = \frac{1}{\sqrt{2}}\right)$

Thus $v_g = \frac{c^2}{v_p} = \frac{c}{\sqrt{2}} \ldots$

this $\left(1 - \frac{\omega}{n}\frac{dn}{d\omega}\right)$ cancels, so take $dk/d\omega$ for $\left(1 + \frac{\omega}{n}\frac{dn}{d\omega}\right)$

also $v_g = \frac{d\omega}{dk}$, we know $v_g = \frac{c}{n}\left(1 - \frac{\omega}{n}\frac{dn}{d\omega}\right) = \frac{d\omega}{dk}$

Thus $\frac{dk}{d\omega} = \frac{n}{c}\left(1 + \frac{\omega}{n}\frac{dn}{d\omega}\right)$

Now $v_g = \frac{1}{dk/d\omega} = \frac{c/n}{1 + \frac{\omega}{n}\frac{dn}{d\omega}} = \frac{c/n}{1 + \frac{\omega}{n^2}\frac{\omega_p^2}{\omega^3}}$

since $\frac{dn}{d\omega} = \frac{1}{2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{-1/2} \times \frac{+2\omega_p^2}{\omega^3} = \frac{1}{n}\frac{\omega_p^2}{\omega^3}$

Thus $v_g = \frac{c/n}{1 + \frac{\omega_p^2}{n^2\omega^2}} = \frac{c\sqrt{2}}{1 + \frac{1}{2} \times 2} = c/\sqrt{2}$

[2004 P(2) Q2(c)]

→ use the same steps as done in Griffiths Ex 2.2 for $\psi(x) = Ax(a-x)$

[2018 NPP Q5 Q4]

expression for magnetic moment of Nucleus having one Nucleon outside core ~ schmidt → refer WE in DCT pg 396 Ex 14

for $_8O^{17}$ ~ $Z = 8$ – paired so no use; $N = 9$ ~ unpaired

(since paired Nucleons have no contribution)
(only unpaired Nucleon contribute)

$N = 9$ ~ $(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^1$ <odd Neutron>

Thus $I = 5/2$ ~ $l = 2,\ s = 1/2$
$j = l + 1/2$

sketch case.

$\vec{\mu} = \left[(j \pm \frac{1}{2})g_l + \frac{1}{2}g_s\right]\mu_N$

for $_8O^{17}$ ~ $\vec{\mu} = \frac{1}{2}g_s\mu_N = -1.913\,\mu_N$

odd Neutron!
$g_l = 0$
$g_s = -3.826$ (given in exam)

[2019 Q6](Q4)

Origin of Nuclear magnetic moment
- Intrinsic spin of Neutron & proton (I – angular mom^n)
- Shell & JJ coupling (Nucleon revolve in shells) (A−1 Nucleons create a potential for unpaired Nucleon outside the core)

[2019 Q7] (Q4)

spin & $\vec{T}$ are added vectorally

↳ Three possible states are 0 and 1

$\vec{L}$ – can take any value based on Parity.

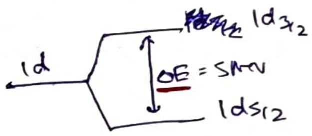
$(-1)^l$ ~~even~~ even parity …
$(-1)^l$ – odd parity …

(eg: deuteron has even parity $I = 1^+$)

[2019 Q4]

(involved ls coupling)

$\Delta E = \left(l + \frac{1}{2}\right)V_{ls}$



Now $\Delta E = 5$MeV given

Thus $V_{ls} = \frac{2}{2l+1}(\Delta E)$ ($l = 2$ for d orbital)
$= \frac{2}{5} \times 5 = 2$MeV

[2022 Q3 Q4] and [2016 Q3 Q4] → Basic assumption of single particle shell model

↳ JJ coupling, LS, revolve around shells, form shells, (A−1) for potential for unpaired outside core, independent particle motion, discrete energy level in shells, Nuclear magic numbers

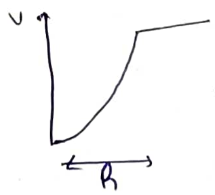
↳ show for (n,l), there exists large degeneracy relative to quantum number (N).

How do the centrifugal and spin-orbit terms remove the degeneracy of 3D spherical harmonic oscillator.

Ans in SBP → (from Pagoa / SBP same)

Harmonic oscillator Shell Model

Let potential due to (A−1) Nucleus be of Harmonic oscillator



$V = \frac{1}{2} M\omega^2 r^2$

Use schrodinger eqn → V (Harmonic)

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \left(\frac{1}{2}m\omega^2 r^2\right)\psi = E\psi$$

Let $\psi = R_{nl}(r)\, Y_{lm}(\theta,\phi)$ (Y → spherical Harmonics)

On solving:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 u}{\partial r^2} + \left(\frac{l(l+1)\hbar^2}{2mr^2} + \frac{1}{2}m\omega^2 r^2\right)u = Eu$$

Harmonic (in H-atom, $V = -\frac{e^2}{4\pi\epsilon_0 r}$)

where $\psi = \frac{u}{r}$, $u = \frac{\psi(r)}{r} = \frac{R(r)}{r}$

here $\frac{l(l+1)\hbar^2}{2mr^2}$ = centrifugal potential

splits into 'l' sublevels, thus removing degeneracy of 'n' quantum no. i.e.

$$E_{nl} = \left(2n+l-\frac{1}{2}\right)\hbar\omega = \left(\Lambda+\frac{3}{2}\right)\hbar\omega \quad (\Lambda = 2n+l-2)$$

Harmonic oscillator energy levels.

directly we reach this answer − $(\Lambda+\frac{3}{2})\hbar\omega$

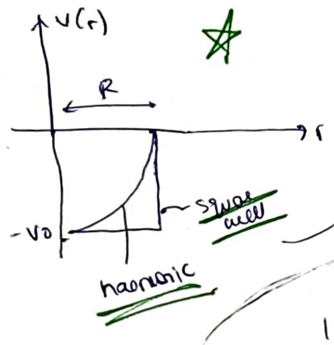
This model can only predict upto magic number 20! and not beyond that

E_N, $N_\Lambda = (\Lambda+1)(\Lambda+2)$

$N = \frac{1}{3}(\Lambda+1)(\Lambda+2)(\Lambda+3)$

(as shown in our table before... 40x)

Basically we can have either $V_{\text{square well}}$ or V_{harmonic}



The spin-orbit coupling inside Nucleus further splits energy levels and as a result potential becomes:

$$V_{nlm}(r) = V_{cent}(r) + V_{ls}(r)(\vec{L}\cdot\vec{S})$$

doublet

$$E_{nl} = \hbar\omega\left(2n+l-\frac{1}{2}\right) - \alpha' \begin{cases} l & I = l+\frac{1}{2} \\ -(l+1) & I = l-\frac{1}{2} \end{cases}$$

This removes the degeneracy of quantum number l by further splitting levels into sub-levels and the splitting increases with increase in l.

This can become so large for a given 'n' that the term with large 'l' can slide down to energies as low as those of degenerate level quantum no. of $(n-1)$

↳ This causes overlap in spode of ply vaala

(basically including spin orbit coupling gives nL_I doublet & correct magic numbers ~ spode of ply dish of ply − 2, 8, 20, 28, 50, 82, 126...)

Now $-\frac{\hbar}{2m}\frac{\partial^2}{\partial r^2}(rR) + \left(\frac{l(l+1)\hbar^2}{2mr^2} + V\right)(rR) = E(rR)$ (since $\psi = \frac{u}{r}$)

here V can be square well (or) Harmonic.

next: w/o spin-orbit coupling, the answer has discrepancies ~ only magic upto 2, 8, 20

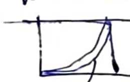
don't put $V = -\frac{e^2}{4\pi\epsilon_0 r}$ that is for H atom & is separate

(after spin-orbit coupling ~ $\Delta E = \left(l+\frac{1}{2}\right)V_{ls}$ & we get nL_I ~ with overlap also possible)

(after spin-orbit is included, we get correct magic number ~ spode of ply vaala)

in both square well & Harmonic potential: we assume single nucleon moving in potential created due to rest of paired Nucleons. This is why the theory is called "Extreme Single Particle Model"

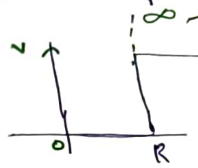
★ The Best form of potential is intermediate b/w harmonic & square well



more realistic

$\frac{1}{1+e^{(r-c)a}} \sim f(r) = \frac{1}{1+e^{(r-c)a}}$ in the middle (Woods-Saxon potential)

Square well potential



assume $V(r) = \begin{cases} 0 & r<R \\ \infty & r>R \end{cases}$

Now $-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left(\frac{l(l+1)\hbar^2}{2mr^2} + V(r)\right)u = Eu$

Now for $r<R$ (inside well)

$V=0$

Thus $-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \frac{l(l+1)\hbar^2}{2mr^2}u = Eu$

eqn of this is soln:

$u(r) = rJ_l(kr)$ ~ spherical Bessel function

Now $u(0)=0$, $u(R)=0$ ~ since wavefunction is at Nodes on the Boundaries. ($V(r)=\infty$)

also each l ~ $-m_l$ to $+m_l$ and $m_s = \pm\frac{1}{2}$

Thus total $(2l+1)\times 2$

$= 2(2l+1)$

$\psi = \frac{u}{r} = \cancel{J_l(kr)}$

$= J_l(kr)$

Now shell enclosures predicted by Square well

this order is IMP

l-state	2(2l+1)	Shell closure
1s ($l=0$)	2	2 ✓
1p ($l=1$)	6	8 ✓
1d ($l=2$)	10	18 ✗
2s ($l=0$)	2	20 ✓
1f ($l=3$)	14	34 ✗

- only 2, 8, 20 correct (exact same for Harmonic) since $\frac{1}{3}(n+1)(n+2)(n+3)$ was from $2(2l+1)$ only!

For Harmonic: $V(r) = \frac{1}{2}m\omega^2 r^2$

For square well $V(r)$ ~ $0, r<R$; $\infty, r>R$

For H-atom V ~ $-\frac{e^2}{4\pi\epsilon_0 r}$

don't put $-\frac{e^2}{4\pi\epsilon_0 r}$ — This is for H-atom Not for Harmonic/square-well Nuclear potential.