CIVIL SERVICES EXAMINATION (MAINS)

PHYSICS PAPER - I: CLASSICAL MECHANICS

TUTORIAL SHEET: 1

(Conservation laws)

1) What is the recoil energy in electron - volts of mass 10^{-23} gm after emission of a r ray of energy of 1 Mev? (1990)

2) Define differential scattering cross-section. Write down the dependence of Rutherford scattering cross-section σ (θ), on the scattering angle θ and sketch this dependence graphically. In the present case the total scattering cross section $\sigma = \int \sigma(\theta)\, d\Omega$ turns out to be infinite. Comment on this result. (1990)

3) A neutron of energy 1 MeV collides with a stationery helium nucleus and is scattered. Deduce the momentum of the neutron and of the helium nucleus in their center of mass system. (1990)

4) Define differential scattering cross section for a scattering process. The differential scattering cross-section for neutrons scattered elastically from a solid is of the form

$\sigma(\theta) = \mathrm{A} e^{-B\left(\vec{K}_i - \vec{K}_f\right)^2}$ where A & B are constants and $\vec{\mathrm{K}}\mathrm{i}$ and $\vec{\mathrm{K}}\mathrm{f}$ are respectively the wave vectors of the incident and scattered neutron Determine the total Scattering cross - sections, given $\left|\vec{\mathrm{K}}\mathrm{i}\right| = \left|\vec{\mathrm{K}}\mathrm{f}\right|$. (1991)

5) Prove that if E and E^1 are respectively the neutron energies in the laboratory system, before and after collision with a nucleus of mass number A, then

$$\frac{E^1}{E} = \frac{1 + A^2 + 2A\,Mc}{(1 + A)^2}$$

Where Mc is the cosine of the scattering angle in the centre of mass system. (1991)

6) What do you mean by centre of mass of a system of particles? Derive expressions for the instantaneous position vector and velocity of the centre of mass of such a system of particles (1992)

7) Using Rutherford's observation that the number of α - particles scattered at angle ϕ and falling on unit area of the screen varied as $\left[\mathrm{cosec}\left(\frac{\phi}{2}\right)\right]^4$, deduce an expression for the probability of scattering between angles ϕ & ϕ + dϕ . (1992)

8) A rocket of mass 1000 kg. is ready for a vertical take off. The exhaust velocity of its fuel is 4.5 km/s. Deduce
(a) The minimum rate of fuel ejection so that the rocket weight be just balanced
(b) The velocity acquired in 8 seconds if the fuel ejection rate is 2.50 kg/s. (You may neglect the effect of changing mass of the rocket in the given conditions). (1993)

9) A radioactive nucleus of mass m_o amu emits alpha particle with kinetic energy E_α. If the disintegration occurs when the nucleus is free, deduce an expression for the energy evolved (E_T) during the disintegration. (1993)

10) Find the fractional decrease of kinetic energy of a of mass m_1 when a head on elastic collision takes place with another particle of mass m_2 initially at rest. In this context show why hydrogen would be best to be used for slowing down. Actually D_2O, not H_2O is used. Why?

(1993)

11) What is centre of mass? Show that there exists only one centre of mass in a system of particles. Discuss the usefulness of centre of mass in studying motion of a system of particles.

(1994)

12) The distance between the centres of Oxygen and Carbon atoms in a CO molecule is $1.2A^0$. Determine the position of the centre of mass of the molecule relative to Carbon. (Assume atomic masses of Carbon and Oxygen as 12 & 16 respectively).

(1995)

13) A particle of mass m, moving with an initial velocity V_0 is acted on by a central repulsive inverse square force, $F = \frac{k}{r^2}$. Show that the Scattering angle θ depends on the impact parameter `b'as

$$\cot\left(\frac{\theta}{2}\right) = \frac{mV_0^2}{k} b \qquad (1990)$$

14) A particle of mass m_1 moving with a velocity V_1 undergoes an elastic collision with a particle of mass m_2 at rest, in laboratory - frame. After the collision the first particle moves at a certain angle to the direction of its initial velocity, and this angle is θ in laboratory-frame and ϕ in centre of mass-frame. If the ratio of masses $\frac{m_2}{m_1}$ is A, show that θ & ϕ are related as $\tan\theta = \frac{\sin\phi}{\cos\phi + \frac{1}{A}}$

(1996)

15) In the NH_3 molecule, the three hydrogen atoms forms an equilateral triangle. The distance between the centre of this triangle from each hydrogen atom is 0.939 A°. The nitrogen atom is at the apex of the pyramid with the three hydrogen atoms forming the base. The distance between the hydrogen nd nitrogen atoms is 1.014 A°. Find the position of the centre of mass relative to the nitrogen atom.

(1997)

The two atoms of a molecule of a gas interact according to the potential $\phi(r) = \frac{A}{r^6} + \frac{B}{r^{12}}$ r being the separation distance between the two atoms. Determine A and B if the potential energy $\phi(r) = \phi(r_0)$ at the equilibrium separation $r = r_0$.

(1998)

16) How do we infer the law of conservation of linear momentum from Newton's laws of motion?
A stationery bomb explodes and on explosion, it fragments into three parts. Two of these parts, which are of equal masses, fly apart perpendicular to each other with a velocity of 60m/s each. The third part has a mass four times the other two. Find the magnitude and the direction of the velocity of the third part

(1999)

17) Consider the motion of a rocket in a gravitational field and derive an expression for its final velocity when the fuel burns at a constant rate till it is fully consumed.

(1999)

Distinguish between elastic and inelastic collision. Find the minimum distance an α - particle with kinetic energy of 0.4 Mev can approach a stationary but free Lithium nucleus in a head on collision.

(1999)

18) The mass of the moon is about 0.13 times the mass of the earth. The distance from the center of the moon to the center of the earth is 60 times the radius of the earth. Taking the earth ' s radius to be 6378 km, find out the distance of the center of mass of the earth-moon system from the center of the earth.

(2002)

19) Ram and Shyam are two skaters weighing 40 kg and 60 kg respectively. Ram traveling at 4 m/s meets Shyam traveling at 2 m/s in opposite direction and collides headon.

a. If they remain in contact, what is there final velocity?

b. How much kinetic energy is lost? (2000)

20) Define scattering cross-section. A charged particle of mass m and charge Ze is scattered by another charged particle of charge Ze at rest. Deduce the expression for the scattering cross-section.

(2000)

21) Using the rocket equation and its integral, find the final velocity of a single stage rocket. Given that (a) the velocity of the escaping gas is 2500 m/s (b) the rate of loss of mass is (m /200)/sec. (where m is the initial mass and 0.27 m is the final mass .)

(2002)

22) Derive the relationship between the impact parameter and the scattering angle for the scattering of an alpha particle of charge + 2e by a nucleus of charge +Ze.

Calculate the impact parameter for an angle of deflection of 30 if the kinetic energy of the alpha particle is 6x10-13 joules.

(2002)

25) If a single stage rocket fired vertically from rest at the earth's surface burns its fuel in a time of 30 sec and their relative velocity v_r=3km Sec^{-1}, what must be the mass ratio **mo/m** for a final velocity is of 8Km /sec? (2004)

26) Considering the scattering of α-particles by the atomic nuclei, find out the Rutherford scattering cross- section. Explain the physical significance of the final expression. (2005)

27) Derive an equation of motion for a variable mass. Explain how it is applied in the motion of a rocket. (2006)

28) What is center of mass? Show that the total linear momentum of a system of particles about the centre of mass is zero.

(2006)

29)

A force field is given by

$$\vec{F} = \left(2xy + z^3\right)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}\,.$$

Is it a conservative field? If so, what is the scalar potential? **(2008)**

TS2 Rotating frame of references

1. What is understood by the term 'Coriolis force'? Obtain expressions for velocity & acceleration of a particle in rotating coordinate system.
(1988)
2. Define Coriolis force and write an expression for it, through suitable examples, explain the way this force varies in different parts of the earth's surface and for different velocities of the concerned particle.
(1989)
3. Write short notes on
(I) Coriolis force (1991, 1993, 1997)
(II) Inertial forces in a rotating frame (1992)
4. Explain what Coriolis force means. Discuss the action of Coriolis force on a body falling freely on the earth at latitude 'λ'
(1994)
5. If the earth were to rotate at 'n' times its present speed of rotation about its axis, the apparent weight of a body at the equator would assume zero value. Find an expression for 'n'.
(1995)
6. Define Coriolis force. Obtain an expression for the Coriolis force. How does it account for the whirling of winds in opposite directions in Northern and southern Hemispheres?
(1996)
7. Obtain the equation of motion of a particle moving relative to a rotating frame of reference Explain the term representing Coriolis force in this expression. (2001)
8. For a freely falling body from the height 'h' on the surface of the earth in the northern hemisphere with a latitude 'θ', show that the deviation of the body towards east at the final stage is given by 1/3 w $\cos\theta(8h^3/g)^{1/2}$, where w is the angular velocity of the earth and 'g' is the acceleration due to gravity.
(2004)
9. Derive the relation $\vec{V} = \vec{V}_0 + \omega \times \vec{r}$, where $\vec{V}$ is the velocity of a particle located at $\vec{r}$ in a fixed frame of reference S and $\vec{V}$ that observed in frame S rotating with angular velocity $\vec{\omega}$ with respect to S but having the common origin. (2007)
10. Show using the above relation that the equation of Motion of the particle in S gets modified in S giving rise to Various fictitious force. Identify the coriolis force and describe its effect on the flow of rivers. (2007)

Tutorial 3 gravitation and central force motion

01) State Kepler's law of planetary motion. Assume the law of gravitation to be of the form $F = \frac{GmM}{r^n}$, where n is some number. Find the value of n which will be consistent with Kepler's third law. For this you may assume planetary orbit to be circular.

2) A satellite moves round the earth at a distance of 3.884×10^5 km from its centre. Find its period of revolution in days. Proceed to deduce the distance of geo-stationary satellite.

3) A body falls from a great height towards the surface of the moon. Write down its equation of motion and solve it to determine the speed with which body strike the surface. Also find its numerical value.

4. Find acceleration of Moon at new Moon/Solar eclipses.

05) A satellite is launched in to a circular orbit close to the surface of the earth. Find an expression for the additional velocity that has to be imparted to the satellite to overcome the gravitational pull.

6) A star of sise of our sun having a radius 7×10^5 km and having a mass 2 times as that of the sun is rotating at a speed of one revolution every 10 days, if the star undergoes a gravitational collapse to a neutron star of rad. 10Km, assuming that the star is a uniform sphere at all times, calculate the

7.find the time period of a satellite rotating at a distance of 20,000 KM from centre of earth, w.r.t. a point on the earth.

8.find velocity of a satellite. Of mass 1.9 *10^27 Kg, at a distance of 70,000 km from centre of body.
9.additional velocity required to escape a body from near the surface of earth

10.Find a of a elliptical orbit, in diagram shown in r. gupta solution

11) A particle is moving under a central force field. obtain the equation of motion and prove that in such a field the areal velocity of the particle remains constant.

14) Neptune's period of revolution is 165 yrs. what is the average radius of neptune's orbit in terms of the sun-earth distance.

15) Two bodies of masses M_1 and M_2 are placed at a distance d apart. Show that at this position where the gravitational field due to them is zero, the potential is given by $V = -\frac{G}{d}\left(M_1 + M_2 + 2\sqrt{M_1 M_2}\right)$ **(2009)**

12. Time period given, mass of sun given, find the distance of aphelion and perihelion.

13. on Central force of Motion
(a) find trajectory, semi latus rectum, e of conic section
(b) prove path of particle lies in a plane.

Tutorial Sheet: 4
Rigid Body Dynamics

DIAS Delhi Institute for Administrative Services

Tutorial Sheet: 4
Rigid Body Dynamics

1. What are precession and nutation? Show that the angular velocity of precession Ω is related to angular momentum $\vec{L}$ and external torque $\vec{T}$ as : $\vec{T} = \vec{L} \times \vec{\Omega}$. Assume $\Omega << \omega$.

(1995)

2. Explain precessional motion of a top. A solid sphere of radius 2 cm and mass 50gm has a thin nail of length 5mm fixed perpendicular to its surface. When this sphere spins like a top with a speed of 20 rev/sec. What will be its precessional speed?

(1995)

3. Show that the angular momentum of a particle located at position r relative to the origin of co-ordinates is given by where $\vec{L} = \vec{r} \times \vec{p}$, where p is the linear momentum of the particle. Using this result prove that the angular momentum of a system of particles can be expressed as the sum of their angular momentum around the center of mass and the angular momentum around the origin of a single particle of mass equal to the total mass of the system located at the center of mass.

(1998)

4. Explain the precession of a spinning top and show that precessional velocity is independent of the angle of inclination. A top is spinning with 20 rev/s about an axis inclined at 30^0 with the vertical. Its radius of gyration is 5 cm. The center of mass is π cm from the pivot point. Calculate the frequency of precession.

(1999)

5. Starting from the definition of angular momentum of a single particle, obtain an expression for the angular momentum of a rotating rigid body. Hence, discuss the time derivative of the angular momentum and deduce the law of conservation of momentum

(1999)

6. Show that the combined effects of translation of the center of mass and rotation about an axis through the center of mass are equivalent to a pure rotation with the same angular speed about an axis through the point of contact of a rolling body.

(2000)

7. Write the Euler's equations for the rotational motion of a right body with one point fixed, under the action of a torque N. Apply these equations to discuss the rotational motion of a symmetrical top on the absence of any force other than the reaction at the fixed point.

(2003)

8. What do you mean by the moments and products of inertia? Show that the angular momentum vector is related to the angular velocity components by linear transformation relations?

(2004)

7) Derive Euler's equations of motion for a rigid body rotating about a fixed point under the action of a torque. When a rigid body is not subjected to any net torque, write down Euler's equations of motion of the body with one point fixed. **(2006)**

8) The angular momentum $\overrightarrow{M}$ of a rigid body comprising of N particles and rotating with angular velocity $\vec{\omega}$ is given by $\overrightarrow{M} = \sum_{k=1}^{N} m_k \vec{r}_k \times \left(\vec{\omega} \times \vec{r}_k\right)$ where the origin coincides with the centre of mass. Express the components of $\overrightarrow{M}$ in terms of components of the inertia tensor. Hence, show that the most general free rotation of a spherical top is a uniform rotation about an axis fixed in space. **(2007)**

9) Derive an expression for the moment of inertia of a rigid body about any axis. What is an "ellipsoid of inertia"? Explain clearly what you mean by the terms "principal axes" and "principal moments of inertia"? Find the moment of inertia of a thin rectangular lamina about an axis passing through the center of the lamina and perpendicular to its plane. Hence determine the moments of inertia about axes passing through the midpoints of its both sides and perpendicular to its plane. **(2008)**

10) Show that for any rigid body consisting of at least three particles not arranged in one straight line, number of independent degrees of freedom is six. Define Euler's angles θ, ϕ and ψ to describe the configuration of such a rigid body. Consider two frames of reference, one fixed to the body and the other to the space defined as $S' = (x', y', z')$ and S = (x, y, z) respectively, Show that the angular momentum $(\vec{L})$ of the rigid body in the two frames are related by $\left(\frac{d\vec{L}}{dt}\right)_S = \left(\frac{d\vec{L}}{dt}\right)_{S'} + \vec{\omega} \times \vec{L}$ Where $\vec{\omega}$ is the angular velocity of rotation. **(2009)**

Mechanics of continuous body. TS5

1. Show that the Bulk modulus K, Young's modulus Y and Poisson's ratio σ are connected by the relation $K = \frac{Y}{3(1-2\sigma)}$ **(2008)**

2. What do you understand by streamline motion and critical velocity of a viscous liquid through a capillary tube. Capillaries of lengths l, $2l$ and $\frac{1}{2}$ are connected in series. Their radii are r, $\frac{r}{2}$ and $\frac{r}{3}$ respectively. If the streamline flow is maintained and the pressure across the first capillary is P_1, deduce the pressures across the second and the third capillaries. **(2008)**

3. Show that the total energy per unit mass of liquid flowing from one point to another without any friction remains constant throughout the displacement. **(2009)**

4. When a sphere of radius r falls down a homogeneous viscous fluid of unlimited extent with the terminal velocity v, the retarding viscous force acting on the sphere depends on the coefficient of viscosity η, the r and its velocity v. Show how Stokes law was arrived at connecting these quantities from the dimensional considerations. **(2010)**

TUTORIAL SHEET: 6
Special Relativity

1. Show from the Lorenz transformation that two events (t_1=t_2) at different points (x_1=x_2) in reference frame S are not in general simultaneous in reference frame S^1 which is moving in the +x direction with the constant velocity V with respect to S. (1998)

2. What is the momentum of a proton having kinetic energy 1 Be V? the energy equivalent to proton rest mass is 0.938 Mev. (1988)

3. Write down Lorenz transformation relations and prove that $x^2+y^2+z^2-c^2t^2$ is invariant under this transformation. (1990)

4. An event occurs at x^1=60m, at t^1=$8x10^{-8}$ s in a reference frame S^1which is moving along the common X or X^1 axis with a speed of 3c/5 with reference to a stationery frame S. The origins of two frames coined at t=0, t^1=0. Deduce the space time coordinates of the event in the frame S. (1990)

5. Prove that addition theorem of velocities in special theory of relativity. Two bodies A&B are moving away in opposite directions, each with a speed of 0.70C with respect to a stationery observer. Deduce the speed of B as measured by A? (1990)

6. The half of μ-meson at rest is $2x10^{-8}$ seconds. Determine the half life of μ-meson while traveling with half the speed of light in vacuum. (1992)

7. Write down the expression for the dependence of mass of a particle on its velocity in special relativity. What will be the speed of a particle if its mass becomes double of its rest mass. (1992)

8. State Lorenz transformation equations. Show that for V/c<<1, the Lorenz transformation equations reduces to Galilean transformation equations. Explain the physical significance of Lorenz transformations. (1993)

   Write short notes on "mass-energy equivalence". (1993)

9. State and explain variation of mass with velocity, hence find an expression for the density of a body in an arbitrary inertial frame of reference. (1994)

10. The density of gold in its proper frame of reference is $19.3x10^3$ kg/m^3. Determine its density in a frame of reference where its velocity is 0.9C, C is the velocity of light in free space. (1995)

11. In the relativistic region, compare the relative increase in velocity with the relative increase in energy of a particle. (1995)

12. Determine the speed of an election, which has kinetic energy 1 Mev. The energy equivalent to the rest mass of an election is 0.51 me V. (1995)

13. The coordinates of an event in an inertial frame S are (25m,o, o, $5x10^{-8}$S). What will be the coordinates of this event in another inertial frame S^1 moving with a velocity 0.6C 0 in +x direction with rest x to s. The origin of two frames coincide at t=t^1=0. (1996)

14. Obtain the relativistic transformation relation for density in inertial frames. What is the equivalent energy corresponding to 1 amu of mass? (1997)

15. A μ meson travels towards the earth's surface from high up in the atmosphere with a speed of 0.99C. It decays after traveling a distance of 6km. In what time does the μ meson decay as measured by observers in reference frames (i) bound to the earth (ii) bound to the meson itself. (1998)

16. How is an election volt connected to other units of energy like the joule or the erg? Determine the space of an election of energy 1.3 Me V assuming its rest energy to be 0.l5 Me V. (1998)

17. Prove that the expression $x^2+y^2+z^2-c^2t^2$ is invariant under Lorenz transformation. (1999)

18. Describe the set-up of Michelson-Morley experiment. Why was a fringe shift expected in it? How are its negative results understood? (1999)

19. What is 'Lorentz Contraction'? A spaceship of rest length 100m takes 4μs to pass an observer on Earth. What is its velocity relative to the Earth? (2000)

20. Derive an expression for the mass-energy equivalence using the principle of special relativity. (2000)

21. The mass of muon at rest is 207 Me where Me is the election rest mass (0.511 Me V). The mean life time at rest for muon is 2.2 μs. The life time of muon emerging from an accelerator is measured in the labat 6.9 μs. Estimate the speed of these muons in the laboratory. (2001)

22. A person is a space ship is holding a rod of length of 0.5 m, the space ship is cruising at a speed V parallel to the earth's surface. What does the person in the space ship notice as the rod is rotated from parallel to perpendicular to the space ship's motion? What does an observer on earth's surface notice? (2001)

23. Two spaceships are moving at a velocity of 0.9c relative to the Earth in opposite directions. What is the speed of one spaceship relative to the other? (2002)

24. An observer A sees two events at the same space point (x=y=z=0) and separated by $t=10^{-6}$s. Another observer B sees them to be separated by $t^1=3x10^{-6}$s. What is the separation in space of the two events as observed by B? What is the speed of B relative to A? (2002)

25. An observer S_1 sees two bodies A and B having equal rest mass approach each other with equal but opposite velocity of the body 4c/5. To a second observer S_2, the body A is at rest. What is the velocity of the body B as seen by observer S_2? What are the kinetic energies of the body B in the frames of S_1 and S_2? (2003)

26. How does Doppler effects of light in relativistic physics qualitatively differ from its non-relativistic analogue? Calculate the Doppler shift in the frequency of a photon traveling along y-axis, with respect to an observer moving along the x-axis with a constant speed u. (2003)

27. A meson of rest mass π comes to rest and disintegrates to a muon of rest mass μ and a neutrino of zero rest mass. Shoe that the kinetic energy of motion of the muon is

$$T=(\pi-\mu)^2 c^2/ 2 \pi$$ (2004)

28. Write down the expression for the relativistic mass of a particle moving with a velocity *v* in terms of its rest mass. Establish from the above expression Einstein's mass energy relation $E=mc^2$. (2004)

29. Show that the length L of an object moving with a velocity υ is given in the direction of motion by $L = L_0(1-\partial^2 / c^2)^{1/2}$,Where L_0 is the proper length and c is the velocity of light is free space. What will be the shape of a spherical ball while moving under relativistic regime?

31. Prove that two successive Lorentz transformations are equivalent to another Lorentz transformation. Hence write down the Einstein's velocity addition relation. (2006)

32. The source Ś moves along the x'-axis at a speed v, and emits light at an angle θ'to the x'-axis of its own frame. In th S-frame the emitting angle with the x-axis is θ. Hence x and x'-axis are coincident. Show that the exact relativistic aberration formula can be derived from the velocity transformation relations. (2006)

$$\tan\theta = \frac{\sin\theta' \sqrt{1 - v^2/c^2}}{\cos\theta' + v/c}$$

33. A body of rest mass m_0 is moving in the positive y-direction at a velocity of 0.6 C relative to the laboratory frame. Calculate the components of the four dimensional momentum vector in the laboratory frame and in the frame of an observer who is travelling in the positive x-direction at a speed of 0.8 C relative to the laboratory frame. (2007, 20 marks)

34. State the Postulates of the Special theory of relativity and based on these obtain Lorentz as well as inverse Lorentz transformations. Hence, obtain an expression to conclude that a moving clock runs more slowly than a stationary clock. (2007, 35 marks)

35. An unstable particle has a lifetime of 5 microseconds in its own frame of reference and is moving towards the earth at a speed of 0.8C.What will be the lifetime of the particle to an observer on the Earth? (2007, 10 marks)

36.

The length of a moving rod can be defined as the product of its velocity and the time interval between the instants that both the end points of the rod pass a fixed mark in S system. Show that this definition leads to the space contraction **(2008)**

37.

A meson of rest mass π comes to rest and disintegrates into a muon of rest mass μ and a neutrino of zero rest mass. Show that the kinetic energy of motion of the muon is $T = \frac{(\pi - \mu)^2 c^2}{2\pi}$ **(2008)**

38.

Show that a four-dimensional volume element dx dy dz dt is invariant to Lorentz transformation. **(2009)**

39.

Obtain the relativistic equation for aberration of light using velocity transformation equations. **(2009)**

ELECTRICITY & MAGNETISM
TUTORIAL SHEET: 14
(Electrostatics)

1. Two point charges each of magnitude +2 mill coulomb are placed at A and B in front of an infinite conducting plane, which is grounded. The line OAB is perpendicular to the plane with the point O on the plane. If OA = 1m and AB = 2m, calculate the force on the charge at A.

   (1989)

2. The components of an electrostatic field in vacuum are give as

   $$Ex = \frac{a}{r^3} + \frac{bx^2}{r^5}$$

   $$Ey = \frac{cxy}{r^5} \qquad Ez = \frac{fxz}{r^5},$$ where a, b, c and f are constants; x, y, z the rectangular Cartesian coordinates and $r^2 = x^2 + y^2 + z^2$. Using the basic equations obeyed by the electrostatic field in Vacuum, find the relations between a, b, c, and f and determine the charge density at a general point in space. Would that explain the observed field?

   (1990)

3. Show that the electric field intensity due to any distribution of charges at rest can be expressed as the gradient of a potential. What is the relation between potential and potential energy?
   A thin disc of radius R is uniformly charged, σ being the charge per unit area. Find the potential and the electric intensity at points on the axis of the disc. How do these change as one crosses the disc? Explain the changes physically. (1991)

4. Two charges are placed at a distance 1 meter. The magnitude of one charge is double that of the second charge. Find the neutral points in the two cases (i) the charges are of the same sign (ii) the charges are of opposite sign. What happens to the neutral point if the two charges are of equal magnitude and opposite sign?

   (1992)

5. State Coulomb's law and show that the electric field can be derived from a potential function. If the charge distribution is continuous, find the integral formula for determining its field.

   (1992)

6. Using Laplace's equation obtains an expression for the potential between two coaxial cylinders.

   (1994)

7. An electric dipole is placed in an external electric field. Find the interaction energy between the electric dipole and the field. And hence find the force and the torque acting on the dipole

   (1995)

8. An electric dipole moment $\vec{p}_2$ is placed at (r, θ) in the electric field of another dipole moment $\vec{p}_1$, placed at the origin. Assuming that the electric potential, V, at (r,θ) due to P_1 is $V(r,\theta) = \frac{p_1 \cos\theta}{4\pi\varepsilon_0 r^2}$, find the dipole – dipole interaction energy.

   (1996)

9. Using Gauss law find the electric field inside a cylindrical capacitor and hence derive the expression for its capacitance. Find the dielectric constant of the material inside a 50mm long capacitor of capacitance 40 μF having inner conductor of radius 1mm, outer conductor of radius 10mm. Which of the materials has such a value of the dielectric constant? (1997)

10. Determine the energy of attraction an electric dipole and a plane Conducing Surface at zero potential.

    (1998)

11. A potential field is given by

$$\phi = (x^2 + y^2 + z^2) \text{ volt}$$

Find the electric field at a point (x,y,z) and the charge density in the region. (1999)

12. Calculate the electric field as a function of position due to a dipole whose potential is $\cos\theta/4\pi r^2$ where $r=\sqrt{x^2+y^2}$. The dipole is at the origin of x,y system. (2001)

13. A Conducting sphere of radius a is placed in uniform Eo. Using the method of images show that potential is given by $\phi Eo\left(r - \frac{a^3}{r^2}\right)\cos\theta$ (2001)

14. Calculate the electric field for a point on the axis of a uniform ring of charge 'q' and radius 'a'. Show that the maximum value occurs at $x = \pm a/2$.

(2002)

15. Show that the potential energy of a charge Q uniformly distributed through the sphere of radius R is given by $p=3/5\ Q^2/4\pi\varepsilon_0 R$

(2002)

16. A Geiger tube consists of a wire of radius 0.2 mm and length 12cm and a coaxial metallic cylinder of radius 1.5cm and length 12cm. Find
    i. The capacitance of the system and
    ii. The change per unit length of the wire when the potential difference between the wire & the cylinder is 1.2 kv. (Assume the dielectric constant of the gas in the tube is 1).

(2002)

17. A cylinder of length L and radius b has its axis coincident with z-axis. The electric field in the region is $E = 100\ \vec{k}$. Find the electric flux through:
    i. The top circular end
    ii. The curved wall of the cylinder
    iii. The closed surface of sphere. (2003)

18. Consider an infinite grounded conducting plane .If a point charge is held at a distance d from the plane, compute by method of images the electric potential above the plane & the induced charges on the conductor. (2003)
19. What is the volume density of the charge in a region of space, where the electrostatic potential is given by $V= a - b\,(x^2+y^2) - c \ln (x^2+y^2)$ where a, b, c are constants. (2004)
20. A point charge q is held at a distance d in the front of an infinite grounded conducting plane. What is the electric potential in front of the plane? (2004)
21. Derive approximate expressions for the potential and the radial as well as the azimuthal components of the field due to an electric dipole at points far away from it. Also derive expression and hence describe the effect of a uniform electric field on a dipole which can rotate freely. (2005)
22. What is molecular polarizability? Derive Clausius - Mosotti equation relating the molecular polarizability with the dielectric constant of a dielectric material. (2006)
23. Starting from Maxwell's equation, $\nabla\ D = \rho$, where D is the electric displacement density and ρ is the charge density, derive Poisson's equation. Deduce Laplace's equation for charge-free region from Poisson's equation. (2006)
24. A potential in cylindrical coordinates is a function of r and ø but not of z. Obtain the separated differential equations for R and Φ, where $V= R(r)\ \Phi\ (ø)$ and solve them. (2006)
25. Derive poisson equation starting from the Coulomb's law for a set of point charges. (2007, 20 marks)
26. Obtain the solution of the Laplace equation in cylindrical coordinates. (2007, 20 marks)

27.

State Gauss's Law of electrostatics both in integral form and differential forms. Two charged spheres of radius R each, have their centers a distance d apart such that d < 2R. One of the spheres has a uniform positive charge density ρ per unit volume while the other has opposite charge density - ρ . Show that the electric field in the region of overlap between two spheres is uniform. **(2008)**

28.

A point charge +q is held above a grounded conducting plane located at z = 0. If the position of the charge is (0, 0, d) obtain an expression for the induced charge density on the plane as a function of coordinates x and y. **(2008)**

29.

A thin dielectric cylindrical rod of cross-section A is situated along z-axis. The polarization in the rod is along its length and it is given by $\vec{P} = (2z^2 + 5)\hat{z}$. Calculate bound volume charge density at each end of the rod. **(2009)**

30.

For an arbitrary localized charged distribution, obtain an expression of electrostatic potential V in terms of multipole expansion. **(2009)**

31.

Obtain Poisson's equation in electro-statics from Gauss' law. What from does it takes the charge density is zero? **(2010)**

32.

What is meant by a dielectric? Define polarization vector P and relate it with the average molecular dipole moment. Obtain expression for the potential due to a polarized dielectric in terms of the polarization vector. **(2010)**

TUTORIAL SHEET: 15
(Biot-Savart's law & applications)

1. A wire of length 1 is bent into the form of a rectangle of sides a and b and carries a current I. Calculate the magnetic field intensity at the centre of the rectangle. Show further that the intensity is minimum for a = b
(1989)
2. Starting from Biot-Savart's law, calculate the magnetic field at the centre of a solenoid of length 1 metre, radius 2 cm and having 25 turns per cm., the current through the solenoid being 1 Ampere.
(1990)
3. A charged particle moving horizontally towards the east with a velocity of 10^5 m/sec enters into a region where there is a horizontal electric field E of intensity 100 volt/cm directed towards the north as also a magnetic field B. The particle continues to move in the same direction as before what can you say about the field B? Is it completely determined? What will be the path of the particle if the magnetic field be switched off?
(1990)
4. State Biot-Savart's law. Derive an expression for the magnetic field at points along the axis of a circular coil.
(1992)
5. Obtain expressions for electrostatic scalar potential, magneto static Vector potential and electromagnetic potential. How are the first two related to the third? Explain the ideas of retarded potential.
(1994)
6. Starting from Biot-Savart's law, find vector potential at a distance r, from a current carrying wire.
(1996)
7. Calculate the Magnetic moment of a coil having 100 turns, each of area $10 cm^2$ carrying a current of 1 mA, flowing for 1 milli second. The coil is placed in a magnetic field of 0.5 wb, which is perpendicular to the magnetic moment. Calculate the torque acting on the coil and the angular momentum transferred to it.
(1997)
8. Current is flowing in a single-turn circular coil of radius 0.1m such that at a point 5mm from the centre of the coil on the axis perpendicular to the plane of the coil the magnetic field is 0.1 mT. Find the current in the coil. Hence calculate the magnetic moment of the current carrying coil. (2000)
9. Consider two long and straight current carrying wires placed parallel to each other a certain distance apart. Derive an expression for the force per unit length experienced by these wires. Discuss that the attractive (repulsive) nature of this force is related to the directions of flow of currents in the two wires.
(2005)
10. Find the magnetic field $\vec{B}$ at the point P due to a short straight length of wire carrying current 'i'. Length of the wire is l. Point P is at a distance r away from the center of the wire. Angle between $\vec{l}$ and $\vec{r}$ is θ.
(2006)
11. Write Ampere's circuital law, and obtain a generalized form of this law, for non-stationary case. (2006)
12. State the Ampere's law of magnetostatics.Using this law, find the magnetic field at a point due to an infinitely long filamentary current. (2007)
13. What is a magnetic shell? Define the strength of a magnetic shell. A 2mm thick magnetic shell weighing 100 gm has magnetic moment of 1000 units. The density of the shell material is 10gm/c.c.Calculate the intensity of magnetization and the strength of the shell. (2007)
14. Find the vector potential due to a line segment from $x=a$ to $x=b$ carrying a current I at a point P which is at a distance d from the line segment. **(2008)**

15.

Show that the interaction energy of two magnetic dipoles $\vec{m_1}$ and $\vec{m_2}$ separated by a displacement $\vec{r}$ is given by $U = \frac{\mu_0}{4\pi} \cdot \frac{1}{r^3}\left[\vec{m}_1 . \vec{m}_2 - 3\left(\vec{m}_1 . \hat{r}\right)\left(\vec{m}_2 . \hat{r}\right)\right]$ If two magnetic dipoles are held at a fixed distance apart, but allowed to rotate freely, what would be the configuration for stable equilibrium? **(2008)**

16.

State Biot-Savart law. Calculate the magnitude of axial magnetic induction due to a circular loop of area A carrying current I. **(2009)**

TUTORIAL SHEET: 16
(EMI & A.C.)

A 220 volt 50 cycle AC supply is connected to a circuit containing a resistance of 20 ohms, in series with a 100μF Capacitor. Determine the current and the phase.

(1988)

An electromotive force E_0 Sin pt + E_1 Sin 2pt is impressed on a circuit containing an inductor and a resistor. Set up the differential equation obeyed by the current and show that in the steady state the current comprises two sinusoidal terms. Calculate the average power dissipated in the circuit. What is the difference, if any, between a varying current and an alternating current?

(1989)

A circular coil of wire having 100 turns and radius 10cm is rotating about a Vertical axis in its own plane uniformly at the rate of 480 revolutions per minute. There is a horizontal Magnetic field of intensity 0.01 wb/m^2. The terminals of the coil are connected to the ends of an inductor having inductance 0.01 Henry. Assuming that the resistance in the circuit can be neglected. Find the current in the circuit at the instant the plane of the rotating coil is perpendicular to the magnetic field.

(1990)

1. Set up the equation for the discharge of a capacitor C connected in series with a resistor R and an inductor L. If R_0 stands for $2\sqrt{\frac{L}{C}}$, discuss three cases (I) $R<R_0$. What will you observe if the discharge takes place at a low temperature when the material of resistor has become super conducting?

(1991)

2. The total energy U in an oscillating L-C circuit is given by U = U_B + U_E=$\frac{1}{2}LI^2 + \frac{q^2}{C}$, when the resistance of the circuit is zero. From this show that it is an oscillatory circuit and find the time period.

(1992)

3. If steady voltage is applied to an L-R circuit, show how voltage across the inductance and the current in the circuit changes with time. Explain the term inductive time constant.

(1992)

4. A harmonic e.m.f is applied to a series circuit, containing resistance, inductance and capacitance. Derive the expression for the current and condition for resonance.

(1993)

5. A series circuit consisting of 4.10 ohms of resistance, 810 μ H of inductance and 225 μμF of capacitance is excited by a constant voltage amplitude generator of variable frequency. At what frequency is the maximum power delivered?

(1993)

6. A harmonic e.m.f is applied to a parallel resonant circuit, containing resistance R, inductance L and Capacitance C. Derive expressions for resonance, angular frequency w_0 and the bandwidth Δw.

(1995)

7. Calculate w_0, Δw and quality factor Q for LCR parallel resonant circuit given the values:
$C = 0.4\mu F$, $L = 4$ mH, and $R = 1\ K\Omega$

(1994)

8. Show that the energy stored in a Capacitor is $\frac{q^2}{2C}$ while the energy stored in the inductor is $\frac{1}{2}LI^2$, where the symbols have their usual meanings. (1994)

9. An alternating voltage is applied to CR circuit connected in series. Under what conditions it acts as an (i) integrator and (ii) a differentiator?

(1995)

10. An alternating voltage of varying frequency is applied across a two-branch parallel circuit of R_1 and L in series and R_2 and C in series as shown in Fig. Below:
Find its resonant frequency. If R_1 and R_2 are not zero, state the conditions when the resonant frequency is given by $f_0 = \frac{1}{2\pi\sqrt{LC}}$

(1995)

11. Self-inductance of two coils, A and B, connected in series is 25 m H or 10mH depending on the relative current directions in the coils. Self Inductance of A is 10mH calculate mutual inductance M of the pair of coils, coupling factor and leakage factor. If the current in coils is changing at the rate of 1000 A/S, find the induced electromotive forces across the coil A.

(1996)

12. An alternating voltage is applied to the circuit shown in Figure. Show that it has two resonant frequencies f_p and f_g inter-related as

$$f_p = fs\sqrt{\left(1+\frac{1}{r}\right)}; \qquad \text{where} \quad r = \frac{C_b}{C_a}$$ (1996)

Q. A relay has a coil resistance of 10Ω and inductance of 1H. It is energized by a single voltage pulse of 10V, which remains constant for 250 ms and then falls to 0V. Relay contact closes when the increasing current reaches 200 mA and opens when the decreasing current is at 100 mA. Calculate the time for which the relay contact is closed.

(1996)

13. The Self-inductance of primary and secondary coils of a R.F transformer is 10mH each. When the coils are connected in series, self-resonating frequencies are 132.6 KHZ and 108.3 KHZ. Calculate the mutual inductance between the coils & the winding capacitance.

(1997)

14. Define quality factor for an A.C. circuit and discuss the meaning of electrical resonance in a series LCR circuit. Explain the term sharpness of resonance

(1998)

15.Two coils are connected in series and their total self-inductance is 4.40 mH. When current is reversed then total self-inductance is 1.60 mH. All the flux due to the first coil links the second coil, but only 40% of the flux due to the second coil links the first coil. Find the self-inductance of each of the coils and their mutual inductance.

(1998)

16. A potential difference with a frequency of 50 cycles per second is applied to a coil of resistance 1 K ohms and inductance 2H. Calculate the power factor of the circuit.

(1998)

17. An L – C – R circuit has a resistance of 100 ohms, a capacitance of 0.2 μF and an inductance of 5 H. An a.c. source E = 50 Sin 1000t Volt is connected in the circuit. Calculate the average power dissipated.

(1999)

18. In a series L – C – R circuit connected to an alternating constant voltage source the current amplitude is $\left(\frac{1}{n}\right)$ times the amplitude at resonance at frequency w_1 and w_2. Obtain an expression for its quality factor at resonant frequency. (1999)

19. Two inductance coils having inductances L_1 and L_2 and negligible resistances are connected in parallel. The coils have a mutual inductance M. Obtain an expression for the effective inductance of the combination.

(1999)

20. For an *R-L-C* series resonant circuit, show that $f_0 = \sqrt{f_1 f_2}$, where f_0 is the resonance frequency and f_1 and f_2 are the half-power frequencies. Is this relation true for a parallel resonance circuit in which *R*, *L* and *C* are connected in parallel to each other? Explain.

(2000)

21. A series *R-L* circuit has a constant d.c voltage *V* applied at the time t = 0 by closing a switch

(a) Derive an expression for the current in the circuit
(b) Calculate the voltage drops across *R* and across *L*.
(c) At what time are these voltage drops equal?
(d) Find the power dissipated by *R* and power stored in *L*.
(e) Show that the steady-state energy is stored in the magnetic field.

22.What value of inductance has to be used so that a lamp with rating of 200 volt & 10 amperes lights the same way with 250 V source at 50Hz.

(2001)

23.An inductance is connected to a 6volt battery through a resistance R. What is the steady state current in the circuit? After what time the battery would be delivering one half its steady state current?

(2001)

24.Explain the use of a parallel resonance circuit: 1) as a rejecter circuit 2) for current amplification.

(2001)

25. A series LCR circuit with L=2H, C=2 μF and R=20 is powered by a source of 100 volts and variable frequency. Find the resonance frequency fo The value of Q

i. The width of resonance Δf and
ii. The maximum current at resonance.

(2002)

26.A bridge network with resistance capacitance and inductance is given in the above figure. Show that the condition for balancing the bridge is independent of the frequency of applied voltage.

(2003)

In an a.c. circuit a resistance (R= 100Ω) and a capacitance (C= 1 μ F) in series are connected to an a.c. source V= 200 sin (100 π t). Calculate the current through the circuit and voltages across R and C. Draw a vector diagram representing the magnitudes and phases of the voltages. (2007)

27.

State Faraday's Law of induction in terms of emf and magnetic flux. Use Stoke's theorem to express the law in differential form. A uniform, time varying magnetic field $\vec{B} = \vec{B}_0 (1+\alpha t)\hat{k}$ fills a circular region of radius R lying in the $x-y$ plane with the centre at origin. Here $\vec{B}_0$ and α are appropriately dimensioned constants. Find the induced electric field at a distance from the centre where r <R. **(2008)**

28.

A long solenoid of radius R and n turns per unit length carries a sinusoidal current $I = I_0$ cos ωt . Determine the magnitude of induced electric field (E) outside the solenoid. **(2009)**

29.

A series R-L-C circuit is connected across a voltage source V = 100 sin 300t. If R = 500Ω, L = 1 H and C = 2 μF, calculate the average power delivered to the circuit. **(2009)**

30. Discuss the growth of current when an e.m.f. is suddenly applied to a circuit containing resistance, inductance and capacitance in series. What is the time constant of the circuit? **(2010)**

31. A series circuit has an inductance of 200 micro henries, a capacitance of 0·0005 microfarad and a resistance of 10 ohms. Find the resonant frequency and quality factor of the circuit. **(2010)**

TUTORIAL SHEET: 17
(Kirchoff’s laws & applications)

1. State Kirchoff’s laws of the distribution of currents (a) in the usual form for steady currents and (b) in a form applicable to alternating current networks. Discuss the method for comparing inductances by using Maxwell’s bridge. State the disadvantages of the method. (1998)
2. An inductance in connected to 6 volt battery through a resistance R. What is the steady state current in the circuit? After what time the battery would be delivering one half its stead state current? (2001)
3. Using Kirchoff’ laws find currents in each branch of the circuit shown in the following diagram.

(2002)

4. A network PQRS is connected as shown in the figure below. Apply Kirchoff’s law and show that the current flowing through the 20 Ω resistor PR is 0.029 A.

(2004)

TUTORIAL SHEET: 18
(Electromagnetic Waves)

1. Derive an expression for the Poynting's Vector and explain its significance. A light source (1 Kilowatt) is radiating energy uniformly. Determine the intensity of the electric field at a distance of one meter from the source.

(1988)

2. Write down Maxwell's equations in vacuum. Show that in the source-free case, both the electric and magnetic vectors obey wave equations of identical form and considering a plane wave solution, prove that the electric and magnetic vectors and the direction of propagation are mutually perpendicular. Show further that there is a propagation of energy given by the poynting Vector

(1989)

3. In a certain region of space in vacuum, the components of the magnetic induction are (in wb/m^2)
   i. $Bx = A\ e^{-ay} + bx$
   ii. $By = A\ e^{-ax} + cy$
   iii. $Bz = 0$

   Where x, y, z are in metres and A, a, b, c are constants. Find the relation, if any, between these constants and also the current distribution that gives rise to this field. (No electric field is present). Is the current distribution consistent with the charge conservation principle?

(1991)

4. Write down Maxwell's equations for an isotropic homogenous dielectric and point out their relations with observational laws.
   How do these equations lead to the concept of electro-magnetic waves?

(1992)

5. Give the four basic experimental laws of electro magnetic in words and in mathematical forms. Explain how they are modified to obtain the Maxwell's equations. Show how these, equations lead to prediction about the speed at which e.m waves are propagated in vacuum.

(1993)

6. What is Pointing Vector? Explain how the transport of electro magnetic energy is expressed quantitatively by Poynting Vector?

(1993)

7. Show that the electro magnetic waves in free space are transverse in nature.

(1994,1996)

8. State and prove Poynting's theorem. Discuss the physical significance of each term in the resulting equation.

(1994)

   If the average distance between the sun and earth is 1.5×10^{11} meter, find the average solar energy flux on the earth (solar constant) given the power radiated by the sun = 3.8×10^{26} watt.

(1994)

9. Show that the Ampere's circuital law fails for time varying currents. How has the difficulty been overcome by Maxwell by introducing the concept of displacement current density? How is the expression for $\vec{\nabla}\times\vec{B}$ modified for magnetized material magnetization vector $\vec{M}$?

(1995)

10. Starting from Maxwell equations, obtain, differential equations for scalar and vector potentials. Solve the equation for scalar potential in a dielectric medium having source of charge density ρ.

(1995)

11. Show that attenuation constant α and phase fact β for an electromagnetic wave, propagating in a perfectly conducting medium are equal. At what frequency the skin depth in silver is 1 μm, when the conductivity of silver is 30×10^6 siemens/meter and relative permeability is 1.0?

(1996)

12. Write down Maxwell's equations in free space and hence show that the phase velocity of the electromagnetic wave is equal to the velocity of light in free space.

(1997)

13. Obtain the characteristic impedance of the vacuum. Derive the expression used.

(1997)

14. Derive the wave equations for $\vec{E}$ and $\vec{B}$ and solve one of these for plane wave propagation in an unbounded, homogeneous dielectric medium. Further show that in a plane wave $\left(\vec{E}, \vec{B}, \vec{K}\right)$ form a mutually orthogonal right handed system.

(1998)

15. A traveling electromagnetic wave is described by the equation E_x (Z, t) = 0.5 cos. (20t–2Z). Determine (i) speed of the wave, V (ii) wave length, λ (iii) Time period, T (iv) Direction of propogation.

(1998)

16. The electric field vector of a plane electromagnetic wave is given by

$$\vec{E} = E_0 \cos\left(kz - wt + \hat{\delta}\right)\hat{x}$$

Write the magnetic field vector. Calculate the average energy per unit volume stored in electromagnetic field and the average energy flux density.

(1999)

17. Calculate the peak values of $\vec{E}$ and $\vec{B}$ for a laser beam of power 2×10^{10} watt and radius 0.1mm.

(1999)

18. 'The introduction of the displacement current is one of the major contributions of Maxwell'. Discuss.

(2000)

19. Using Maxwell's equations, derive the electromagnetic wave equations for a conducting medium and solve it.

(2000)

20. What is the limiting case of a metallic conductor in the above case? Explain.

(2000)

21. What is gauge transformation? Define coulomb gauge. Derive the equation for vector Potential under coulomb gauge.

(2001)

22. Define scalar and vector potentials. Recast Maxwell's equations in terms of these potentials.

(2001)

23. Derives the energy continuity equation for electromagnets waves using the pouting vector.

(2001)

24. Why did Maxwell have to introduce the idea of displacement current? Derive the wave equation from Maxwell's equations. Obtain Fresnel's formula for reflection and transmission coefficients of the electric field vector where it is perpendicular to the plane of incidence.

(2002)

25. What are vector and scalar potentials for the electro magnetic field? Are they unique? Explain what are coulomb's Lorentz gauges. Derive the electromagnetic wave equation in Lorentz gauge and show that it is equivalent to Maxwell's equation.

(2002)

26. fine pointing vector and explain its significance. The electric field vector for an electromagnetic field travelling in vacuum is given by $\vec{E} = E_0 \cos(kz - wt)\ \hat{L}$

Calculate the poynting vector for the wave and show that its magnitude is equal to the energy density of the wave time the velocity of light.

(2003)

27. A plane wave of frequency ω travels into two linear dielectric media. It has a normal incidence at the interface of the media. Giving appropriate boundary conditions, obtain expressions for the intensities of reflected and transmitted rays. (2003)

What are the Vector and scalar potentials? Derive Maxwell's equation in terms of these potentials.

(2004)

30. Show that the pointing Vector $S = \vec{E} \times \vec{H}$ represents the energy flow per unit time both in magnitude and direction in case of a plane electromagnetic wave. (2004)

31. Show that the electric and magnetic field vectors, $\vec{E}$ and $\vec{B}$, plane electromagnetic waves are mutually perpendicular in a plane normal to the direction of propagation. How are phases of $\vec{E}$ and $\vec{B}$ related to each other? (2005)

32. Write down the macroscopic from of the Maxwell's equation in any isotropic (but inhomogeneous) medium and define the symbols appearing therein. Convert these equations in the integral forms to highlight the laws represented by these equations. (2005)

33. Describe physical significance of the displacement current considering the example of current flow through a capacitor. (2005)

34. What do you mean by a gauge transformation? What is its importance? Show that the Lorentz gauge condition $\nabla \cdot \vec{A} + \frac{1}{c^2} \cdot \frac{\partial \Phi}{\partial t} = 0$ is Lorentz invariant. Here $\vec{A}$ and Φ are the vector and scalar potentials. (2006)

Starting from Maxwell's equations of electromagnetic field in vacuum obtain the classical wave equations for the four field vectors $\vec{E}$, $\vec{D}$, $\vec{B}$ and $\vec{H}$. Show that the field vectors can be propagated as waves in free space with the velocity of propagation equal to 3×10^8 m/s, where for free space we have the vacuum permittivity $\varepsilon_0 = 8.854 \times 10^{-12}$ farad/m and vacuum permeability $\mu 0 = 1.257 \times 10^{-6}$ henry/m. (2006)

35. Write down the different components of the electromagnetic field tensors $F_{\mu\nu}$ and further prove that Maxwell's equations of electrodynamics are invariant to Lorentz transformations. (2006)

36. Explain the Rayleigh Scattering of light. Show that the energy density of light scattered from an isotropic homogeneous medium of a gas is inversely proportional to the fourth power of wavelength of the incident light. (2007, 35 marks)

38.

(a) A plane electromagnetic wave is traveling in vacuum along a direction which makes an angle of 45^0 each with the positive x and z-axes. The electric field at time t is given by the expression $\vec{E} = E_0\left(2\cos\omega t\hat{j} - \sin\omega t(\hat{i} - \hat{k})\right)$. Find out magnetic field and Poynting vector.

(b) Explain how Maxwell modified Ampere's law of magnetostatics by introducing the concept of displacement current. How does it resolve the paradox of a charging capacitor?

(c) A plane electromagnetic wave traveling in z-direction and polarized in x-direction is incident normally from left on to an interface between two media at z = 0. The medium to the left (z < 0) has a refractive index n_1, while that to the right is of refractive index n_2. The magnetic penne abilities of both the media are equal and $\mu_1 = \mu_2 = \mu_0$. Obtain expressions for reflection and transmission coefficients (R and T) and show that R + T =1. **(2008)**

39.

Consider in the region $0 \le z \le 1$ m an infinite slap made of a material with relative permeability, $\mu_r = 3\cdot 5$. If $\vec{B} = \left(2y\hat{i} - 5x\hat{j}\right)\times 10^{-3}\, Wb/m^2$ Within the slab, determine magnetization $\vec{M}$. **(2009)**

40.

Consider the incidence of a plane- polarised electromagnetic wave at the interface of two media having dielectric permittivity and magnetic permeability (ε_1, μ_1) and (ε_2, μ_2) respectively. The interface is chosen to be x = 0 plane. $\vec{K}_1, \vec{K}_2$ and $\vec{K}_3$ represent the propagator vectors associated with the incident, refracted and reflected waves respectively. Using the boundary conditions on them, establish the Snell's laws of refraction. **(2009)**

41.

In a non-charged current-free dielectric, $\rho = 0$ and $\vec{J} = 0$. Show that in this medium, electric $(\vec{E})$ and magnetic $(\vec{H})$ fields satisfy three-dimensional wave equations

$$\nabla^2 \vec{E} = \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} \text{ and } \nabla^2 \vec{H} = \varepsilon\mu \frac{\partial^2 \vec{H}}{\partial t^2}$$

Using Poynting theorem of electromagnetic theory, describe the significance of the vector $\vec{P} = (\vec{E} \times \vec{H})$ and the scalar $u = \frac{1}{2}\left[\vec{B}\cdot\vec{H} + \vec{D}\cdot\vec{E}\right]$

(2009)

42.

A plane-polarised electromagnetic wave is incident on the interface of two dielectrics having dielectric permittivity ε_1 and ε_2. Assume that the electric vector $\vec{E}$ lies in the plane of incidence. Using the boundary conditions at the interface obtain the expressions for the amplitude reflection coefficient (r_{11}) and the amplitude transmission constant (t_{11}). **(2009)**

43.

A wire of length 2 m is perpendicular to X-Y plane. It is moved with a velocity $\vec{V} = (2\vec{i} + 3\vec{j} + \vec{k}) ms^{-1}$) through a region of uniform induction $\vec{B} = (\vec{i} + 2\vec{j}) W\ m^{-2}$. Compute the potential difference between the ends of the wire. **(2010)**

44.

Using Maxwell's field equations for a homogeneous non-conducting medium derive the wave equation for the electric field. Calculate the velocity of EM wave in free space. **(2010)**

45.

Explain the term 'Poynting vector' and state the significance of Poynting theorem. **(2010)**

46.

Calculate the skin depth for radio waves in free space of wavelength 3 m in copper, given that electrical conductivity for copper is $6\times10^{7}\Omega^{-1}m^{-1}$. **(2010)**

TUTORIAL SHEET: 19
Black Body Radiation

1. Write down the expression for energy distribution of a black - body radiation at temperature T and deduce Wien's displacement law. (1988)
2. Define solar constant and say which of the values 1.34 W/m^2, 1.34 × 10^3 W/m^2, 1.34 × 10^5 W/m^2 is valid for it? Calculate the total energy radiated by the sun in one second and hence the decrease in its mass per second. (1990)
3. Write down Planck's law of radiation and establish Wien's law from it. (1991)
4. State Planck's formula for black - body spectrum. Show that Planck's formula reduces to Wien's formula at short wave lengths. (1992)
5. In a nuclear explosion, the maximum temperature reached was of the order of 10^8 K. Estimate the order of wavelength at which the maximum of radiation energy occurs. (1992)
6. Show that the temperature 'T' of a planet varies inversely as the square root of its distance 'R' from the Sun. (The Sun and planets are considered to be black bodies in radioactive equilibrium.) (1992)

8. The solar constant on the lighted surface of the earth is given as 'J'. Assuming Solar radiation as black body radiation, find an expression for the temperature of the Photosphere of the Sun. (1995)

9. Derive the distribution law which explains the black body spectrum over the entire wave length region. Using this relation determine the value of Wien's constant b. (1997)

10. Write down the expression for the energy distribution function for the black body radiation at temperature T. Show that expression goes into the Rayleigh - Jeans distribution at one end of the frequency spectrum and the Wien's distribution at the other end. (1998)

11. Calculate the maximum amount of heat which may be lost per sec. by radiation from a sphere of 10 cms. diameter at a temperature of 227^{o}C when placed in and enclosure at a temperature of 27^{o}C. Given that σ = 5.7 × 10^{-2} watt/cm^2/deg^{-4}/sec.

12. Calculate the temperature of Sun if solar const. is 1.9 cals per minute, angular diameter is 30^o and σ = 5.7 × 10^{-5} C.G.S. units.

13. Earth receives 1.3 Kw/m^2 of radiant energy from the sun. Assuming sun to be a spherical black body of radius 7x10^{8}m and Earth-Sun distance to be 1.5x10^{11}m, Calculate the surface temperature of Sun.[Stefan-Boltzmann constant σ=5.67x10^{-8} w m^{-2} k^{-4}] (2003)

14. Use the Planck formula for the black body radiation

$$u(\omega,T) = \frac{h^2}{\pi^2 c^3}\frac{\omega}{\exp(\beta h\omega)-1}$$

with $\beta = \frac{1}{k_B T}$ to derive Wien's law, Rayleigh-Jeans law and Stefan-Boltzmann law.

(2005)

15. Using Planck's radiation formula
$u(\nu)d\nu = \frac{8\pi h}{C^3} \frac{\nu^3 d\nu}{e^{h\nu/kT}}$, where the symbols have their usual meaning, find the wavelength of the

region where energy density is the greatest. Also calculate the total energy density over all the frequencies. (2006)

16. State Rayleigh-Jeans law. Show that the intensity of emissions at a particular wavelength is proportional to the temperature *T*. Discuss the limitations of this law in describing the intensity distribution of emission spectrum of a blackbody. (2007, 25 marks)
17. Using Planck's radiation formula, deduce Wien's displacement law. **(2009)**

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WAVES
TUTORIAL SHEET: 7A
Damped and Forced Oscillations

1. Calculate the rate of energy dissipation by a damped harmonic oscillator, in the weak damping limit With $w_0\tau >> 1$, so that $\omega \cong \omega_0$. Symbols have their usual meanings. $\tau = \frac{1}{2\gamma}$
(1988)

2. Write down the differential equation for a damped simple harmonic oscillator. Solve it and discuss the characteristics of dead - beat motion.
(1990)

3. Give a mathematical analysis of forced vibration and hence explain the phenomenon of amplitude resonance.
(1992)

4. Show that for forced oscillations amplitude resonance and energy resonance do not occur at the same frequency.
(1995)

5. Write the equation of motion for an oscillator driven by a simple harmonically varying force. Obtain the condition for maximum energy transfer to the oscillator.
(1996)

6. The amplitude of a damped Oscillator of frequency 300 Hz reduces to one - tenth of its initial amplitude after 3000 Oscillations. Calculate the damping constant and the time in which its energy will reduce to one - tenth of its initial energy. 1500 osc.
(1997)

7. What are damped oscillations? Obtain the differential equation for damped oscillations and write its possible solution. Explain, with corresponding sketches, when there can be very heavy damping, critical damping and weak damping.
A resistive force -bv proportional to the
(1999)

8. An ideal massless spring of force constant k has a mass m attached to one of its ends, the other end being fixed to a rigid support. The spring is horizontal and the mass moves on a horizontal floor. Velocity v acts on the mass. Assuming the damping to be light, obtain the frequency of oscillation.
When m= 0.1 kg and k=10 n/m, it is found that the frequency of oscillation is $\sqrt{1/2}$ times the frequency in the absence of damping. Calculate the value of constant b.
(2003)

9. Write down the equation of motion for a damped harmonic oscillator assuming the damping force proportional to the velocity of the particle. Obtain the general solution for its displacement as a function of time. Discuss the cases of over damping, under damping and critical damping.
(2004)

10. In the steady state forced vibration a point particle of mass 'm' moves under the influence of an external force (F sin pt) $\hat{i}$ in addition to the restoring force – (kx) $\hat{i}$ and damping force – ($\beta\dot{x}$) $\hat{i}$. Show that (i) the amplitude is maximum when $p = \sqrt{\omega^2 - 2b^2}$, where $k/m = \omega^2$ and (ii) the value of

the maximum amplitude is $\dfrac{f}{2b\sqrt{\omega^2 - b^2}}$. What do you mean by the sharpness of resonance? (2006)

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TUTORIAL SHEET: 7B

Beats, Stationary waves, Phase & Group velocity, Huygen 's Principle

1. The phase velocity of surface waves of wave length λ is $V_p = \left(\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right)^{1/2}$ where T is the surface tension and ρ the density of the liquid and g is acceleration due to gravity. Find the group velocity and express it in terms of the phase velocity. For which wavelength is the phase velocity a minimum? (1991)

2. Explain the laws of refraction of light on the basis of Huygens principle. (1991)

3. The refractive indices of a material of wavelengths 5090 A°, 5340 A° and 5890 A° are equal to 1.647, 1.640 and 1.630 respectively. Estimate the phase and group velocities of light near λ = 5340 A°. (1993)

4. Distinguish between phase velocity and group velocity. Calling group velocity V_g and phase velocity V_p in a medium of refractive index n, establish the relation

$$V_g = V_p\left(1+\frac{\lambda}{n}\frac{dn}{d\lambda}\right)$$ where λ refers to the wavelength of the related light in vacuum. (1994)

5. Certain string has a linear mass density of 0.25 kg/m and stretched with a tension of 25 N. One end is given a sinusoidal motion, its frequency 5 Hz and amplitude 0.01 metre. If at t=0, the end has zero displacements and is moving along the positive y direction, derive the wave speed, the wave length and the wave equation of the wave in the string. (2001)

6. The phase velocity in a material is $\sqrt{g/k}$ where k is the propogation constant. Prove that the group velocity will be half of the phase velocity. (2001)

7. A wave is represented by Ψ_1=10 cos (5x+25t). Find wave length λ, velocity v, frequency f and the direction of propogatron. If it interferes with another wave given by Ψ_2 = 20 cos (5x+25t+π/3), find the amplitude and the phase of the resultant wave (2002)

8. The phase velocity of the surface wave in a liquid of surface tension T and density ρ is given by

λ is $V_p = \left(\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right)^{1/2}$ Show that the group velocity V_g of the surface wave is given by

9. Two transverse harmonic waves, each of amplitude 5mm, wave length 1 m and speed 3m/s are traveling In opposite directions along a stretched string fixed at both ends. Obtain an expression for the standing wave produced. Locate the positions of nodes and antinodes. (2003)

10. A siren of frequency 900 Hz is going towards a wall away from an observer at a speed of 10 m/sec. Determine
    - (i) Frequency of sound directly heard from the siron.
    - (ii) Frequency of sound reflected from the wall.
    - (iii) Number of beats per second heard by the observer. (velocity of sound = 330 m/sec). (2004)

11. For a transverse sinusoidal wave of wavelength λ propagating along negative x direction through a string fixed at a point, show that the nodes are located at x =0, λ/2, λ, 3λ/2, ... while the kinetic energy/unit length at the antinodes is given by

$$E = 2\rho A^2 \omega^2 \cos^2 \omega t$$

Where ρ, A and ω are the mass density/unit length, amplitude of transverse displacement and angular frequency of the wave, respectively. (2005)

12. Establish the relationship between the phase velocity V_p and the group velocity V_g of waves. Under physical conditions $V_g < V_p$, and $V_g > V_p$ can be possible ? (2007)
(2007)

13. For stationary waves on a string whose ends are fixed, show that the energy density is maximum at antinodes and minimum at nodes. (2009)

14. In the propagation of longitudinal waves in a fluid contained in an infinitely long tube of cross-section A, show that (2010)

$$\rho = \rho_0\left(1-\frac{\partial\xi}{\partial x}\right)$$

Where, ρ_0 =equilibrium density, $\frac{d\xi}{dx}$ = Volume strain and $\left|\frac{\partial\xi}{\partial x}\right| << 1$

ρ = density of the fluid in the disturbed state

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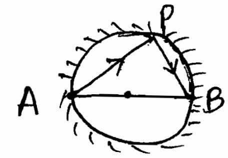
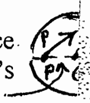
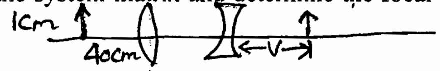


TUTORIAL SHEET: 8
Geometrical Optics

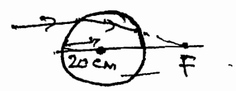
1. A ray of light starts from point A and after reflection from the inner surface of sphere reaches to diametrically opposite point B. Calculate the length of a hypothetical path APB and using Fermat's principal, find the actual path of length. Is the path minimum? (Ans. 2 dia, No)
2. In figure, P is a point source of light. If the distance of P from the center O of the spherical reflecting Surface is 0.8r and if the light ray starting from P and after being reflected at reaches at point Q, Show by Fermat's principal; $\cos\theta/2=3/4$.



3. Consider a lens of thickness 1cm, made of a material of refractive index 1.5, placed in air (refractive index of air=1). Let the radii of curvatures of the two surface be +4cm and –4cm (negative sign corresponds to a concave surface). Obtain the system matrix and determine the focal length and the position of unit points and nodal points.



4. Consider a system of two thin lenses as shown in figure For a 1cm tall object at a distance of 40cm from the convex lens, calculate the position and size of the image. Ans.: v=-14.5cm, 1/2.2cm
5. Consider a sphere of radius 20cm of μ=1.6. Find the position of paraxial focal point



6. An achromatic doublet of focal length 20cm is to be made by placing a convex lens of borosilicate crown glass in contact with a diverging lens of dense flint glass. Assuming n_r = 1.51462, n_b= 1.52264, n_r^1 =1.61216, n_b^1 =1.62901, calculate the focal length of each lens; here the unprimed and primed quantities refer to crown and flint glass respectively.
Ans. F=8.61cm, f_1= -15.1cm
7. A lens with spherical surfaces and aperture of diameter 6cm shows spherical aberration of 1.8 cm. If the central portion of diameter 2cm alone is used, deduce the aberration.
(Ans.: 0.2cm).
8. The spherical aberration of a lens is given by $x = h^2/f$ Φ is a constant. Compare the aberration in the following three cases:
(i) When central zone h=0 to 5 mm is used.
(ii) When peripheral zone h=10 mm to 12mm is used.
(iii) When the whole lens h=0 to 12mm is used. (Ans.25:44:144)
9. State Fermat's principle. Apply it to get the laws of reflection from a plane surface.
(2002)
10. Two thin convex lenses of focal length 0.2m and 0.1m are located 0.1m apart on the axis of symmetry. An object of height 0.01m is placed at a distance of 0.2m from the first lens. Find by the matrix method, the position and the height of image.
(2003)
11. Show that the ratio of the focal length of the two lenses in an achromatic doublet is given by $f_1/f_2 = -w_1/w_2$, where w_1 and w_2 are the dispersive powers of the lenses of focal length f_1 and f_2 respectively.
12. A thin converging lens and a thin diverging lens are placed coaxially at a distance of 5cm. If the focal Length of each lens is 10cm, find for the combination (i) the focal length (ii) the power (iii) the position of the principal point. = node point
(2004)
13. What do you understand by paraxial rays? Show that the effect of translation of a paraxial ray while travelling along a homogeneous medium is represented by a 2 ×2 matrix if the ray is initially defined by a 2×1 matrix.
(2005)
14. Derive Snell's law of refraction index related to the velocity of light? Light of wavelength 600 nanometer (in vacuum) enters a glass slab of refractive index 1.5. What are the values of wavelength, frequency and velocity of light in glass?
[Velocity of light in vacuum = 3×10^8 ms^{-1}]
(2006)
15. Derive the condition for achromatism of two thin lenses separated by a finite distance and made up of same material.
(2009)

$$w = \left[\frac{\mu_b - \mu_r}{\left(\frac{\mu_b+\mu_r}{2}\right) - 1}\right]$$

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TUTORIAL SHEET: 9

Interference

1. A soap-film of refractive index 1.33 is illuminated with light of different wave lengths at an angle of 45°. There is complete destructive interference for $\lambda = 5890$ A°. Find the thickness of the film. (1991)
2. An interference pattern is obtained by using two coherent sources of light, and the intensity variation is observed to be ± 10% of the average intensity. Determine the relative intensities of the interfering sources. (1993)
3. Show that the interference fringes in uncoated thin films are distinct when seen in reflection, but very indistinct in transmission. (1994)
4. In a biprism experiments the fringe-width with light of wavelength $\lambda = 5900$ A° is 0.43 mm. On introducing a mica sheet in the path of one of the interfering rays the central fringe shifts by 1.89mm. If refractive index of mica is 1.59, calculate the thickness of the sheet. (1995)
5. Show that the interference obtained in young's two-slit experiment are hyperbolic in shape. Under what conditions these are expected to appear straight? (1996)
6. Why does a Soap film appear coloured when it is viewed by reflected white light? A thin film is illuminated by sodium light of wavelength 5900 A°. Its refractive index is 1.42. Calculate its minimum thickness so that it appears dark in reflected light. (1997)
7. What are the essential conditions for observing the interference of light? Two Coherent sources with intensity ratio 4:1 interfere. Find I_{max}/I_{min} ? (1999)
8. Why an extended source is necessary to see colours in a Soap-film? Non-reflecting surfaces are made by coating very thin films of a transparent material. Find the ~~maximum~~ minimum thickness of such thin coatings given that $\lambda = 5.5 \times 10^{-5}$cm and $\mu = 5/4$. (1999)
9. Explain in detail how one can obtain fringes with the Michelson Interferometer using incandescent lamps. (2000)
10. Monochromatic light from a distance source of wavelength λ falls on a double slit. A glass plate of thickness t is inserted between one slit and the screen. Calculate the intensity at a central point as the function of thickness t'. (2001)
11. In a experiment using a Michelson interferometer, explain with the help of suitable ray diagrams:
    (i) Why do we need extended sources of light,
    (ii) Why do we get circular fringes, and
    (iii) Shifting of fringes inwards or outwards as we shift the movable mirror. (2002)
12. Two microscope slides of length 10cm each form a wedge. At one end they are in contact and at the other end they are separated by a thin wire of diameter d. (see the diagram below). Interference fringes are obtained when illuminated vertically by a monochromatic light of $\lambda=500$ nanometers. The fringe spacing is found to be 1.25 mm. Estimate the diameter of the wire. d (2004)
13. Explain the working of Michelson interferometer using appropriate optical diagram. Also draw paths of the rays. (2006)
14. Obtain the relation to find radii of the rings and the wavelength of light in Newton's circular ring. Calculate the radius of curvature of the convex glass surface where diameter of 5th and 15th bright rings formed by sodium yellow light are measured to be 2.303 mm and 4.134 mm. Given μ=1.5 and λ_{yellow} =5282A°. (2006)
15. Describe the working of a Fabry – Perot interferometer. Determine the intensity of the fringes of the transmitted light. Why the fringes obtained in the Fabry – Perot interferometer are comparatively sharper than those obtained from the Michelson interferometer ? (2007)
16. Let the two waves with parallel electric fields be given by

$$E_1 = 2Cos\left(\vec{k}_1.\vec{\partial} - \omega t + \frac{\pi}{3}\right) kV/m,$$

$E_2 = 5Cos\left(\vec{k}_2.\vec{\partial} - \omega t + \frac{\pi}{4}\right) kV/m$.

Find the intensity of each beam l_1, l_2 and also the interference term l_{12} at a point where their path difference is zero. Calculate the visibility

$V\left(= \frac{l_{max} - l_{min}}{l_{max} + l_{min}}\right)$ for the interference pattern.

$\left[\epsilon_0 = 8.85\times10^{-12} C^2/.Nm^2, \mu_0 = 4\pi\times10^{-7} N/A^2\right]$ (2008)

17. Let

$E_A = E_1 \sin\omega t$ and $E_B = E_2 \sin(\omega t + \wp)$

By using analytical method, obtain an expression to explain interference. Also show that intensity varies along the screen in accordance with the law of cosine square in interference pattern (2009)

18. Explain the phenomenon of interference in thin films. Why is the contrast better in brightness off fringes obtained from the interference of reflected light rays compared to the transmitted light rays? (2009)

19. Describe Michelson interferometer for evaluation of coherence length of an optical beam. Calculate coherence length of a light beam of wavelength 600 nm with spectral width of 0.01 nm. (2010)

20. Show that two light beams polarized in perpendicular directions will not interfere. (2010)

21. An optical beam of spectral width 7·5 GHz at wavelength λ=600 nm is incident normally on Fabry-Perot etalon of thickness 100 mm. Taking refractive index unity find the number of axial modes which cane supported by the etalon. (2010)

In general,

avg. wrt. time

$I \propto \langle E^2 \rangle$

$= \langle \vec{E}.\vec{E} \rangle$

$= \langle (\vec{E}_1 + \vec{E}_2).(\vec{E}_1 + \vec{E}_2) \rangle$

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TUTORIAL SHEET: 10

* First image does not mean position wise. It means image corresponding to 1st focus

Diffraction

1. The diameter of the central zone of a zone-plate is 2.3 mm. If a point source of light (λ = 589.3 nanometer) is placed at a distance of 6 metres from it, calculate the position of the first image. (1988)
2. In double - slit Fraunhofer diffraction; calculate the fringe spacing on a screen 50 cm away from the slits. If they are illuminated with blue light $\lambda = 4800$ A°, slits separation d = 0.10 mm, and slit - width a = 0.020mm. What is the linear distance from the central maximum of the first minimum of the fringe - envelope? (1989)
3. A single slit of width 0.14mm is illuminated normally by monochromatic light and diffraction bands are observed on a screen 2m away. If the centre of the second dark band is 1.6cm from the middle of the central bright band, deduce the wavelength of light. (1990)
4. Show schematically the intensity distribution for a 2-slit Fraunhofer diffraction-interference, if slit-widths are 2λ each and centres of slits have separation 6λ. Assume incident light falling normally, and limit the discussion to the central diffraction band range. (1990)
5. Distinguish between Fresnel and Fraunhofer classes of diffraction of light. Discuss the theory of plane grating and hence find an expression for the angular dispersion of a plane-grating. $(d\theta/d\lambda)$ (1992)
6. What is Fraunhofer diffraction? Under what conditions may it be observed? Find an expression for the intensity distribution in double slit Fraunhofer diffraction, taking the result for diffraction at a single slit as given. (1993)
7. Obtain the intensity pattern due to Fraunhofer diffraction at two parallel slits. Each slit has a width 'a' and the separation between the slits is 'd'. How many interference fringes will appear in the central diffraction maximum, if d = 4a? Bright fringes.... (1995)
8. Give the concept of Fresnel's half period zones. Describe the salient features of Fresnel's diffraction pattern due to a straight edge, showing the intensity distribution. How are these features explained? (1997)
9. Differentiate between Fresnel and Faunhoffer diffractions. How can one explain the Fresnel diffraction pattern due to a straight edge? (1999)
10. Discuss the Fresnel diffraction pattern formed by a straight edge using the cornu's spiral. (2002)
11. Obtain an expression for the intensity of light in the fraunhofer diffraction pattern due to a circular aperture. What is Airy pattern? Explain with a neat diagram. (2003)
12. A narrow slit illuminated by monochromatic light of λ=6400Å is placed at a distance of 3 metres from a straight edge and the screen is 6 metres, Calculate the distance between the first and the fourth dark bands. (2004)
13. What is the essential difference between interference and diffraction of light? How can you achieve grating Fraunhofer diffraction in the laboratory? Using the concept of Fraunhofer diffraction at a single slit, find out the intensity distribution produced by two slits of equal width. (2005)
14. The radius of the first zone in a zone plate is 2mm. What will be the position of the first image of a point source of light of wavelength $\lambda = 500$ nm placed at a distance of 5 m from the zone plate? (2007)
15. Obtain the expression for the primary focal length of Fresnel zone plate. (2010)

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TUTORIAL SHEET : 11

(Resolving Power of Instruments)

1. State Rayleigh criterian for limit of resolution. Show that $\frac{I_{middle}}{I_{max}} = \frac{8}{\pi^2}$ (1992)

2. A diffraction grating with 3×10^4 lines is used in the second order in the range of wavelength 6000 A°. Find the smallest ($\Delta\lambda$) it can resolve. (1992)

3. Discuss the theory of diffraction grating ~~and~~ find conditions for the absent spectra. Distinguish between resolving power and dispersive power of a grating. (1994)

4. The angular separation between two distant stars is 1 arc - second. If the effective wavelength of light is 5500 A° , what should be the diameter of the objective of a telescope so that the stars are just resolved? (1996)

5. A plane transmission grating has 6000 lines per cm. Determine the angular separation between the two lines of sodium of wavelengths 5896 A° and 5890 A° in the second order spectrum. If the width of the grating is 2,5 cm., will these lines be resolved? (1997)

6. Define dispersive and resolving powers of a plane transmission grating and obtain expressions for the two. Show that the first and second order spectra produced by such a grating will never overlap when the incident light contains wavelengths in the range of 4000 A° to 7000 A°. (1998)

7. Explain the terms resolving power and magnifying power of an optical instrument. On what parameter do these physical quantities depend in case of a telescope? For a given resolving power what is the optimum magnifying power in this case? (1998)

8. Sodium light is incident normally upon a plane transmission grating having 5000 lines/cm. Calculate the angular separation of the D_1 and D_2 lines in the first order spectrum. As their angular separation is very small, how can one magnify it 10 times?
(Given $\lambda D_1 = 5890$ A°, and $\lambda D_2 = 5896\ A^0$ (1999)

9. Give Rayleigh criterion for resolution. Why telescopes with larger objectives are better? The objective of the telescope at St. Palomer has a diameter of 5.08m. What is the least distance on Moon, which can be resolved by it? (Given: distance Moon from the earth = 3,84,000 km. & λ = 5890 A°). (1999)

10. Derive the expression for resolving of power of a diffraction grating with N lines. Calculate the minimum number of lines in the diffraction grating if it has to resolve the yellow lines of sodium (589 nm and 589.6 nm) in the first order. (2002)

11. Define resolving power and dispersive power of a grating. Two spectral lines of wavelengths 500 nm & 500.5 nm are seen clearly resolved in second order spectrum of a grating. If the grating has 250 lines per cm, what should be the minimum width of the grating ? (2007)

12. Calculate the Fraunhoffer diffraction pattern from a grating of N slits with width e, separated by equal opaque spaces d. Find the condition for principal maxima and the corresponding values of intensity.
A parallel beam of Na light is incident normally on a plane grating with 4250 lines per cm. The second order spectral line is observed to be deviated through 30^0. Calculate the wavelength of light. (2008)

13. (i) Distinguish between high dispersive power and high resolving power.
(ii) Obtain an expression for the resolving power of a plane transmission grating.
(iii) Deduce the missing orders for a double-slit Fraunhofer pattern, if the slit widths are 0.16 mm and 0.8 mm apart. (2009)

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TUTORIAL SHEET: 12

Polarization

1. A beam of linearly polarized light is changed into circularly polarized light by passing it through a slice of crystal 0.003 cm. thick. Calculate the difference in refractive index of two rays in crystal assuming this to be minimum thickness that will produce the effect and that the wavelength of light is 6×10^{-7}m.

(1988, 1989)

2. Explain mathematically how left and right circularly polarized light is produced by combining two linearly polarized beams. Given a beam of light, how can one experimentally test whether it is unpolarized or circularly polarized?

(1990)

3. Deduce the possible thickness of a quarter wave plate of quartz which is to be used for Sodium light of wavelength 5890 A°. (μ_o = 1.658, μ_e = 1.486)

(1991)

4. Give an outline of Fresnel's explanation of optical rotation. How does optical rotation due to material vary with λ?
For an optically active material the difference between the refractive indices for right - handed and left - handed vibrations ($\mu_R - \mu_L$) for λ = 4500 A° is 12×10^{-5}. Estimate the optical rotation caused by 1mm thick plate in light of λ = 4500 A°. Assume ($\mu_R - \mu_L$) to be independent of λ.

(1992)

5. Give an account of the origin of optical activity in quartz crystal. A wafer of crystalline quartz of thickness 2.945×10^{-5} m is used to change a beam of linearly polarized light (λ = 589 nm) into circularly polarized light. Find the difference in refractive index for the two waves in the crystal, assuming this to be minimum thickness that will produce the effect?

(1993)

6. Describe how Fresnel has accounted for the rotation of the plane of polarisation of light. Explain the action of a half - shade device.

(1994)

7. A left circularly polarized beam of light (λ = 6000 A°) is incident on a quartz crystal (optic axis parallel to the surface). Find the state of polarisation of the emergent beam. (Thickness of quartz crystal = 2.3×10^{-5}m, μ_e = 1.5538 μ_0 = 1.5444).

(1995)

8. What is a quarter wave plate? Explain its use in the production and detection of circularly polarized light. For calcite λ = 54 72 ^{0}A, μ_0 =1.659 and μ_E = 1.488. If the minimum thickness of a plate that can be cut from calcite is 30 μm, what should be the minimum thickness for preparing a quarter wave plate?

(1996)

9. What is a quarter plate? A phase retardation plate of quartz has a thickness 0.1436 mm. Calculate the wavelength in the visible region for which this plate will act as a quarter wave plate. The refractive indices of quartz for ordinary and extra ordinary rays are 1.5443 and 1.5533 respectively.

(1999)

10. Four perfect polarizing plates are stacked so that the axis of each is turned 30° clockwise with respect to the preceding plate; the last plate is crossed with the first. How much of the intensity of an incident unpolarized light is transmitted by the stack?

(2000)

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11. Why does one get polarized light from Nicol's prism? How should one adjust the polarizer and andyser, so that an intensity of the incident light is reduced by a factor of 0.25.
12. How do you know that the light is a transverse wave? What is a quarter wave plate? How is it constructed? (2002)
13. A quartz quarter wave plate is to be used with the sodium light (λ=5869 Å). What should be its thickness. (2004)
14. Why does one see tow image points for a single object point while viewed through a calcite crystal? What is this property of the crystal known as? What is an optic axis of a crystal? Explain the meaning of positive and negative crystals with one example for each kind. (2005)
15. What is optical activity? Given reasons for the conclusion that optical rotation in liquids has a molecular origin. What do you mean by ordinary and extraordinary rays? What are positive and negative crystals? Give an example of each. Compute the minimum thickness of a quarter-wave plate made from quartz for incident wavelength of 589.3 nanometer. Given μ_0 =1.544 and μ_E= 1.553. (2006)
16. How would you produce plane polarized light by reflection ? What is Brewster's law? Calculate the angular position of the sun above the horizon so that light reflected from a clam lake is completely polarized. The refractive index of water is 1.33. Circularly polarized and unpolarized light are passed in turn through a Nicol prism. The Nicol is rotated about the direction of light as axis. What would you observe in each case? How would you distinguish between them? (2006)
17. Consider superposition of two plane polarized electromagnetic waves:

$$E_y = \hat{y}\, a\cos(kx - \omega t) \text{ and}$$

$$E_z = \hat{z}\, b\cos(kx - \omega t + \phi)$$

Discuss the conditions for the resultant wave to be left circularly and right circularly polarized adopting the convention as seen by an observer traveling with the wave. (2007)

18. (i) Show that the plane of polarization is rotated through

$$\theta = \frac{\delta}{2} = \frac{\pi d}{\lambda}(\mu_L - \mu_R)$$

(ii) Aplane-polarised light is incident perpendicularly on a quartz plate out with faces parallel to optic axis. Find the which introduces phase difference of 60^0 between e- and o-rays. (2009)

this is always the case
st. light emitted
out of crystal is combination
of o + e rays.

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TUTORIAL SHEET: 13

LASERS

1. A ruby laser ~~produces~~ a beam of light of wavelength 6943 A° with a circular cross - section of 1cm in diameter. Calculate the diametre of this beam at a distance of 1000 kilometers.

(1992)

2. Explain the general principle of laser action. What do you mean by population inversion? Discuss the involved in the ruby laser. A pulsed laser is rated at 10 m W. It generates 3 ns wide pulses at frequency 500 HZ. Compute the instantaneous power in the pulse.

(1993)

3. The light ($\lambda = 6000$ A°) from a laser of sectional diameter 1.0cm and power 0.20 watt is focused by a lens of focal length 10cm.Determine the area of the image and intensity in it in watt/cm^2.

(1994)

4. Discuss the working principle of He – Ne laser indicating the transitions involved in the process. Determine the power output of a laser in which a 3.0 J pulse is delivered in 1.0 n second.

(1995)

5. Describe the working principle of a three level solid state laser giving the transitions involved in the laser action.

(1996)

6. Obtain an expression for the ratio of the probabilities of stimulated and spontaneous emissions. What do you infer from this relation? How is population inversion interpreted thermodynamically?

(1996)

7. In a hydrogen atom for the 2p → 1s transition the probability per unit is 6×10^8 s^{-1}. A_{21} Calculate the angular frequency of the emitted photons and the order of Einstein's coefficient B_{21}

(1997)

8. A 3 MW laser beam which has a diameter of 1cm is focused by a lens of focal length 5 cm. The wavelength of laser is 10,000 A°. Calculate the intensity at the focal plane of the lens.

(1997)

9. What is population inversion? Mention the methods of achieving population inversion. Explain the concept of negative temperature.

(1997)

10. A short - focus lens is used to focus a laser beam of wavelength 6328 A°. If the beam width is comparable to the focal length of the lens, calculate the area of cross-section of the region of focus.

(1999)

11. Explain why a two - level system is not adequate for laser operation. Draw the essential parts of a ruby laser and explain the working principle.

(1999)

12. Explain the phenomenon of self focussing of laser beams.

(2003)

Ch nonlinear 23rd Exchange self focusing

10 one of the reason for straight line movement

13. Explain how Einstein's A and B coefficients are related to the phenomena of spontaneous and stimulated emission of radiation, respectively. Derive the relation between A and B. Establish that at very high frequency around X-ray wavelength regime, lasers cannot be made as easily as at low frequencies, e.g., far infra-red regime.

14. Consider an ensemble of two – level atoms in thermal equilibrium. Show that the ratio of Einstein A and B coefficients is given by

$$\frac{A}{B}=\frac{8\pi h\nu^3}{c^3}$$

Why is it not possible to achieve inversion of population in a two- level medium?

(2008)

15. What are the characteristics of stimulated emission? Show that in the optical region stimulated emission is negligible compared to spontaneous emission.

(2009)

16. (i) At what temperature are the rates of spontaneous and stimulated emission equal? (Assume $\lambda = 500$ nm)

(2009)

(ii) What are the important properties of a hologram?

(iii) Optical power of 1 mW is launched into an optical fiber of length 100 m. If the power emerging from the other end is 0.3 mW, calculate the fiber attenuation.

(2009)

17. A laser beam of 1 micrometer wavelength with 3 megawatts power of beam diameter 10 mm is focused by a lens of focal length 50 mm. Evaluate the electric field associated with the light beam at the focal point.

(Dielectric permittivity of free space,

$\varepsilon_0 = 8\cdot 8542\times 10^{-12} C^2/N-m^2$)

(2010)

$uB_{12} = uB_{21}$

$\Rightarrow$ Probability of transition is same.

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TUTORIAL SHEET: 14

(Special Topics)

1. What is a hologram? Explain how the image of the object is formed when one looks through it.

(1990)

2. Define coherent length. A helium - neon laser emits radiation at wavelength $\lambda =$ 623.8 nm with $\Delta\lambda = 2\text{pm}$. Calculate the coherent wavele

(1993)

3. What is meant by temporal and spatial coherence ? Show that the coherence length $L = \lambda Q$ where λ is the mean wave length and Q represents the purity of a

(1996)

4. What is spatial coherence? Considering young's two slit experiment, prove that the distance between the slits must be sufficiently less than $\left(\frac{\lambda}{\theta}\right)$ for obtaining fringes of good contrast λ is the wavelength of light used and θ is the angle subtended by the source upon slits.

(1997)

5. Explain the phenomenon of pulse dispersion in step index fibre.

(2003)

6. What is holography? Describe the experimental set up for Gabor's on-line holographic recording. What are the limitations of Gabor's experiment? How were these overcome by Leigh and Upatheiks?

(2003)

7. Drawing a neat diagram, discuss how light travels through on optical fiber. Show that the numerical aperture of a commercially available optical fiber is around 0·25. Explain its physical significance.

8. Why does one get three-dimensional image in holography? Explain with appropriate figures how can one construct and read a hologram.

(2005)

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PHYSICS PAPER - I: THERMAL PHYSICS
TUTORIAL SHEET: 20
Basic Concepts

1. Calculate the work done in compressing adiabatically 10^{-3}kg of air initially at STP to one - half its original volume. (Given density of air at STP = 1.293 kg./m^3 and γ = 1.4
(1990)

2. Establish that for an adiabatic process in an ideal gas $TV^{\gamma-1}$ = constant, where the symbols have their usual meanings.
(1991)

3. One gram of hydrogen gas at 27° C is compressed isothermally from 100 litres to 25 litres. Calculate the energy needed.
(1991)

4. Show that the work done in a reversible expansion of an ideal gas from volume V_1 to V_2 is greater than the corresponding work done in an irreversible expansion against a constant pressure P_2.
(1994)

copy

5. Explain the concept of 'internal energy' of a system. Formulate mathematically the first law of thermodynamics. Calculate the work done in an isothermal compression of a gas.
(1999)

6. What is an adiabatic process? Give three engineering examples of adiabatic process, which are in common use?
(2000)

7. One mole of an ideal gas is compressed at constant temperature T from a volume V_1 to a volume V_2. Find the work done and heat absorbed by the gas. The gas now expands adiabatically to a volume 2V. Taking the gas to be diatomic, calculate the final temperature of the gas.
(2008)

8. Consider one mole of an ideal gas whose pressure changes with volume as P =αV, where α is a constant. If it is expanded such that its volume increases η times, find the change in internal energy, work done by the gas and heat capacity of the gas.
(2010)

Universal gas constant

$R = 8.314$ SI units

Q9 State 2nd law. Prove no engine operating b/w 2 given temp is more efficient than a Carnot engine between 2 temperature

γ : specific heat ratio $= \left(\frac{f+2}{f}\right)$

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TUTORIAL SHEET: 21
Carnot's Cycle and Entropy

1. A Carnot's engine is made to work between 0° C and -200°C. Calculate its efficiency. Derive the expression you use for calculation.
(1988)

2. A volume of one gm. mole of an ideal gas expands isothermally to four times its initial volume. Calculate the change in its entropy in terms of gas constant.
(1988)

3. 1kg of ice at 0°C is melted and converted to water at 0°C. Compute the change in entropy.
(1989)

4. A and B are two huge blocks of same metal. The blocks are connected by a huge rod of the same material. The temperatures of A and B are 1500 K and 500 K respectively. The rate of heat conduction is 10^4 J/sec. Estimate the rate of entropy increase of the universe due to this process.
(1992)

5. Define entropy. Write a general expression for the elementary entropy change dS for 1 mole of an ideal gas. How would this become for cases where the change is (i) isothermal (ii) isochoric (iii) isobaric? Deduce expressions for $S_2 - S_1$ for each of these cases, where (1) and (2) refer to the initial and final stage entropies respectively. Constants Cp, Cv, R may be used to express the results.
(1993)

6. A 10 ohm resistor carrying 3 ampere current is cooled by running water so as to keep the temperature at 300K. Discuss the change in entropy per second (i) of the resistor (ii) of the universe?
(1994)

7. Define entropy. Derive expressions for the entropy of a perfect gas in terms of (i)T and V (ii) T and P. The symbols have their usual meanings.
(1995)

8. Explain the concept of entropy. The specefic heat Cp of a material depends on temperature T according to the relation $Cp = a + bT + cT^2$ where a, b, c are constants. Derive an expression for the change in its entropy when the temperature changes from T_1 to T_2.
(1996)

9. How is absolute scale of temperature obtained with the help of Carnot cycle? Hence define absolute zero.
(1996)

10. Prove that $\int_A^B \frac{\delta Q}{T}$ evaluated along a reversible path joining the states A and B does not depend on the path chosen. Hence define the entropy function. Calculate the entropy change in an ideal gas undergoing a state change from (V,P) to (2V, P/2) for three suitably chosen different paths and show that the result turns out to be the same in all the cases.
(1998)

11. Define the efficiency of a Carnot Engine. An engine is designed to have an efficiency of 25% and to absorb heat at a temperature of 267°C. Find the maximum temperature at which it can exhaust heat.
(2000)

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12. Calculate the increase in entropy when 1 kg ice melts at zero degree centigrade. The Latent heat of fuse of ice is 3.36×10^{5} Joules/kg. (Assume that the melting is an isothermal reversible process).

(2001)

13. Describe Carnot cycle and show that efficiency is given by $H = Q_1 - Q_2 / Q_1 = T_1 - T_2 / T_1$ where symbols ~~hence~~ their usual meaning.

(2002)

14. Describe Otto cycle and obtain an expression for the efficiency is lower than that of a Carnot cycle operating between the highest and lowest temperature of otto cycle.

(2003)

15. A gas expands isothermally from the pressure P_1 and Volume V_1 to the pressure P_2 and the volume V_2 Calculate:
   (i) the change in internal energy
   (ii) the change in entropy
   (iii) the change in enthalpy
   What will be corresponding quantities when gas expands adiabatically?

(2004)

16. Show that the entropy of one gas mole of an ideal gas is given by $S = C_p \ln V + C_v \ln P + S_0$

(2004)

17. 1 kg of ice at 0^0c floats on 10 kg of water at 30^0c, the whole system being thermally isolated. What will be the change in entropy of the system when thermal equilibrium is reached ?
[Specific heat of water = 4.2 kJ kg^{-1} K^{-1} and latent heat of fusion of ice = 336 kJ kg^{-1}]

(2006)

18. Prove the law of increase of entropy. Show that for a system at fixed temperature and pressure to be in equilibrium, its Gibbs free energy should be minimum.

(2007)

19. A system at temperature T_1 is brought in contact with a reservoir at temperature $T_2 > T_1$. When the system and the reservoir reach thermal equilibrium, calculate the change in entropy of the universe assuming the heat capacity of C_p of the system to be constant. Discuss whether the considered change is positive or not.

(2007)

20. State second law of thermodynamics. Prove that no engine operating between two given temperatures is more efficient than a carnot engine operating between same two temperatures.

(2008)

21. Define entropy. How is it related to disorder? Hence, derive the Boltzmann relation S = k log Ω, where Ω is the probability and k is the Boltzmann constant.
Show that for any type of process, involving a closed system

$$\Delta S \geq \frac{\Delta Q}{T}$$

Where the equality sign applies for internally reversible processes and the inequality for internally irreversible processes.

(2009)

$L_{ice} = 3.36 \times 10^{-5}$ SI units

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TUTORIAL SHEET : 22

Thermodynamic relationships

1. Obtain the clausius clapeyron equation
$$\frac{dp}{dT}=\frac{L}{T(V_2-V_1)}$$ where the symbols have their usual meanings.
(1989, 1994)

2. Establish the relation $Cp - Cv = T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial V}{\partial T}\right)_p$
(1991)

3. Write down the general expression for the Joule - Kelvin effect and define Joule - Kelvin coefficient, μ. Show that for an ideal gas $\mu = 0$
(1991)

4. The density of steam at 100°C is nearly 0.60×10^{-3}kg/litre and its latent heat is 2.3×10^6 J/kg. Calculate the change in the boiling point of water if air pressure changes from 76.0cm to 70.0 cm. of Hg.
(1991)

U, F, H, G

5. Define the thermodynamic energy functions. Using these function establish the following relations:

(i) $\left(\frac{\partial T}{\partial V}\right)_\phi = -\left(\frac{\partial P}{\partial \phi}\right)_V$ (ii) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial \phi}{\partial V}\right)_T$

(iii) $\left(\frac{\partial T}{\partial P}\right)_\phi = \left(\frac{\partial V}{\partial \phi}\right)_P$ (iv) $\left(\frac{\partial \phi}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$

(1992)

6. Calculate the Joule - Thomson coefficient for Nitrogen gas at 293 K and 100 atm. pressure taking

$Cp = 8.21$ cal deg^{-1} mole^{-1}

a = 1.39 litre^2 - atm - mole^{-2}

b = 3.92×10^{-2} litre mole^{-2}

(1994)

7. Using Maxwell's thermodynamic relations, show that the internal energy of an ideal gas at a constant temperature is independent of its volume while for a real gas it is a function of volume also.
(1996)

8. Define Helmholtz free energy F and Gibb's free energy G. What does decrease in G and F signify? Show that the internal energy U at temperature T is given by $U = F - T\left(\frac{\partial F}{\partial T}\right)_V$
(1997)

9. Using a Carnot cycle, Obtain the Clausius - Clapeyeron equation. At normal pressure the ratio of densities of ice and water is 9 : 10. When 1 kilo - mole of ice is melted, the change in entropy is 22.2×10^3 J/K. If the external pressure is increased by 10^5 N/m^2, what will be the change in the melting point of ice?
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10. Prove the latent heat equation $\left(\frac{dL}{dT}\right)_{sv} - \frac{L}{T} = Cs - Cp$, Where Cs & Cp are specific heats of saturated vapour and the liquid in contact with it respectively. Given the following values referring to 1gm. Of water at 100°C, L = 539 Cal/gm., $\left(\frac{\partial L}{\partial T}\right)_{sv} = -0.64$ Cal K^{-1} gm^{1} Cp = 1.01 Cal K^{-1} gm^{-1} . Calculate Cs. Explain why the specific heat takes a negative value (1998)

11. Consider the expression $C_p - C_v = -T\left(\frac{\partial V}{\partial T}\right)_p^2\left(\frac{\partial P}{\partial V}\right)_T$

and give reasoning regarding the value of T when $C_p = C_v$ for water. Also, evaluate $C_p - C_v$ for a Van der Waals gas to elaborate that its value is larger for any real gas as compared to an ideal gas.

(2005)

12. Using thermodynamic principles show that the Joule- Thomson coefficient μ can be expressed as

$$\mu = \frac{1}{Cp}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]$$

Calculate the value of μ for an ideal gas and interpret your result physically.

(2006)

13. Establish the relation

$$C_p - C_v = \left[p + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_p.$$

Use it to find out an expression for C_p of one mole of a gas whose internal energy is given by $U = cT - \frac{a}{V}$ and which satisfies the equation of state $\left(P + \frac{a}{V^2}\right)(V - b) = RT$.

Here a, b and c are constants. (2007)

14. Calculate the change in pressure for a change in freezing point of water equal to -0·91 °C. Given, the increase of specific volume when 1 gm of water freezes into ice is 0·091 cc/gm and latent heat of fusion of ice is 80 cal/gm.

(2010)

CLAUSIUS CLAPEYRON

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TUTORIAL SHEET: 23

Special topics

1. Define thermodynamic temperature of a magnetic system. Making use of Gibb's equation, derive an expression for cooling produced due to adiabatic demagnetisation process. Why is the method used only after pre - cooling to a low temperature?

(1993)

2. What is an adiabatic demagnetisation cycle? Discuss the cycle in terms of T-S indicator diagram?

(1994)

3. Define thermodynamic temperature of a magnetic system. Discuss how cooling takes place due to adiabatic demagnetisation. Explain, why cooling due to adiabatic demagnetisation is important at low temperatures.

(1995)

4. Explain how very low temperature can be produced by adiabatic demagnetisation.

(1999)

5. Obtain Vander Waal's equation of state for real gases. What is the value of critical Coefficient for an ideal gas?
Show that the Value of the critical coefficient for Vander Waal's gas is independent of the type of the gas.

(2001)

6. Show that the chemical potential of a system is an intensive quantity and is a function of temperature and pressure only

(2005)

extra

8. (a) What led van der Waals to modify the ideal gas equation? Using the concepts of critical temperature T_c, pressure P_c and volume V_c, show that the critical constant for a real gas is 8/3. 5+15=20

(b) Find out the expressions for van der Waals constants a and b. 20

(c) Calculate the values of van der Waals constants a and b for oxygen with $T_c = 154 \cdot 2$ K, $P_c = 49 \cdot 7$ atmosphere and $R = 80$ cm^3 atmosphere/K. 20

(2011)

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TUTORIAL SHEET: 24
Specific heat of solids

1. Thermal energy of a solid is given by the relation

$$E = \int_0^{\nu_m} \frac{h\nu \, g(\nu) \, d\nu}{e^{h\nu/K_B T} - 1}$$

where $\nu m = \frac{K_B \theta_D}{h}$ θ_D being the Debye temperature.

Given $g(\nu) = 6Nh^2\nu/(K_B\theta_D)^2$, deduce the expression for E for $T \ll \theta_D$ and discuss the temperature variation of specific heat for $T \ll \theta_D$ (Given $\int_0^\infty \frac{x^2 dx}{e^x - 1} = 2.404$)

(1990)

2. Derive the expression for the specific heat of a solid on the Einstein model. Comment on its shortcomings. (1991)

3. Derive an expression for the specific heat of solids in Einstein's model. Explain Einstein temperature (T_E). Find the value of specific heat when
(i) $T >> T_E$ (ii) $T << T_E$ (iii) $T = T_E/2$

(1993)

4. Write the expressions obtained from Einstein's and Debye's theories for the specific heat of solids: What is the basic difference between the two theories? Calculate the heat required to raise the temperature of a solid of mass m from T_1 to T_2 in the low temperature region.

(1995)

5. State the basic assumptions of Debye theory of specific heat of solids and write the expression for Cv derived from this theory. Show that this expression yields the famous T^3 law of specific heat at low temperatures. Discuss the extent to which the theory agrees or disagrees with observations on specific heat through the variation of the Debye characteristic temperature θ_D with temperature, in general? (1998)

6. The specific heat of a substance is found to vary with temperature in the following way $C(T) = aT + bT^2$, where C (T) is the specific heat at the temperature T and 'a' and 'b' are constants. Compare the average specific heat of the substance in the temperature range 0 - T to the specific heat at the mid - temperature $\frac{T}{2}$ (1999)

7. Mention the assumptions made by Einstein in explaining the variation of specific heat of solids with temperature. Show how these assumptions were used to derive the formula for the specific heat of solids. How and why Einstein's theory fails at very low temperatures. (1999)

8. Discuss the differences in the assumptions underlying Einstein and Debye theories of specific hear C_v. Give schematic plots of C_v versus reduced temperature for these theories and elucidate the differences therein. Elaborate the meaning of the "law of correspondence states" for these plots. (2005)

9. State Dulong and Petit's law How does it agree with experiment ? Discuss the limitations of the classical theory and success of the quantum theory in explaining the specific heat of solids. (2006)

10. What are the limitations of Einstein's theory of specific heat of solids when compared with experiments at low temperature? Outline the assumptions made in Debye's theory and show that the specific heat at low temperature follows $C_v \propto T^3$ law. What is the significance of Debye's temperature, T_D ? (2009)

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

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TUTORIAL SHEET : 25

Maxwell- Boltzmann's Statistics

1. A gas possess a Maxwellian velocity distribution function show that the fraction of molecules in a given volume that possesses a velocity (+Vx) in one direction only and whose magnitude is greater than some selected value V_0 is

$$\int_{V_0}^{\infty} f(v_x) dV_x = \frac{1}{2}\left[1 - \text{erf}\left(\frac{1}{2}\frac{mv_0^2}{KT}\right)^{1/2}\right]$$

symbols have their usual meanings.

(1988)

2. Find the temperature at which r.m.s. velocity of nitrogen molecules in earth's atmosphere equals the velocity of escape from the earth's gravitational field. ~~Mass of~~ N_2 atom = 23. 24 $\times 10^{-24}$gm. Mean radius of earth = 6370 km.

(1988)

3. Calculate the mean free path of helium atoms at NTP, the co-efficient of viscosity being 190×10^{-7} kg $m^{-1} s^{-1}$. Atmospheric pressure = $0.076 \times 13.6 \times 10^3 \times 9.81$ N/m^2.

76 mm of Hg

(1988)

4. Explain, using the kinetic model, why peaks in the curves showing Maxwell distribution of molecular speeds move towards higher speed at higher temperature.

(1989)

5. Calculate the mean free path of molecules of H_2 gas at 20^0C at atmospheric pressure. Assume the molecular diameter to be 2.00×10^{-10}m.

(1990)

6. Write down the Maxwell - Boltzmann law for the distribution of speeds C of molecules in a gas. Show the distribution graphically for the temperature T and 2T; also write down the expression for average value of c^3.

(1990)

7. A system is composed of two-level atoms, the excited state 1 being 0.10 eV above the ground state, O. Find the fraction of all atoms which will be in state; 1 if the system is in thermal equilibrium at temperature 300K.

(1990)

8. Write down Maxwell - Boltzmann distribution for the energy of molecules of a gas at temperature T. Find the energy at which this distribution peaks. Compute in eV the mean energy of molecules of a gas at 27°C.

(1991)

9. Calculate the temperature at which the average speed of H_2 molecules equals that of O_2 molecules at 350K.

(1992)

10. A shower of 6×10^3 molecules, each travelling initially with same velocity, traverses a gas. Estimate the number of molecules which will travel unaffected even after traversing a distance equal to twice the mean free path.

(1992)

$N_{nc} = N_0 e^{-\left(\frac{x}{\lambda}\right)}$

non collision

$$\boxed{\frac{dN}{N} = -\frac{dx}{\lambda}}$$

11. Derive an expression for the pressure exerted by an ideal gas on the walls of the chamber in terms of concentration of molecules 'n', gas temperature T and a universal constant. Specifically discuss how T comes into the picture? (1993)

12. Derive expressions for the Maxwellian distribution of (i) one component of velocities (ii) one component of momenta, in the molecules of an ideal gas.
(1994)

13. Calculate the most probable speed Vp, the average speed $\langle V \rangle$ and the root - mean square speed $\langle V^2 \rangle^{1/2}$ for hydrogen molecules at 273 K.
(1995)

14. What is Brownian motion? Deduce Einstein's formula for translatory Brownian motion of particles suspended in a liquid and hence determine the Avogadro's number.
(1996)

15. State the principle of equipartition of energy. What are its limitations? Show that it is applicable only when the energy is a quadratic function of the associated variable.
(1996)

16. how the distribution of velocities for temperatures T_1, T_2 and T_3 ($T_1 > T_2 > T_3$) of gas molecules according to the Maxwell - Boltzman law. Using this law prove that the most probable velocity is $\sqrt{\frac{2}{3}}$ times the root mean square velocity.
(1997)

17. Obtain an expression for the mean free path. Mention the correction introduced by Maxwell. Calculate the value of Avogadro's number, if the mean free path of nitrogen molecules at STP is 6.85×10^{-8}m. The molecular diameter is 3.5 A^o.
(1997)

18. At the NTP the mass of one litre of H_2 is 0.09 gm. Calculate the (i) RMS (ii) Mean (iii) Most probable speed at 27^{o}C.
(1999)

19. The RMS speed of Oxygen molecules at 0^{0}C is 460 m/sec. What would be the RMS speed of Argon molecules (Mol. wt. = 40 gm/mole) at 40^{0}C and at what temperature this speed would be double that at NTP?
(1999)

20. What is the most probable distribution of speeds in a large number of molecules of a gas and indicate the steps for its derivation.
(2000)

21. Derive an expression for the Maxwellian distribution of velocities for the molecules of an ideal gas. (2001)

22. 100 particles at a temperature T are distributed among three energy levels, $E_0=0$, $E_1 = KT$ and $E_2 = 2KT$. What is total energy of the system?
(2003)

23. State the law of equipartition of energy. Show how this law can be used to calculate specific heat of gases and hence find the ratio $\gamma = C_p / C_v$ for diatomic and triatomic gases.
(2008)

24. Consider N independent particles, each of which can be in either of two energy states $+\epsilon$ or $-\epsilon$. Derive expression for entropy of micro canonical ensemble for this system, where $f = \frac{\sum E_i}{N\epsilon}$ is fixed ~~is fixed and~~ E_i is energy of ith particle

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TUTORIAL SHEET: 26
F-D & B-E Statistics

1. Derive the Bose- Einstein's distribution for an ideal gas. (2002)
2. Disuses the phenomenon of Bose –Einstein's condensation. Obtain the expression for the condensation temperature. Briefly comment on observation of Bose –Einstein's condensate. (2002)
3. Define Fermi energy. For an ideal Fermi gas of N particles at absolute zero temperature, show that the total energy is 3/5 NE_f where E_f is the Fermi energy. (2003)
4. Show that for a distribution of electrons, the number of electron Ni in the energy state Ei are given by

$$N_i = g_i /A \exp (E_i /kt) + 1$$

Where g_i represents the no. of quantum stales in the energy level E_i. Further state under what condition this distribution law goes over to Maxwell-boltzmann statistics.

Show by drawing curves how Fermi –Dirac distribution function varies with the energy at T =0 and also at the other finite temperatures. (2004)

5. Starting from the expression $N = \sum_k < n_k >$, where $< n_k >$ is the average number of particles in the k^{th} quantum state, derive an expression for the average number of particles in the ground state of an ideal Bose gas. (2005)
6. Utilize the above expression to elaborate the concept of the Bose –Einstein condensation and discuss that the phenomenon explains qualitatively the properties in the low –temperature phase of liquid ^{4}He. (2005)
7. Derive the expression for the Fermi – Dirac distribution function. Represent it graphically for T = 0 and T ≠ 0. (2006)
8. Derive a relation between the total number of Fermions in terms of Fermi momentum and hence obtain te expression for the total energy E of the system at absolute zero Combine this expression with the equation of state $PV = \frac{2}{3}E$ to show that the pressure of an ideal Fermi gas at T = 0 is proportional to $\frac{5}{3}$ power of its number density. (2007)
9. Show that at $T = 0$, the Fermi distribution function has a value 1 for energies less than the Fermi energy ϵ_F and is zero above it For a system of non-interacting electrons at T = 0, show that the ground state energy of the system of N particles is $\frac{3}{5}NE_F$. (2008)
10. Energy distribution for n_i particles in classical statistical mechanics is given by

$$n_i = g_i e^{-\alpha-\beta\varepsilon_i}$$

Where α and β are constants. g_i is the single particle states in the i th level. Using equipartition theorem, show that correct thermodynamic interpretation is

$$\beta = \frac{1}{kT} \qquad (Use. \int_0^\infty e^{-x} x^{1/2} dx = \frac{\sqrt{\pi}}{2} \text{ and}$$

$$\int_0^\infty e^{-\beta x} x^{3/2} dx = \frac{3\sqrt{\pi}}{4\beta^{5/2}})$$

(2009)

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11. Calculate the number of different arrangements of 10 indistinguishable particles in 15 cells of equal a priori probability, considering that one cell contains only one particle.

(2010)

12. Consider the following statement:
"The Fermi energy of a given material is the energy of that quantum state which has the probability equal to $\frac{1}{2}$ of being occupied by the conduction electrons."
Is the above statement correct? Give reasons for your answer.

(2010)

Q. Show that Fermi Dirac distⁿ leads to Pauli Exclusion Principle.

या तो g_i को Constant मान के $\langle E \rangle = \frac{1}{2} kT$ Prove करो

या फिर g_i को $A e^{\frac{1}{2}}$ मान के $\langle E \rangle = \frac{3}{2} kT$ Prove करो